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EVALUATION OF A DISTRIBUTED PARAMETER MODEL FOR COUNTERFLOW HEAT EXCHANGERS

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KEYWORDS

Heat exchanger model, minimisation, evaluation.

ABSTRACT

The paper deals with simulation of the heat exchanger and model evaluation. There are presented partial differential equations of heat exchanger model, which assume liquid mediums. Presented, in the paper, model is used as an approximation for three different type heat exchangers. The purpose of these investigations was to explain weather the same model could match well to different of shape and type exchangers by adjusting some model parameters.
There is shown a method of determining model parameters. Results of simulation are compared with measured data.

INTRODUCTION

Counterflow heat exchanger is often used in industry as a device which has to heat up or cool some medium. It occurs as an self-contained apparatus as well as a concurrent apparatus, which has to improve energetic efficiency of installation. Analyse of dynamic properties of heat exchanger, using mathematical model, makes easier choosing control systems in technology projecting phase.
At the Institute of Automatic Control of Technical University of Silesia there was developed and worked out a pilot heat distribution plant. The installation represents real heating system with complicated connections of heat receivers. The installation consist of three different type heat exchangers, mixer and water heater. There are: plate-type, "double-pipe" and spiral-tube type heat exchangers. The installation is fully equipped with measurement points:

- 9 temperature measure points,
- 1 level measure point,
- 4 flow rate counting points,
- 10 water pressure-meter points.

Such instrumentation allows evaluating mathematical models of heat exchange processes.

MODEL DESCRIPTION

The model of heat exchanger discussed in the paper is used as an approximation of each exchanger in the installation.
Mathematical model of a heat exchanger is based on the following assumptions:

- liquid mediums inside heat exchanger,
- fluid density and specific heat are constant,
- the thermal conductivity of the metal walls is high enough so that temperature gradients in the wall are negligible,
- there is no heat conduction in the direction of flow in the metal between the fluids nor in the fluids themselves.

Considered model bases on following equations:

$$\frac{\partial \theta_1}{\partial \tau} = -w_1 \frac{\partial \theta_1}{\partial y} - a_1(\theta_1 - \theta_2)$$
$$\frac{\partial \theta_2}{\partial \tau} = \frac{w_2}{r}\frac{\partial \theta_2}{\partial y} - \frac{a_2}{r}(\theta_2 - \theta_1) - \frac{b}{r}(\theta_2 - \theta_{out})$$

where:
a_1, a_2, b - model parameters,
w_1, w_2 - normalised flow velocities ($w_1 = W_1/W_{10}$)
r - ratio W_{10}/W_{20},
θ_1, θ_2 - temperature in first and second circuit [°C],
For solving the system of two partial differential equations the method of lines (CTDS method) is used. The heat exchanger is divided into k sections.

$$\frac{d\theta_{1,j}}{d\tau} = w_1\left(\theta_{1,j-1} - \theta_{1,j}\right)k - a_1\left(\theta_{1,j} - \theta_{2,j}\right); \qquad j = 1,\dots,k$$
$$\frac{d\theta_{2,j}}{d\tau} = \frac{w_2}{r}\left(\theta_{2,j+1} - \theta_{2,j}\right)k - \frac{a_2}{r}\left(\theta_{2,j} - \theta_{1,j}\right) - \frac{b}{r}\left(\theta_{2,j} - \theta_{out}\right)$$
$$j = 0,\dots,k-1$$

