

# Mixing Time Constant for Jet Flow in Rooms

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## Abstract

This paper presents a study of the dynamics of the turbulent mixing of a hot or cold air stream with the air in the interior of a building zone. Observations and CFD results of transient temperature behaviour in a fully developed jet flow field are presented. A simple model for the characteristic time-constant of the mixing process in a room is derived. The mixing of a turbulent jet, as a function of position inside the room, is also discussed. This mixing time-constant plays an important role in total system dynamic behaviour and stability. For a turbulent jet, it is shown that the mixing time is short in the flow regions where strong shear forces are present and increases rapidly outside the jet. This implies that temperature sensors placed outside the shear flow regions respond slowly to supply air temperature or flow rate changes. This slow response may compromise the dynamic behaviour of systems where the temperature sensor's output is the primary feedback variable for indoor temperature control.

## Nomenclature

### Symbols

$A_d$	diffuser area [m <sup>2</sup> ]
$D_m$	mass diffusivity [m <sup>2</sup> /s]
$K$	turbulent kinetic energy [J]
$k$	thermal conductance [W/m.K]
$L$	turbulent eddy size [m]
$L_s$	macro length scale [m]
$M$	fluid mass [kg]
$P$	flow power [W]
$P_r$	Prandtl number

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### 1 INTRODUCTION

$Sc$	Schmidt number
$T_i$	zone interior air temperature [K]
$T_{id}$	interior air temperature distribution [K]
$t$	time coordinate [s]
$U_0$	diffuser air speed [m/s]
$U_{max}$	jet centreline mean velocity [m/s]
$v$	room volume [m <sup>3</sup> ]
$u$	mean velocity [m/s]
$x$	x space coordinate [m]
$y$	y space coordinate [m]

## Greek

$\alpha$	thermal diffusivity [ $m^2/s$ ]
$\varepsilon$	velocity energy dissipation [ $W/kg$ ]
$\eta$	efficiency of turbulent production
$\lambda_s$	micro scale of turbulence [ $m$ ]
$\omega$	radian frequency [ $rad/s$ ]
$\tau_m$	mixing time-constant [ $s$ ]

## 1 Introduction

"Indoor air mixing" is a term used to describe the physical process of the mixing of hot and cold air in a room. Usually this term refers to the steady inhomogeneities in room temperature distribution. The control system of an HVAC system always attempts to maintain the interior temperature in a prescribed band. The ability of the controller to succeed in this aim depends on the accuracy with which the interior temperature is measured. In practice, however, the interior air will not be at a uniform temperature and the actual variable sensed by the controller is not well defined. The position of the sensor in the room is important. E.g. if the sensor is placed directly in the jet of the supply air diffuser, it will read the supply air temperature rather than the room air temperature.

Another consideration is the time it takes for the interior air to mix, that is, the time it takes to arrive at the final steady distribution. This time delay is important for the proper design of the control system. During the mixing process the local temperature may vary considerably as blobs of hot and cold air move through space. Eventually the blobs are broken down in smaller blobs until complete mixing is achieved.

Almost all HVAC simulation models, and all the popular design methods, assume the room air is well-mixed. This implies that the interior air temperature distribution is homogeneous, and also, that the mixing time is very short and can be ignored. Standard design procedure is to separate the system dynamics into two levels, a fast local level and a slower second level. The local level time-constants

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determine the local dynamics of the coils, dampers etc. The second level time constants are important for the overall system control dynamics. According to the ASHRAE manual: "for supervisory (second level) control, the local loop (first level) may be considered to meet the set points exactly" [3]. However, many HVAC systems display on-off hunting behaviour which is usually assumed to be due to limit cycles from the many non-linearities present. However, the behaviour may also be due to design deficiencies because important dynamical considerations are neglected. The HVAC system simulation program which represents the state of the art as far as dynamical aspects are concerned is probably HVACSIM\* [9]. In HVACSIM\* (element type 15) the interior space is modelled with two regions: a fully-mixed space near the supply diffuser and a piston flow part near the exhaust [10]. This model includes the flow delay for the piston flow part of the space but it still ignores the time it takes for the mixing to take place. The main conclusion of this paper is that the mixing time is highly dependent on the spatial coordinates and often not negligible compared to other first-level time-constants.

A zone model which accurately reflects the room temperature dynamics must model the flow inside the room in detail. This requires a flow solver—a CFD code running on a powerful computer. In recent years the literature [4, 6, 13, 7, 12, 2] indicates that these methods are finding application (for research purposes mostly) in HVAC system analysis. However, CFD codes are difficult to use and expensive. They are currently only used in critical applications. For design purposes a good understanding of the physical mechanism is important. In this regard the recent work reported by Chow et al. [8] is important.

The main objective of this paper is to present a highly simplified method for estimating the time constant of the mixing process. The method is based on the well known solution of the turbulent flow field in the self-similar region of a turbulent jet. Before this method is discussed, the non-steady development of the temperature field in a room is investigated with the aid of a CFD code.

