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Simplified analysis of combined
heat, moisture and air transport
for one-dimensional cases

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ANNEX 24 HAMTIE: Heat- Air- Moisture Transfer in New and Retrofitted Insulated Envelope Parts

**Simplified analysis of combined heat, moisture and
air transport for one-dimensional cases**

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1 Introduction

This report presents a simplified computational model for combined air-, moisture and heat transport for one-dimensional cases.

The model is based on finite difference technique with explicit forward differences in time. Analytical solutions for the coupling, the conductances, between the computational cells for a given air flow through the construction are used. Moisture is transferred by vapour diffusion and vapour convection. No liquid water transport occurs.

The computational model has been implemented into a PC-program 1D-HAM in order to solve the IEA, Annex 24, Second Common Exercise, (1-Dimensional Air Flow through a Wall Structure).

2 Steady-state heat flow in a homogeneous layer

Figure 1 shows a layer with the thickness D (m). The thermal conductivity of the layer is λ (W/mK). The temperature at the left side is T_1 and at the right side it is T_2 . The air flow through the layer is denoted by q_a ($\text{m}^3_{\text{air}}/\text{m}^2\text{s}$). It is positive when air flows from the left to the right side. The heat flow q (W/m^2), the sum of the conductive and convective flow, is positive when it flows from the right to the left side.

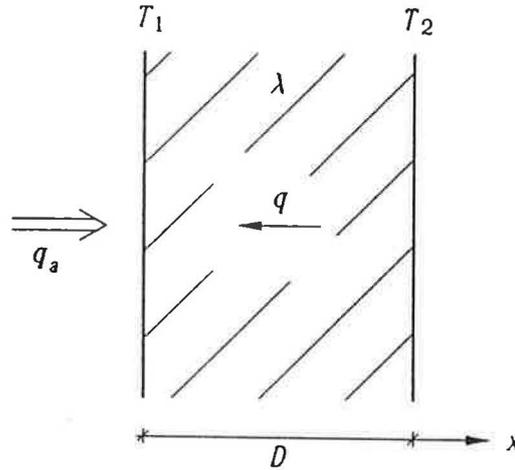


Figure 1: Heat and air flow through a homogeneous layer.

The heat flow through the layer is:

$$q = \lambda \frac{\partial T}{\partial x} - q_a C_a \cdot (T - T_{ref}) \quad (1)$$

Here C_a ($\text{J}/\text{m}^3\text{K}$) is the volumetric heat capacity of the air, and T_{ref} is an arbitrary reference temperature.

The steady-state heat flow through the layer must be constant, i.e. $\frac{dq}{dx} = 0$. The solution is well known:

$$T = T_1 + (T_2 - T_1) \frac{e^{x/\ell} - 1}{e^{D/\ell} - 1} \quad (2)$$

Here the length ℓ is given by:

$$\ell = \frac{\lambda}{q_a C_a} \quad (3)$$

By insertion of $x = 0$ and $x = D$ in (2), we find the correct boundary temperatures T_1 and T_2 . Insertion of (2) in (1) gives:

$$q = q_a C_a \cdot \left(\frac{T_2 - T_1}{e^{D/\ell} - 1} - (T_1 - T_{ref}) \right) \quad (4)$$

We have obtained a constant value of the heat flow which is required.

The following function will be used:

$$\chi_D(q_a) = \frac{1}{e^{D/\ell} - 1} \quad (5)$$

For large ℓ , i.e. small air flow rates, we have the following behaviour of χ_D :

$$\chi_D(q_a) \rightarrow \frac{\ell}{D} = \frac{1}{q_a C_a} \cdot \frac{\lambda}{D} \quad \ell \rightarrow +\infty \quad (6)$$

Using this asymptotic value we get:

$$q \rightarrow \frac{\lambda}{D} \cdot (T_2 - T_1) \quad q_a \rightarrow 0 \quad (7)$$

This is the heat flow for pure heat conduction.