where:
$\theta_{1,0} = \theta_{1in}$ and $\theta_{2,k} = \theta_{2in}$

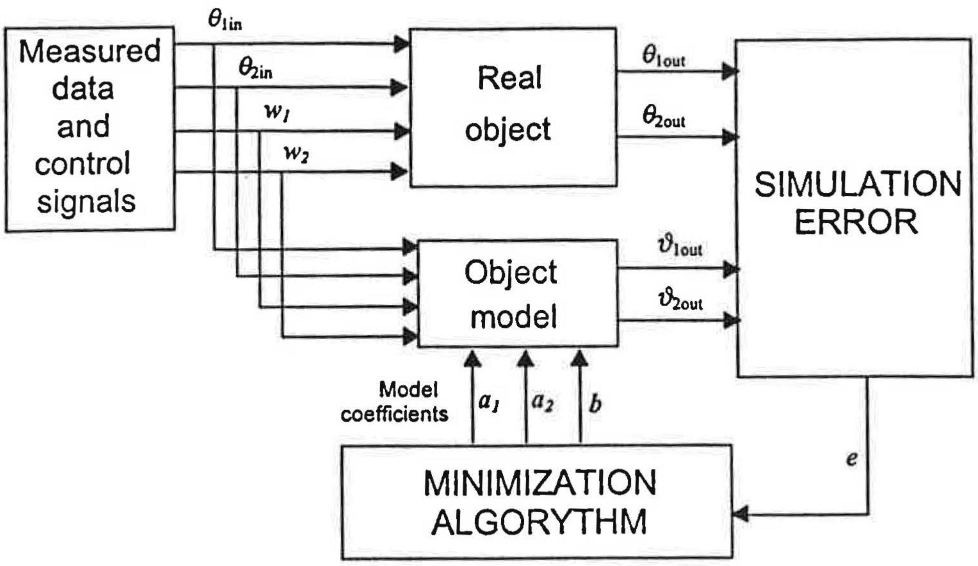

Fig. 1. Scheme of minimisation method.

That operation transforms each partial differential equation into k-order system of the ordinary differential equations. Obtaining initial profiles for that system is rather difficult. To solve the problem the initial profiles are assumed as linear and a number of first samples are neglected in calculations.

Although the structure of a model is well known, there is some uncertainty of model parameters. The uncertainty is about some medium parameters (fluid density, specific heat) and ratios (heat exchange ratio) which varies during the lifetime of installation. That values are contained in model parameters a_1,a_2 and b.

MINIMISATION METHOD

There were designed method of matching the best values of coefficients so to minimise the error of difference between simulation data and data acquired from real object. Scheme of the method is presented at figure 1. There is no possibility of separating one heat exchanger from whole installation furthermore there is no possibility of giving any input signals. Because of that evaluation should base on the signals acquired from installation during changes of flow velocities.

In that algorithm input data acquired from installation is given to the model. The result of simulation is compared with the output data also acquired from installation. The error e is calculated using given formula:

$$e = \frac{1}{n-m}\sum_{i=m}^{n}\left[\left(\theta_{1out,i} - \vartheta_{1out,i}\right)^2 + \left(\theta_{2out,i} - \vartheta_{2out,i}\right)^2\right]$$

where:

$\theta_{1out,i}$ - output temperature of the first circuit of heat exchanger at time i,

$\theta_{2out,i}$ - output temperature of the second circuit of heat exchanger at time i

$\vartheta_{1out,i}$ - output temperature of the first circuit of simulated model at time i,

$\vartheta_{2out,i}$ - output temperature of the second circuit of simulated model at time i,

n - number of samples in a data set,

m - number of first samples neglected for calculations.

The minimisation algorithm is changing model coefficients so to minimise the difference between outputs of real and simulated object. Because the number of adjusted parameters is greater then one, that operations are iterated unless given final condition is satisfied.

RESULTS

As an example there are presented the results of simulation compared with data acquired from installation for one heat exchanger (figure 2).

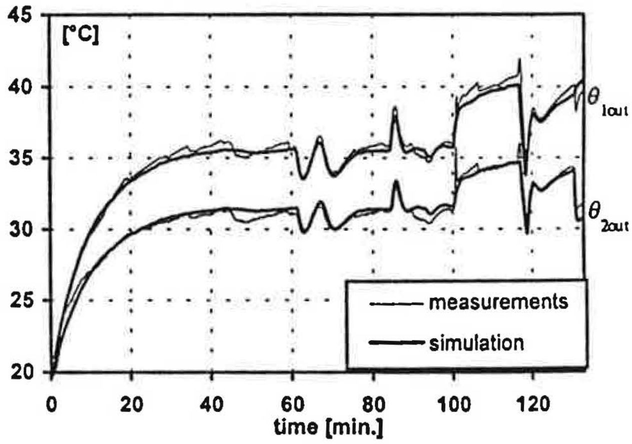

Fig. 2. Results for plate type heat exchanger (k=15). Output temperatures.