CFD Results for a simple 2-D turbulent jet The standard K- $\epsilon$  turbulence model can be used to investigate the temperature response of a flow field to a sudden increase in inlet air temperature. The K- $\epsilon$  model is well verified and is known to be fairly reliable, except for inaccuracies very close to boundaries [1, 16]. A highly simplified situation, consisting of a two-dimensional Jet in a large enclosure, is investigated with the the FloVent (TM) [11] code. To reduce the complexity of the flow field, a constant pressure boundary on the opposite wall opposite to the is used. The steady response obtained in this manner, is shown in figure 1. The zone has a length of 6 m and is 3 m high. In this calculation the temperature is uniform at 15 °C in the zone. The supply jet and outside air is also at 15 °C.

The jet inlet speed is chosen at 15 m/s which is sufficiently high to form a clearly discernible jet. Note in the figure how the jet attaches to the floor. Zone-air entrainment causes recirculating flow fields below and above the Inlet. The resultant

## 1 INTRODUCTION

Figure 1: Steady two-dimensional flow field with uniform temperature for a jet in a large zone of 6 m by 3 in. The jet is 1 in above the floor and 150 mm wide. The inlet speed is 15 m/s. The boundary opposite the jet is a constant pressure boundary, the other are smooth walls. The K-F- turbulent model is used. Velocity vectors and pressure contours are shown.

low-pressure above the jet causes inflow of outside air through the constant pressure boundary. In figure 2 the calculated turbulent kinetic energy is shown. The CFD result also indicates strong turbulence in the shear layers.

The transient temperature field, obtained after a sudden change of the inlet let temperature to 35 °C is shown in figure 3. After 50 s the temperature distribution is almost fully developed, although further calculation showed that true steady state is not attained even after 80 s. The temperature histories, at various levels, at a distance of 2 m in front of the inlet, are shown in figure 4. The curves show a fast initial rise of temperature at positions close to the ground, which are inside the jet. At higher levels, outside the jet, a more gradual increase is observed.

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**J** is **ty** sudden increase in temperature in the Jet indicates the time of arrival of the hot air. The gradual increase outside the jet indicates the turbulent diffusion of hot air from the jet and mixing with the cold room air.

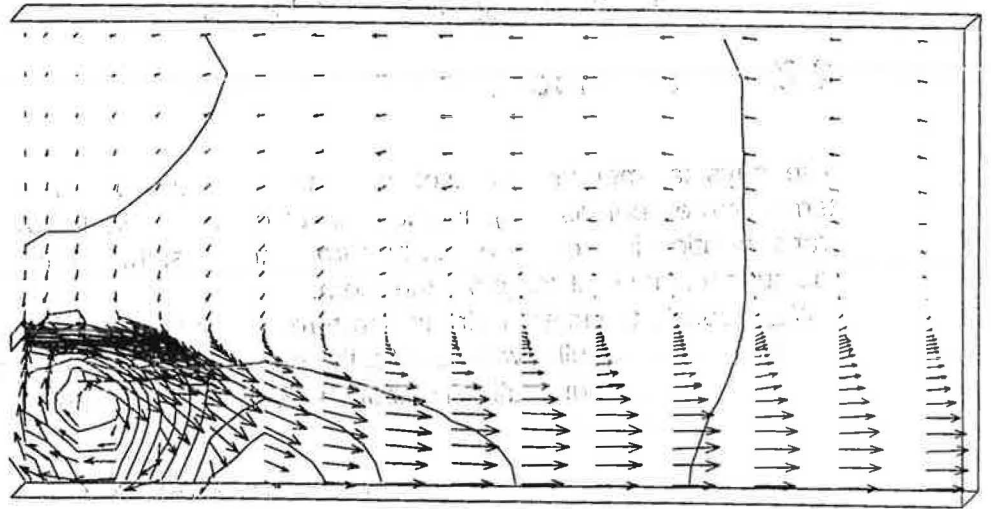


Figure 1: Steady two-dimensional flow field with uniform temperature for a jet in a large zone of 6 m by 3 m. The jet is 1 m above the floor and 150 mm wide. The inlet speed is 15 m/s. The boundary opposite the jet is a constant pressure boundary, the other are smooth walls. The  $K-\epsilon$  turbulent model is used. Velocity vectors and pressure contours are shown.

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Figure 2: Turbulent kinetic energy held obtained from the K-E model. The turbulent energy is mostly in the shear layers.

### 2 Zone air model

The physical mechanism involved in the mixing process is the transportation of heat from the temperature regions, e.g. in the vicinity of a heater, to lower temperature regions elsewhere. transportation is brought about by turbulent diffusion. During cooling, the "coldness" must diffuse from the supply diffuser throughout the room.