The function χ_D has the following property:

$$\chi_D(q_a) + 1 = -\chi_D(-q_a) \quad (8)$$

Using the notation χ_D and relation (8) we have:

$$q = q_a C_a \cdot ((T_2 - T_1) \cdot \chi_D(q_a) - (T_1 - T_{ref})) \quad (9)$$

$$q = -q_a C_a \cdot ((T_2 - T_1) \cdot \chi_D(-q_a) + (T_2 - T_{ref})) \quad (10)$$

3 Steady-state heat flow in two homogeneous layers in contact

Figure 2 shows two layers in contact. The thickness of the layers is D_1 and D_2 . The thermal conductivity of the layers is λ_1 and λ_2 . The temperature at the left side is T_1 and at the right side it is T_2 . The contact surface temperature is denoted by T_{12} . The air flow through the layers is denoted by q_a ($\text{m}_{air}^3/\text{m}^2\text{s}$). The steady-state heat flow is denoted by q (W/m).

The following notations will be used:

$$\chi_1(q_a) = \frac{1}{e^{D_1/\ell_1} - 1} \quad \ell_1 = \frac{\lambda_1}{q_a C_a} \quad (11)$$

$$\chi_2(q_a) = \frac{1}{e^{D_2/\ell_2} - 1} \quad \ell_2 = \frac{\lambda_2}{q_a C_a} \quad (12)$$

3.1 Temperature in the contact surface

We can write down the expression for the heat flow through the two layers assuming that we know the contact surface temperature. With the help of (4) and (11-12) we

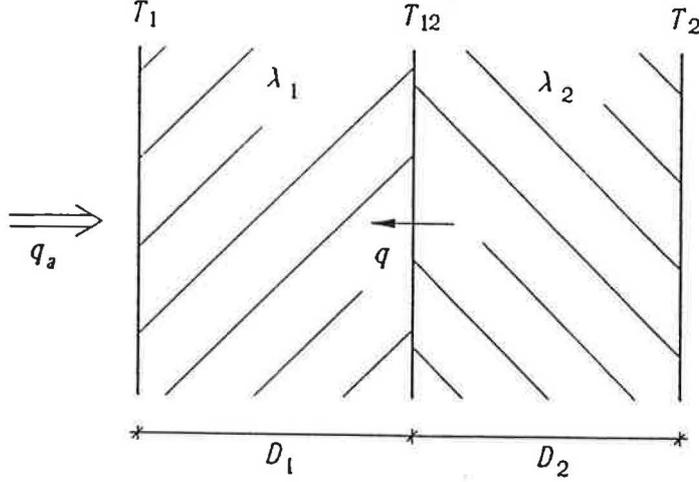


Figure 2: Heat and air flow through two homogeneous layers in contact.

get:

$$\begin{cases} q = q_a C_a \cdot ((T_{12} - T_1) \cdot \chi_1(q_a) - (T_1 - T_{ref})) \\ q = q_a C_a \cdot ((T_2 - T_{12}) \cdot \chi_2(q_a) - (T_{12} - T_{ref})) \end{cases} \quad (13)$$

From the equation system we can determine the temperature T_{12} .

$$T_{12} = \frac{T_1 \cdot \chi_1(q_a) + T_2 \cdot \chi_2(q_a) + T_1}{\chi_1(q_a) + \chi_2(q_a) + 1} \quad (14)$$

Using relations (8) it can also be written as:

$$T_{12} = \frac{T_1 \cdot \chi_1(-q_a) + T_2 \cdot \chi_2(-q_a) + T_2}{\chi_1(-q_a) + \chi_2(-q_a) + 1} \quad (15)$$

3.2 Heat flow through the layers

Insertion of the temperature T_{12} from (14) in (13) gives:

$$q = q_a C_a \cdot \left((T_2 - T_1) \frac{\chi_1(q_a) \cdot \chi_2(q_a)}{\chi_1(q_a) + \chi_2(q_a) + 1} - (T_1 - T_{ref}) \right) \quad (16)$$

Using (8) we also get:

$$q = -q_a C_a \cdot \left((T_2 - T_1) \frac{\chi_1(-q_a) \cdot \chi_2(-q_a)}{\chi_1(-q_a) + \chi_2(-q_a) + 1} + (T_2 - T_{ref}) \right) \quad (17)$$

4 Net energy flow into an inner computational cell

Figure 3 shows three computational cells with the index $i-1$, i and $i+1$. The width of the cells is Δx_{i-1} , Δx_i and Δx_{i+1} . The thermal conductivity of the materials in the cells is λ_{i-1} , λ_i and λ_{i+1} . The temperature at the cell nodes is T_{i-1} , T_i and T_{i+1} .