At table 1 are presented results of experiments for JAD-3.18 (spiral-tube) type heat exchanger for different number of division sections k. Results for

k	a_1	a_2	b	e
5	1.376	1.570	0.0	1.717
10	1.364	1.652	0.0	0.940
15	1.362	1.684	0.0	0.705
20	1.358	1.696	0.0	0.596
25	1.356	1.705	0.0	0.536
30	1.354	1.711	0.0	0.499

Tab. 1. Results for JAD 3.12 heat exchanger.

k	a_1	a_2	b	e
5	1.098	2.169	0.283	0.487
10	1.152	2.117	0.297	0.406
15	1.172	2.106	0.305	0.381
20	1.183	2.102	0.309	0.370
25	1.189	2.095	0.309	0.364
30	1.193	2.096	0.313	0.360

Tab. 2. Results for L25-6C heat exchanger.

k	a_1	a_2	b	e
5	0.734	1.126	0.251	0.465
10	0.736	1.117	0.263	0.392
15	0.737	1.123	0.287	0.372
20	0.741	1.144	0.334	0.362
25	0.742	1.154	0.357	0.357
30	0.743	1.158	0.368	0.353

Tab. 3. Results for double pipe heat exchanger.

plate type heat exchanger (L25-6C made by TAU Energy Products AB) are presented at table 2. Results for double pipe heat exchanger are presented at table 3. The main obstacle is that the shape of the error function is unknown. Introductory experiments showed that the shape of function ***e*** is regular. That allowed to use some standard minimisation algorithm. The most efficient and simplest was method of narrowing the range with golden section of the range where the minimum was expected.

CONCLUDING REMARKS

Simulation studies showed that the shape of error function ***e*** is regular with one minimum point. As an example there are shown profiles of function ***e*** for deviations of parameters near minimum point. Figure 3 shows profiles of function ***e*** for results for plate type heat exchanger. Similar profiles for double pipe heat exchanger are presented at figure 4. In each experiment error ***e*** was more sensitive to changes of parameter a_1 than to other two parameters.

Another problem is to choose optimal number of sections to approximate spatial derivatives. Calculation shows that dividing heat exchanger into more than 20 sections does not give significant drop of error ***e***.

Obtained results both numerical (error ***e***) and visual comparison are in good agreement with expectations. Proposed in the paper method could be employed to any other models to determine parameters values. Although for heat exchanger model it gives quite good results. To obtain better results the constant value of parameters a_1,a_2,b should be replaced with a function dependent on medium flows. The class of heat exchange ratio function of flow velocities is well known in literature. However there is no certainty if the class of that function should be wider when model has to approximate another type of heat exchanger than double pipe.

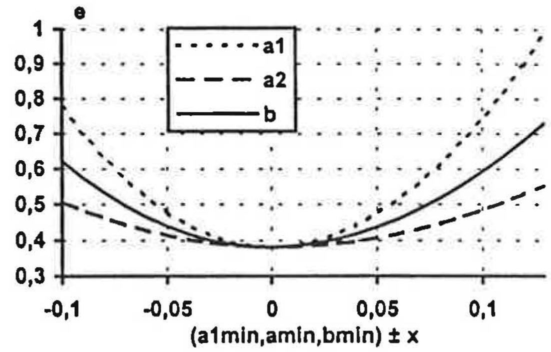

Fig. 3. Profiles of error ***e*** for experiments on plate type heat exchanger

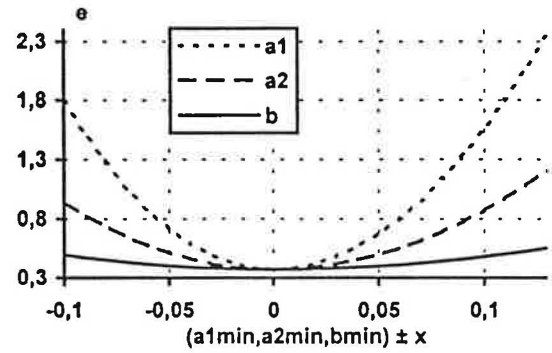

Fig. 4. Profiles of error ***e*** for experiments on double pipe heat exchanger

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