We separate the interior air temperature distribution  $T_{i,1}$ , into three parts, the bulk average  $T_i(t)$ , the steady spatial distribution  $t_{id}(x, V)$ , and the time dependent spatial distribution  $T_{id}(x, V, Z, t)$ , so that

$$T_{id}(X, Z, t) = T_i(t) + t_{id}(X, V, Z, t) \quad + T_{id} \quad 7, t$$

The steady spatial distribution,  $t_{id}$ , makes provision for stratification and other

'deal, long lasting, spatial temperature differences in the room. To design a non-l

proper air distribution system for a zone requires prediction of the steady distribu-

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tribution are usually restricted to steady flow analyses. However, the non-steady

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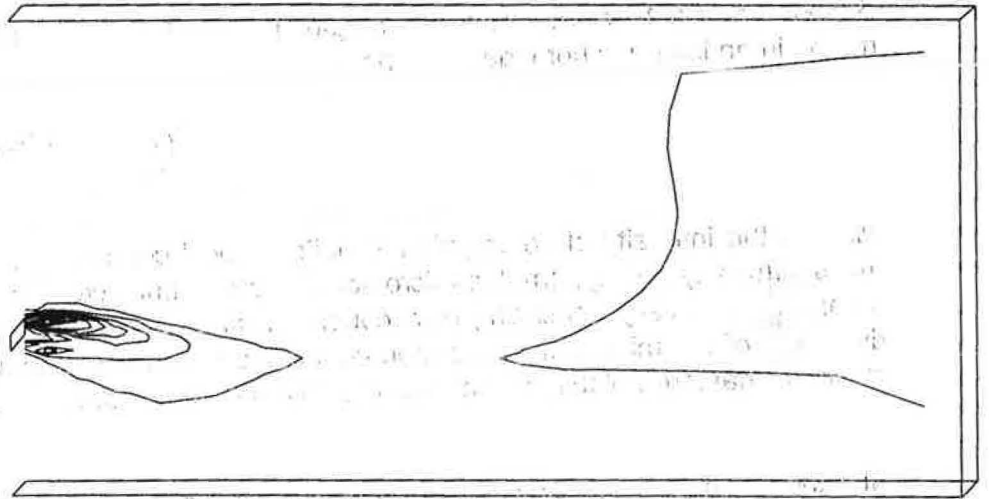


Figure 2: Turbulent kinetic energy field obtained from the  $K-\epsilon$  model. The turbulent energy is mostly in the shear layers.

## 2 Zone air model

The physical mechanism involved in the mixing process is the transportation of heat from the high temperature regions, e.g. in the vicinity of a heater, to lower temperature regions elsewhere. The transportation is brought about by turbulent diffusion. During cooling, the “coldness” must diffuse from the supply diffuser throughout the room.

We separate the interior air temperature distribution  $T_{id}$ , into three parts, the bulk average  $\bar{T}_i(t)$ , the steady spatial distribution  $\hat{T}_{id}(x, y, z)$ , and the time dependent spatial distribution  $\tilde{T}_{id}(x, y, z, t)$ , so that

$$T_{id}(x, y, z, t) = \bar{T}_i(t) + \hat{T}_{id}(x, y, z) + \tilde{T}_{id}(x, y, z, t). \quad (1)$$

The steady spatial distribution,  $\hat{T}_{id}$ , makes provision for stratification and other non-ideal, long lasting, spatial temperature differences in the room. To design a proper air distribution system for a zone requires prediction of the steady distribution to ascertain comfort in the occupied space. CFD programs for room air distribution are usually restricted to steady flow analyses. However, the non-steady distribution, given by  $\tilde{T}_{id}$ , is the focus of this study.

For a well mixed zone  $T_{id}$  and  $T_{i,d}$  vanishes and the fully mixed temperature is  $T_i$ . If, after reaching a mixed state, air at a temperature different from  $T_i$ , or heat, is released locally into the room at a state.  $T_{i,d}$  suddenly changes to a non-zero value in the region where the disturbance takes place. new steady temperature distribution in the room is given by  $T_{id}$ , and the time dependent part, gradually decreases as the new steady state is attained. The local mixing, timeconstant,  $\tau_m$ , (x. V, 7 defined as the time it takes for  $T_{id}(X@ V, \dots t)$  to decrease to 37% of the initial value it 'umped to after disturbance was introduced. This definition is independent of the mechanism which causes disturbance. In general the mixing time-constant includes the time for the flow field to be established the time for the heat to be transported. The flow field may be due to natural or forced flow. In this study assume a steady flow field with time dependent temperature changes of the incoming air cause heaters and cooling coils.

Brodkey [5, p701-derives from a Kdrn@n-Howarth type of equation. a differential equation for turbu mixing in an isotropic homogeneous held

$$1 \frac{d@}{dt} = \frac{12D_1t}{@,2 S} \quad (2)$$

with @, the intensity of segregation in units of local concentration per average concentration,  $D_1$ , [m mass diffusivity and @,, [m] the micro scale of the turbulence. On the assumption that the zone air ca rcaarded as initially consisting of a couple of large blobs of hot and cold air equation 2 describes dynamics of the mixina of the hot and cold blobs to a uniform temperature. Tile presumed mechanis mixing is that large blobs of 'et air break away from the 'et and mix

with the room air. Presumably, the dynamics of the average temperature and the size of the blobs closely coupled so that equation 2 also describes the dynarn~ ical response of the avera e tempera observed at a point. Note that the CF1) results in figure 4 do not display fluctuations. The nume model is based on calculating the mean flow and a model for the generation and dissipation of turbulent kinetic energy. The temperature traces in the figure are the calculated local average values.