The heat flow through the left side of cell i is denoted by q_i^l (W/m^2), and through the right side it is q_i^r (W/m^2).

The following notation is introduced:

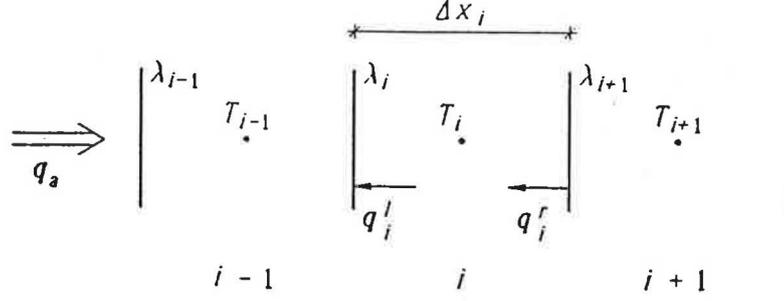


Figure 3: Heat flow to and from computation cell i .

$$\chi_i = \chi_i(q_a) = \frac{1}{e^{\Delta x_i/(2\ell_i)} - 1} \quad \ell_i = \frac{\lambda_i}{q_a C_a} \quad (18)$$

Using this notation in formula (16) we get the following expressions for the heat flow through and to cell number i .

$$q_i^l = q_a C_a \cdot \left((T_i - T_{i-1}) \frac{\chi_{i-1} \cdot \chi_i}{\chi_{i-1} + \chi_i + 1} - (T_{i-1} - T_{ref}) \right) \quad (19)$$

$$q_i^r = q_a C_a \cdot \left((T_{i+1} - T_i) \frac{\chi_i \cdot \chi_{i+1}}{\chi_i + \chi_{i+1} + 1} - (T_i - T_{ref}) \right) \quad (20)$$

The net inflow of heat into computation cell i becomes:

$$\Delta q_i = q_i^r - q_i^l = (T_{i+1} - T_i) \cdot K_{i+1} + (T_{i-1} - T_i) \cdot K_i + q_a C_a \cdot (T_{i-1} - T_i) \quad (21)$$

Here we have introduced the conductance K :

$$K_i = q_a C_a \frac{\chi_{i-1} \cdot \chi_i}{\chi_{i-1} + \chi_i + 1} \quad (22)$$

Using (6), (18) and (22) we find the following asymptotic values for small air flow rates:

$$K_i \rightarrow \frac{1}{\Delta x_{i-1}/(2\lambda_{i-1}) + \Delta x_i/(2\lambda_i)} \quad q_a \rightarrow 0 \quad (23)$$

This conductance corresponds exactly to the one we get for pure heat conduction.

5 Energy flow at the boundary cells

We will assume that there is a boundary layer of air of the thickness δ_{al} at the left hand side boundary and a layer of thickness δ_{ar} at the right hand side. The thermal conductivity of the air is denoted by λ_a . The layer of air will be treated as a material layer in the same way as the interior of the studied structure. Outside the air layer the ambient temperature is denoted by T_l and T_r on the left and right hand side respectively.

In the energy balance of the boundary cell a net radiant heat flow of the magnitude I_l (W/m^2) will be accounted for at the left hand side and a heat flow I_r at the right hand side.

5.1 Left boundary

Figure 4 shows the left hand side boundary cell.

The temperature at the left surface of cell number one the temperature is denoted by T_l . The heat flow from the cell to the surface is q_1^l and from the surface to the air layer is q_1^r .

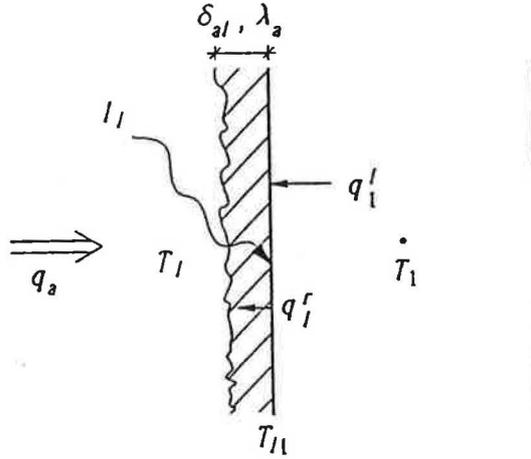


Figure 4: Heat flows and temperatures at the left hand side boundary.