Presumably  $T_{i,d}$  is given by a law similar to 2

$$\frac{dT_{id}}{dt}$$

$$\tau_m \frac{dT_{id}}{dt} + T_{id} = T_i + T_{id} \quad (3)$$

The mixing time for the room air introduces an additional time-constant in the room thermal response measured by a temperature sensor. The time-constant depends on the air flow pattern in the room and position of the diffuser, exhaust inlet, and sensor. The mixing time-constant is

$$\tau_m = \frac{R11C11}{12D_1} \quad (4)$$

For a well mixed zone  $\hat{T}_{id}$  and  $\tilde{T}_{id}$  vanishes and the fully mixed temperature is  $\bar{T}_i$ . If, after reaching a fully mixed state, air at a temperature different from  $\bar{T}_i$ , or heat, is released locally into the room at a steady state,  $\tilde{T}_{id}$  suddenly changes to a non-zero value in the region where the disturbance takes place. The new steady temperature distribution in the room is given by  $\hat{T}_{id}$ , and the time dependent part,  $\tilde{T}_{id}$ , gradually decreases as the new steady state is attained. The local mixing time-constant,  $\tau_m(x, y, z)$ , is defined as the time it takes for  $\tilde{T}_{id}(x, y, z, t)$  to decrease to 37% of the initial value it jumped to after the disturbance was introduced. This definition is independent of the mechanism which causes the disturbance. In general the mixing time-constant includes the time for the flow field to be established and the time for the heat to be transported. The flow field may be due to natural or forced flow. In this study we assume a steady flow field with time dependent temperature changes of the incoming air caused by heaters and cooling coils.

Brodkey [5, p70] derives from a Kármán-Howarth type of equation, a differential equation for turbulent mixing in an isotropic homogeneous field

$$\frac{1}{I_s} \frac{dI_s}{dt} = -12 \frac{D_m}{\lambda_s^2} \quad (2)$$

with  $I_s$  the intensity of segregation in units of local concentration per average concentration,  $D_m$  [m<sup>2</sup>/s] mass diffusivity and  $\lambda_s$  [m] the micro scale of the turbulence. On the assumption that the zone air can be regarded as initially consisting of a couple of large blobs of hot and cold air equation 2 describes the dynamics of the mixing of the hot and cold blobs to a uniform temperature. The presumed mechanism of mixing is that large blobs of jet air break away from the jet and mix with the room air. Presumably, the dynamics of the average temperature and the size of the blobs are closely coupled so that equation 2 also describes the dynamical response of the average temperature observed at a point. Note that the CFD results in figure 4 do not display fluctuations. The numerical model is based on calculating the mean flow and a model for the generation and dissipation of the turbulent kinetic energy. The temperature traces in the figure are the calculated local average values.

Presumably  $T_{id}$  is given by a law similar to 2

$$\tau_m \frac{dT_{id}}{dt} + T_{id} = \bar{T}_i + \hat{T}_{id} \quad (3)$$

The mixing time for the room air introduces an additional time-constant in the room thermal response, as measured by a temperature sensor. The time-constant depends on the air flow pattern in the room and the position of the diffuser, exhaust inlet, and sensor. The mixing time-constant is

$$\tau_m = R_h C_h = \frac{\lambda_s^2}{12 D_m} \quad (4)$$



where the value of either CI, or RI, can be chosen arbitrarily.

To calculated... requires knowledge of the turbulent flow-field structure. Brodkey [51 gives for a Sch number,  $Sc \leq 1$ ,

$$T_{tn} = \frac{(5 \dots)^2}{71 \dots 3 - Sc^2 \dots c} \quad (5)$$

In this equation  $F_{\dots}$  [W/kg] is the velocity energy dissipation per unit mass, and  $L$ , [m] is the macro scale of the scalar blobs. For mixing of hot and cold air  $Sc = Pr \dots 0.7$ , so

$$T_{\dots} \dots 1.09 \quad (6)$$

The structure of the turbulence is characterised by the local scale lengths and energy dissipation. The variables depend on the flow configuration and a general solution is impossible. According to White p4721 the K-F-model can be modified to give predictions for turbulent jets; a procedure which require CF1) code. For a turbulent eddy of size L, and kinetic energy K, dimensional analysis [16, p444] give

$$F_{\dots} \dots (constant) L \quad (7)$$

According to Schlichting [15, p5931, experiments indicate that the numerical value of the constant is 0.165. Since  $L$  (the flow scale equals the concentration scale),

$$\dots \dots 2 \quad (8)$$

To estimate the mixing time-constant, the local macro scale of turbulence and the local turbulent kinetic energy, must be obtained. These two parameters are not independent, they are coupled through turbulent kinetic energy equation, [1.61. For circular jets the calculation can be completed.

**Circular Jets** If turbulent energy production equals dissipation, Schlichting [15, p5921 gives the following relationship for axially symmetric flows; with  $y$  the radial distance from the axis, and  $a$  the mean flow the axial direction

$$\dots \dots 0.165 \dots (9)$$

This equation is probably not valid near walls where the diffusion term of the kinetic energy equation is important [15]. The mean flow profile for a circular jet, in the self-similar region, is given by White p4741

$$U = U_{ax} \left( 1 - \frac{r^2}{R^2} \right)^{1/2} \quad (10)$$

*[Faint, illegible text and mathematical derivations follow, likely bleed-through from the reverse side of the page.]*

where the value of either  $C_h$  or  $R_h$  can be chosen arbitrarily.

To calculate  $\tau_m$  requires knowledge of the turbulent flow-field structure. Brodkey [5] gives for a Schmidt number,  $Sc \leq 1$ ,

$$\tau_m = \left(\frac{5}{\pi}\right)^{2/3} \frac{2}{3 - Sc^2} \left(\frac{L_s^2}{\varepsilon}\right)^{1/3}. \quad (5)$$

In this equation  $\varepsilon$  [W/kg] is the velocity energy dissipation per unit mass, and  $L_s$  [m] is the macro scale of the scalar blobs. For mixing of hot and cold air  $Sc = Pr \approx 0.7$ , so

$$\tau_m \approx 1.09 \left(\frac{L_s^2}{\varepsilon}\right)^{1/3}. \quad (6)$$

The structure of the turbulence is characterised by the local scale lengths and energy dissipation. These variables depend on the flow configuration and a general solution is impossible. According to White [16, p472] the  $K$ - $\varepsilon$  model can be modified to give predictions for turbulent jets; a procedure which requires a CFD code. For a turbulent eddy of size  $L$ , and kinetic energy  $K$ , dimensional analysis [16, p444] gives

$$\varepsilon \approx (\text{constant}) \frac{K^{3/2}}{L}. \quad (7)$$

According to Schlichting [15, p593], experiments indicate that the numerical value of the constant is 0.165. Since  $L = L_s$  (the flow scale equals the concentration scale),

$$\tau_m \approx 2 \frac{L_s}{\sqrt{K}}. \quad (8)$$

To estimate the mixing time-constant, the local macro scale of turbulence and the local turbulent kinetic energy, must be obtained. These two parameters are not independent, they are coupled through the turbulent kinetic energy equation [16]. For circular jets the calculation can be completed.

**Circular Jets** If turbulent energy production equals dissipation, Schlichting [15, p592] gives the following relationship for axially symmetric flows, with  $y$  the radial distance from the axis, and  $\bar{u}$  the mean flow in the axial direction

$$\frac{K}{L^2} = 0.165^2 \left(\frac{\partial \bar{u}}{\partial y}\right)^2. \quad (9)$$

This equation is probably not valid near walls where the diffusion term of the kinetic energy equation is important [15]. The mean flow profile for a circular jet, in the self-similar region, is given by White [16, p474]

$$\frac{\bar{u}}{U_{\max}} \approx \text{sech}^2 \left(10.4 \frac{y}{x}\right) \quad (10)$$

with the centreline mean velocity,  $U_{c,m}$ , proportional to  $x^{-1}$ , and  $x$  the distance along the axis. relationship between  $K$  and  $L$ , is

$$K = 11.8 \frac{x}{R} \tanh(10.4y/x)$$

In the centre of the jet the scale approaches zero, indicating that no mixing-only diffusion, takes place here. The time-constant is

$$\tau_{mix} = 0.58 \frac{x}{U_{c,m} \text{sech}^2(10.4y/x) \tanh(10.4y/x)} \quad (12)$$

This result is presented graphically in figure 5. The time-constant is short in the shear layers where of the mixing takes place. This observation is probably generally true and can be used to estimate mixing time-constant for more complicated geometries. The mixing time-constant is clearly a very strong function of the position of the sensor relative to the diffuser etc. For  $y > x/5$  the mixing time-constant increases rapidly. In practice it is limited by recirculating zones near the boundaries and can not become infinite. Note that the observed time-constant in the centre of the jet is very small while the theoretical result in figure 5 gives a very large value. The infinite theoretical result indicates that no mixing takes place in the centre, and not, that the observed temperature response should change slowly. As soon as the unmixed hot air stream comes into contact with the thermocouple the temperature changes rapidly.

**Whole Room Characteristic** Brodkey [5, p721 gives for the rate of turbulent energy production

$$P = 11 \rho \epsilon \quad (13)$$

where  $P$  is the flow power,  $\epsilon$  the efficiency of turbulent production and  $M$  the fluid

. From equation 6 then follows

$$\tau_{mix} = 1.09 \frac{L}{U_0} \quad (14)$$

$$1.09 \left( \frac{L}{U_0} \right)$$

A time-constant characteristic of a whole room is obtained by taking  $L = \frac{V}{A_d U_0}$ , where  $V$  is the room volume, and  $P = \frac{1}{2} \rho A_d U_0^3$ , where  $A_d$  is the diffuser area and  $U_0$  the air speed at the diffuser.