An energy balance for the left hand side surface of cell number one gives:

$$q_1^r = I_l + q_1^l \quad (24)$$

Using (9) and (18) we can write down expressions for the heat flow to and from the surface:

$$q_1^l = q_a C_a \cdot ((T_1 - T_{l1}) \cdot \chi_1 - (T_{l1} - T_{ref})) \quad (25)$$

$$q_1^r = q_a C_a \cdot ((T_{l1} - T_l) \cdot \chi_{al} - (T_l - T_{ref})) \quad (26)$$

Here χ_{al} is given by:

$$\chi_{al} = \frac{1}{e^{\delta_{al}/\ell_a} - 1} \quad \ell_a = \frac{\lambda_a}{q_a C_a} \quad (27)$$

Using (24-26) we get:

$$T_{l1} = \frac{I_l + q_a C_a \cdot (T_l \cdot \chi_{al} + T_1 \cdot \chi_1 + T_l)}{q_a C_a \cdot (\chi_{al} + \chi_1 + 1)} \quad (28)$$

The net heat flow into cell number one is equal to:

$$\Delta q_1 = q_1^r - q_1^l \quad (29)$$

The formula for q_1^l is given by (25) and by (20) for q_1^r . Using (28) in (25) we get the following expression for the net inflow (29):

$$\Delta q_1 = (T_2 - T_1) \cdot K_2 + (T_l - T_1) \cdot K_0 + q_a C_a \cdot (T_l - T_1) + I_l \cdot K_{II} \quad (30)$$

Here K_0 and K_{II} is:

$$K_0 = q_a C_a \frac{\chi_{al} \cdot \chi_1}{\chi_{al} + \chi_1 + 1} \quad K_{II} = \frac{1 + \chi_1}{\chi_{al} + \chi_1 + 1} \quad (31)$$

5.2 Right boundary

Figure 5 shows the right hand side boundary cell.

The temperature at the right surface of cell number N is denoted by T_{Nr} . The heat flow from the ambient air to the cell surface is q_r^l and the heat flow from the surface to the cell is q_N^r .

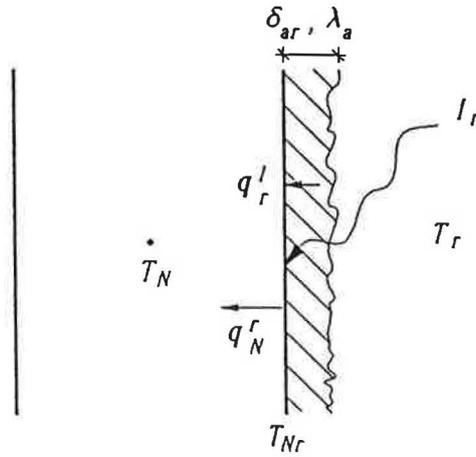


Figure 5: Heat flows and temperatures at the right hand side boundary.

An energy balance for the right hand side surface of cell number N gives:

$$q_N^r = I_r + q_r^l \quad (32)$$

Using (9) and (18) we can write down expressions for the heat flow to and from the surface:

$$q_N^r = q_a C_a \cdot ((T_{Nr} - T_N) \cdot \chi_N - (T_N - T_{ref})) \quad (33)$$

$$q_r^l = q_a C_a \cdot ((T_r - T_{Nr}) \cdot \chi_{ar} - (T_{Nr} - T_{ref})) \quad (34)$$

Here χ_{ar} is given by:

$$\chi_{ar} = \frac{1}{e^{\delta_{ar}/\ell_a} - 1} \quad \ell_a = \frac{\lambda_a}{q_a C_a} \quad (35)$$