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with the centreline mean velocity,  $U_{\max}$ , proportional to  $x^{-1}$ , and  $x$  the distance along the axis. The relationship between  $K$  and  $L_s$  is

$$\frac{K}{L_s^2} = 11.8 \left[ \frac{\bar{u}}{x} \tanh(10.4 \bar{y}/x) \right]^2 \quad (11)$$

In the centre of the jet the scale approaches zero, indicating that no mixing—only diffusion, takes place here. The time-constant is

$$\tau_m = 0.58 \left| \frac{x}{U_{\max} \operatorname{sech}^2(10.4 \bar{y}/x) \tanh(10.4 \bar{y}/x)} \right| \quad (12)$$

This result is presented graphically in figure 5. The time-constant is short in the shear layers where most of the mixing takes place. This observation is probably generally true and can be used to estimate the mixing time-constant for more complicated geometries. The mixing time-constant is clearly a very strong function of the position of the sensor relative to the diffuser jet. For  $y > x/5$  the mixing time-constant increases rapidly. In practice it is limited by recirculating zones near the boundaries and can not become infinite. Note that the observed time-constant in the centre of the jet is very small while the theoretical result in figure 5 gives a very large value. The infinite theoretical result indicates that no mixing takes place in the centre, and not, that the observed temperature response should change slowly. As soon as the unmixed hot air stream comes into contact with the thermocouple the temperature changes rapidly.

**Whole Room Characteristic** Brodkey [5, p72] gives, for the rate of turbulent energy production

$$\varepsilon = \eta \frac{P}{M} \quad (13)$$

with  $P$  the flow power,  $\eta$  the efficiency of turbulent production and  $M$  the fluid mass. From equation 6 then follows

$$\tau_m = 1.09 \left( \frac{L_s^2 M}{\eta P} \right)^{1/3} \quad (14)$$

A time-constant characteristic of a whole room is obtained by taking  $L_s = \sqrt[3]{V}$ , where  $V$  is the room volume, and  $P = \frac{1}{2} \rho A_d U_0^3$ , where  $A_d$  is the diffuser area and  $U_0$  the air speed at the diffuser.

$$\tau_{ma} = 1.37 \left( \frac{V^{5/3}}{\eta A_d} \right)^{1/3} \frac{1}{U_0} \quad (15)$$

If  $\tau = 1$  all the kinetic energy at the diffuser becomes turbulent kinetic energy in the room as the air becomes almost stationary. If the air retains some kinetic energy a smaller value for  $\tau$  should be used. For a room with  $V = 54 \text{ m}^3$ ,  $A_d = 7 \text{ m}^2$ ,  $U_0 = 15 \text{ m/s}$  and  $T_1 = 1$ ,  $\tau = 2 \text{ s}$ . The results of the CF1) analysis in figure 4 indicate a response time between 5 and 15 seconds, outside the jet. However, figure 1 shows that, due to the constant pressure boundary, very little of the kinetic energy at the diffuser is dissipated inside the room.

The time-constant given by equation 15 is a spatial average and is useless for predicting the local time-constant in the vicinity of the temperature sensor. Nevertheless, it gives an indication of the magnitude of the mixing time-constant as a function of zone volume and diffuser size.

In air-conditioning industry, diffusers are often characterised by a characteristic jet length, the *throw*, [14, chapter 10], which gives some idea of the flow distribution in the room. To estimate the mixing time-constant, the throw can be used to get an idea of the mean velocity profile  $\bar{u}$ . Hopefully, in similar fashion as equation 8, the mixing time-constant can be related to the mean velocity by a correlation of the form

$$\tau_m \propto (\text{constant}) \frac{L_s}{\bar{u}} \quad (16)$$

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where  $L_s$  is a characteristic length scale which is small in the shear flow regions and can be estimated from the slope of the mean velocity profile. The turbulent kinetic energy,  $K$ , can be taken roughly proportional to the squared mean velocity. Another possibility is to try to relate the time-constant to the *jet momentum number*. Chow et al. [8] measured the steady velocity field in seven railway stations and found that the diffuser performance can be related to a modified jet momentum number

$$J^* = \frac{Q}{g A_{\text{room}} h} \quad (17)$$

where  $Q$  [ $\text{m}^3/\text{s}$ ] is the flow rate,  $g = 9.81 \text{ m/s}^2$ ,  $A_{\text{floor}}$  [ $\text{m}^2$ ] the floor area, and  $h$  [ $\text{m}$ ] the height of the centre line of the diffuser above floor level. Future investigation must determine the viability of these approaches.

### 3 Conclusion

HVAC system dynamics, although very important for maintaining acceptable indoor comfort, is often neglected in the design process. One of the reasons for this is the lack of useful physical models for the prediction of the dynamic behaviour of some important physical processes and subsystems. In practice the dynamical behaviour of a newly installed system is fixed by adjusting the time-constants of the controller in a somewhat arbitrary manner. However, completely satisfactory

If  $\eta = 1$  all the kinetic energy at the diffuser becomes turbulent kinetic energy in the room as the air becomes almost stationary. If the air retains some kinetic energy a smaller value for  $\eta$  should be used. For a room with  $V = 54 \text{ m}^3$ ,  $A_d = \pi(0.15)^2 \text{ m}^2$ ,  $U_0 = 15 \text{ m/s}$  and  $\eta = 1$ ,  $\tau_{ma} = 2 \text{ s}$ . The results of the CFD analysis in figure 4 indicate a response time between 5 and 15 seconds, outside the jet. However, figure 1 shows that, due to the constant pressure boundary, very little of the kinetic energy at the diffuser is dissipated inside the room.

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In air-conditioning industry, diffusers are often characterised by a characteristic jet length, the *throw* [14, chapter 10], which gives some idea of the flow distribution in the room. To estimate the mixing time-constant, the throw can be used to get an idea of the mean velocity profile  $\bar{u}$ . Hopefully, in similar fashion as equation 8, the mixing time-constant can be related to the mean velocity by a correlation of the form

$$\tau_m \approx (\text{constant}) \frac{L_s}{\bar{u}} \quad (16)$$

where  $L_s$  is a characteristic length scale which is small in the shear flow regions and can be estimated from the slope of the mean velocity profile. The turbulent kinetic energy,  $K$ , can be taken roughly proportional to the squared mean velocity. Another possibility is to try to relate the time-constant to the *jet momentum number*. Chow et al. [8] measured the steady velocity field in seven railway stations and found that the diffuser performance can be related to a modified jet momentum number

$$J^* = \frac{QU_0}{gA_{\text{room}}h} \quad (17)$$

where  $Q$  [ $\text{m}^3/\text{s}$ ] is the flow rate,  $g = 9.81 \text{ m/s}^2$ ,  $A_{\text{floor}}$  [ $\text{m}^2$ ] the floor area, and  $h$  [ $\text{m}$ ] the height of the centre line of the diffuser above floor level. Future investigation must determine the viability of these approaches.

### 3 Conclusion

HVAC system dynamics, although very important for maintaining acceptable indoor comfort, is often neglected in the design process. One of the reasons for this is the lack of useful physical models for the prediction of the dynamic behaviour of some important physical processes and subsystems. In practice the dynamical behaviour of a newly installed system is fixed by adjusting the time-constants of the controller in a somewhat arbitrary manner. However, completely satisfactory

dynamic behaviour is often not achieved. This study contributes to a better understanding of one of the most **important** dynamical processes; it shows that the position of the temperature sensor in a room is an important consideration from the point of view of system dynamics. A highly simplified model shows that the mixing time constant are highly dependent on the spatial **position**.

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## REFERENCES



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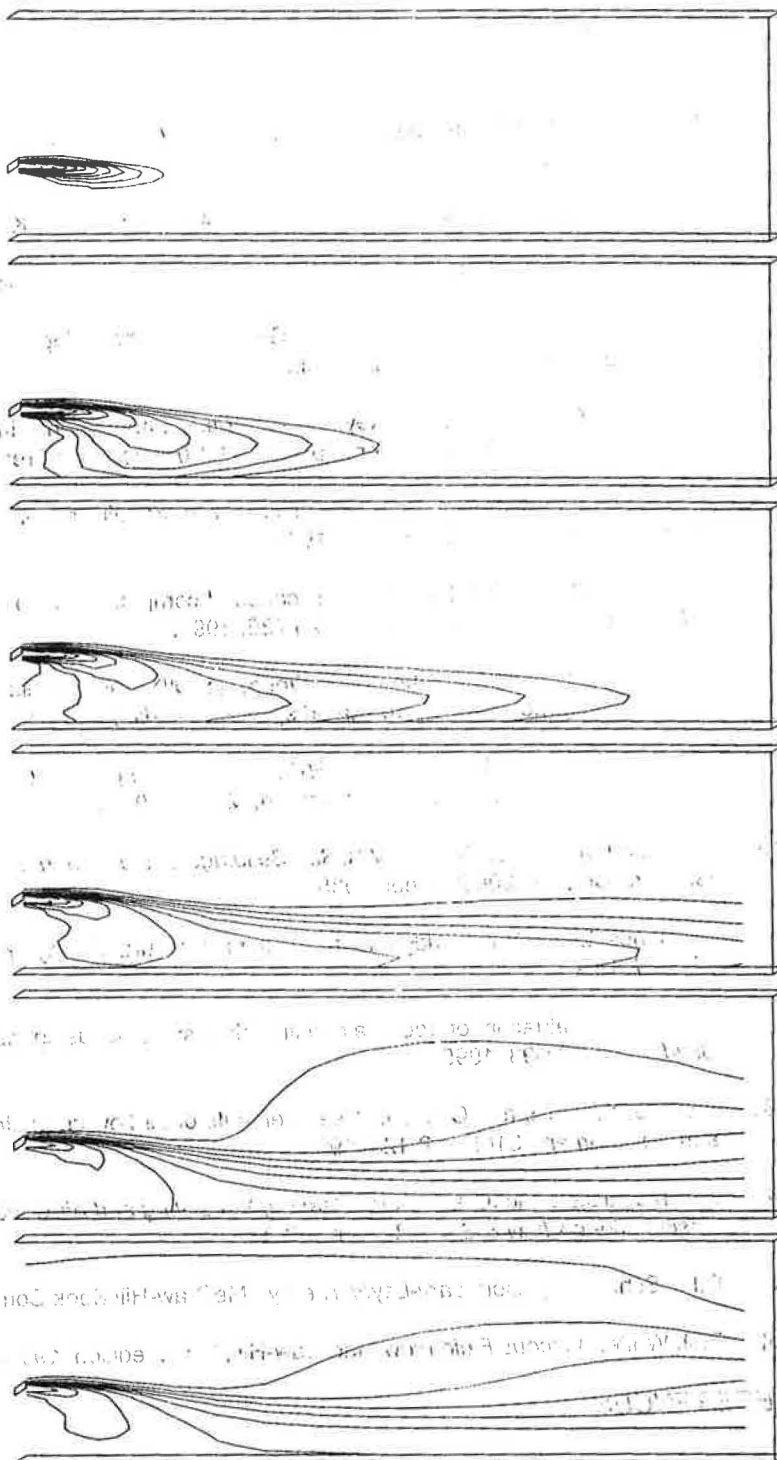


Figure 3: Temperature field development after a sudden jump in the inlet temperature. Temperature contours are shown, from top to bottom, at times 0.1 s, 0.5 s, 1 s, 5 s, 10 s and 50 s after the jump in inlet temperature occurred. The isotherm at the inlet is at 35 °C and the outermost one is at 15 °C.