Using (32-34) we get:

$$T_{Nr} = \frac{I_r + q_a C_a \cdot (T_N \cdot \chi_N + T_r \cdot \chi_{ar} + T_N)}{q_a C_a \cdot (\chi_N + \chi_{ar} + 1)} \quad (36)$$

The net heat flow into cell number N is equal to:

$$\Delta q_N = q_N^r - q_r^l \quad (37)$$

The formula for q_N^r is given by (33) and by (19) for q_N^l . Using (36) in (33) we get the following expression for the net inflow (37):

$$\Delta q_N = (T_{N-1} - T_N) \cdot K_N + (T_r - T_N) \cdot K_{N+1} + q_a C_a \cdot (T_{N-1} - T_N) + I_r \cdot K_{I_r} \quad (38)$$

Here K_{N+1} and K_{I_r} is:

$$K_{N+1} = q_a C_a \frac{\chi_N \cdot \chi_{ar}}{\chi_N + \chi_{ar} + 1} \quad K_{I_r} = \frac{\chi_N}{\chi_N + \chi_{ar} + 1} \quad (39)$$

6 Vapour flow

In the study of the moisture flow we will use the humidity by volume, v (kg/m³) as potential. The transport coefficients is denoted by δ_v (m²/s). The moisture flow rate is denoted by g (kg/m²s). It becomes:

$$g = \delta_v \frac{\partial v}{\partial x} - q_a (v - v_{ref}) \quad (40)$$

Here v_{ref} is an arbitrary reference level of the humidity by volume.

We can directly see the similarities to the heat flow equation (1). By doing the following exchanges in the formulas for the heat flow we get the corresponding moisture flow cases:

$$T \rightarrow v \quad \lambda \rightarrow \delta_v \quad C_a \rightarrow 1 \quad (41)$$

6.1 Net vapour flow into an inner computational cell

Using (21) and (41) the net moisture flow into an inner computation cell becomes:

$$\Delta g_i = g_i^r - g_i^l = (v_{i+1} - v_i) \cdot K_{i+1}^M + (v_{i-1} - v_i) \cdot K_i^M + q_a \cdot (v_{i-1} - v_i) \quad (42)$$

Here we have introduced the conductance K_i^M , where the index M marks that it concerns the moisture flow:

$$K_i^M = q_a \frac{\chi_{i-1}^M \cdot \chi_i^M}{\chi_{i-1}^M + \chi_i^M + 1} \quad (43)$$

$$\chi_i^M = \chi_i^M(q_a) = \frac{1}{e^{\Delta x_i / (2\ell_i^M)} - 1} \quad \ell_i^M = \frac{\delta_{v,i}}{q_a} \quad (44)$$

6.2 Left boundary cell

Using (30) and (41) the net inflow of moisture for the left boundary cell becomes:

$$\Delta g_1 = (v_2 - v_1) \cdot K_2^M + (v_l - v_1) \cdot K_0^M + q_a \cdot (v_l - v_1) + G_l \cdot K_{G_l}^M \quad (45)$$

Here G_l (kg/m²s) represents the inflow of moisture to the surface. This could for instance represent the effect of rain. The moisture conductances K_0^M and $K_{G_l}^M$ are given by:

$$K_0^M = q_a \frac{\chi_{al}^M \cdot \chi_1^M}{\chi_{al}^M + \chi_1^M + 1} \quad K_{G_l}^M = \frac{1 + \chi_1^M}{\chi_{al}^M + \chi_1^M + 1} \quad (46)$$

$$\chi_{al}^M = \frac{1}{e^{\delta_{al}/\ell_a^M} - 1} \quad \ell_a^M = \frac{\delta_{v,a}}{q_a} \quad (47)$$

6.3 Right boundary cell

Using (38) and (41) the net inflow of moisture for the right boundary cell becomes:

$$\Delta g_N = (v_{N-1} - v_N) \cdot K_N^M + (v_r - v_N) \cdot K_{N+1}^M + q_a \cdot (v_{N-1} - v_N) + G_r \cdot K_{Gr}^M \quad (48)$$

Here G_r (kg/m²s) represent the inflow of moisture to the surface. The conductances K_{N+1}^M and K_{Gr}^M are given by:

$$K_{N+1}^M = q_a \frac{\chi_N^M \cdot \chi_{ar}^M}{\chi_N^M + \chi_{ar}^M + 1} \quad K_{Gr}^M = \frac{\chi_N^M}{\chi_N^M + \chi_{ar}^M + 1} \quad (49)$$

$$\chi_{ar}^M = \frac{1}{e^{\delta_{ar}/\ell_a^M} - 1} \quad \ell_a^M = \frac{\delta_{v,a}}{q_a} \quad (50)$$

7 Numerical model for combined heat and vapour transport

In this section we will combine the moisture flow and the heat flow. The potential for heat flow is the temperature T , and for moisture flow it is the humidity by volume v .

7.1 Numerical grid

Figure 6 shows the numerical grid. In total a number of N cells is used.

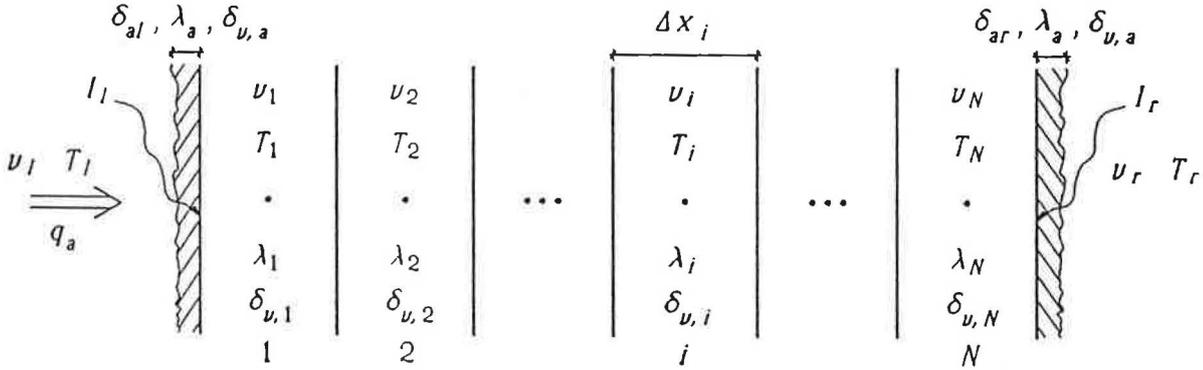


Figure 6: Numerical grid and some parameter associated to the cells and the boundaries.

7.2 Heat flow model

The flow of heat to and from the cells are governed by the temperatures in the cells and the boundary conditions. The net heat flow to the inner cells ($1 < i < N$) are

given by formula (21). The net in flow of heat to the left and right boundary cells ($i = 1, N$) is given by (30) and (38) respectively.

The increase of temperature of a cell due to the net inflow of heat is given by:

$$\Delta T_i = \Delta t \cdot \frac{\Delta q_i}{C_i \cdot \Delta x_i} \quad (51)$$

Here Δt is the time step considered, and C_i (J/m³K) is the volumetric capacity of the material in cell number i .

The stable time step for the thermal process is:

$$\Delta t = \min \left\{ \frac{C_i \cdot \Delta x_i}{K_{i-1} + K_{i+1} + q_a C_a} \quad i = 1, \dots, N \right\} \quad (52)$$

7.3 Moisture transport model

The transports of moisture to and from the cells are governed by the humidity by volumes in the cells and the boundary conditions. The net in flow of moisture to the inner cells are given by formula (42). The net in flow of moisture at the left and right boundary is given by (45) and (48) respectively.

The content of moisture in a material is denoted by w (kg/m³) The increase of the moisture content of cell number i due to the net in flow of moisture is given by:

$$\Delta w_i = \Delta t \cdot \frac{\Delta g_i}{\Delta x_i} \quad (53)$$

Here Δt is the time step considered.

Figure 7 shows the used simplified sorption isotherm.

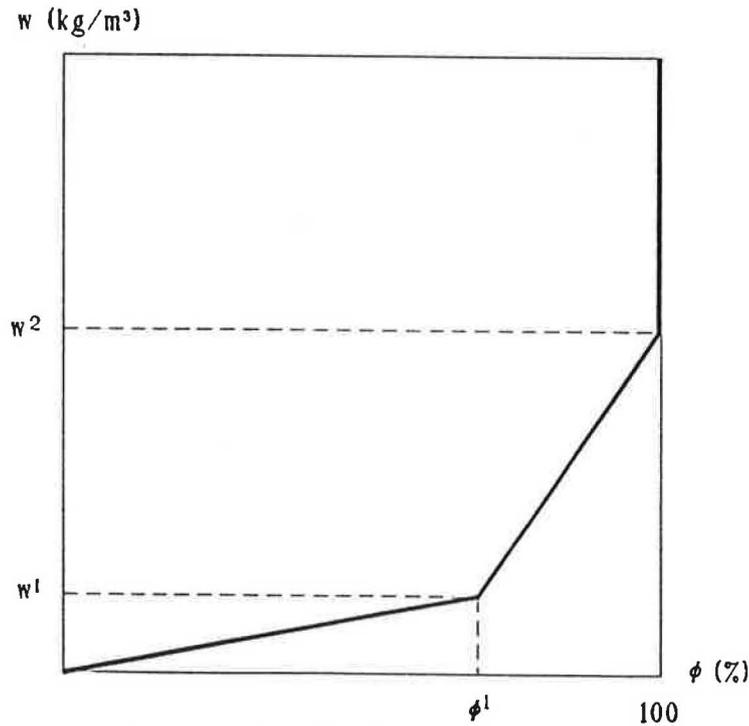


Figure 7: Simplified sorption isotherm.

The relative humidity is denoted by ϕ (%). The sorption isotherm is approximated by two straight lines. These should be chosen in order to fit the true sorption isotherm as good as possible. The level w^2 corresponds to the upper limit of the hygroscopic

region. In the computational model there will be no limit for the amount of stored water in a material. The model will not account for liquid water flow. We have the following relations:

$$w = \frac{\phi}{\phi^1} w^1 \quad 0 < \phi \leq \phi^1 \quad (54)$$

$$w = w^1 + \frac{\phi - \phi^1}{100 - \phi^1} \cdot (w^2 - w^1) \quad \phi^1 < \phi < 100 \quad (55)$$

The inverse relations are:

$$\phi = \frac{w}{w^1} \cdot \phi^1 \quad 0 < w \leq w^1 \quad (56)$$

$$\phi = \phi^1 + \frac{w - w^1}{w^2 - w^1} \cdot (100 - \phi^1) \quad w^1 < w \leq w^2 \quad (57)$$

$$\phi = 100 \quad w > w^2 \quad (58)$$

The stable time step for the moisture transfer process is:

$$\Delta t = \min \left\{ \frac{1}{v_{sat}(T_{max})} \frac{w^1}{\phi^1/100} \frac{\Delta x_i}{K_{i-1}^M + K_{i+1}^M + q_a} \quad i = 1, \dots, N \right\} \quad (59)$$

Here $v_{sat}(T_{max})$ denotes the humidity by volume at saturation at the temperature T_{max} . This temperature should be the highest temperature that occurs in the construction during the time of simulation. Formula (59) is valid during the condition $w^2 > 100 \cdot w^1/\phi^1$. This means that the slope of the sorption isotherm increases at higher relative humidities.

7.4 Transient model

The simulation of the combined air-, moisture- and thermal flow processes is described by the following algorithm. The time step Δt is the smallest of the time step given in (52) and (59).

1. Start ($t = t_{start}$)
2. Input of start values for temperatures and moisture (w, T)
3. Insert the actual boundary conditions (T, I, G, v, q_a)
4. Determine the conductances ($K, K^M, K_{Ir}, K_{Il}, K_{Gr}^M, K_{Gl}^M$)
5. Calculate the stable time step (Δt)
6. Calculate the relative humidity ($\phi(w)$)
7. Calculate the humidity by volume ($v = \phi \cdot v_{sat}$)
8. Calculate the change in moisture content (Δw)
9. Calculate the change in temperature (ΔT)
10. Update the time ($t_{new} = t_{old} + \Delta t$)
11. Update the moisture content ($w_{new} = w_{old} + \Delta w$)
12. Update the temperature ($T_{new} = T_{old} + \Delta T$)
13. Repeat step 3 to 12 until the stop time is reached
14. Stop