Figure 3: Temperature field **development** after a sudden jump in the inlet temperature. Temperature contours are shown, from top to bottom, at times 0.1 s, 0.5 s, 1 s, 5 s, 10 s and 50 s after the jump in inlet temperature occurred. The isotherm at the inlet is at 35 °C and the outer-most one is at 15 °C.

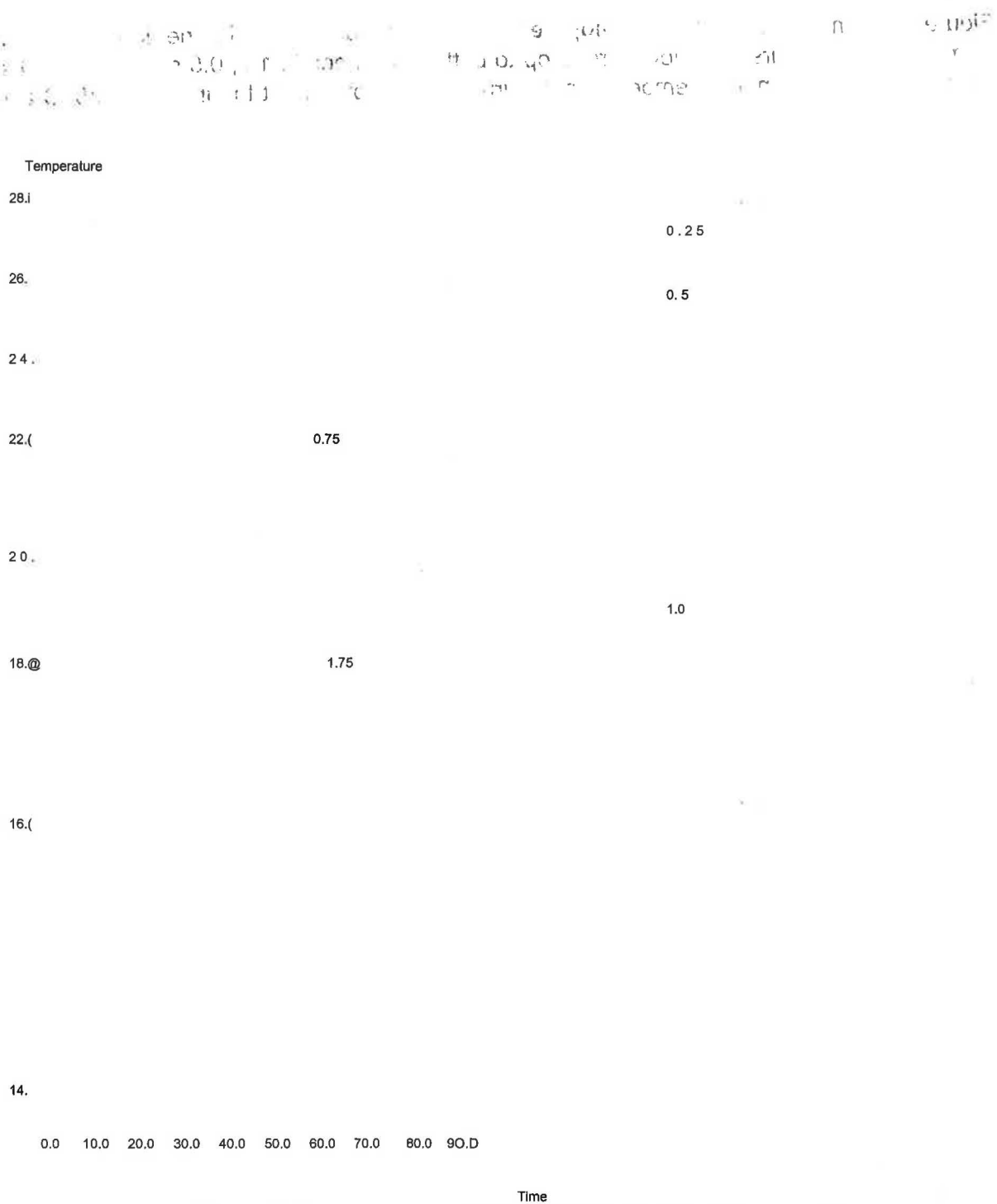


Figure 4: Temperature histories calculated at a distance of 2 m in front of the inlet, at 0.25 m, 0.5 m, 0.75 m, 1 m, 1.25 m, 1.5 m and 1.75 m above ground level.

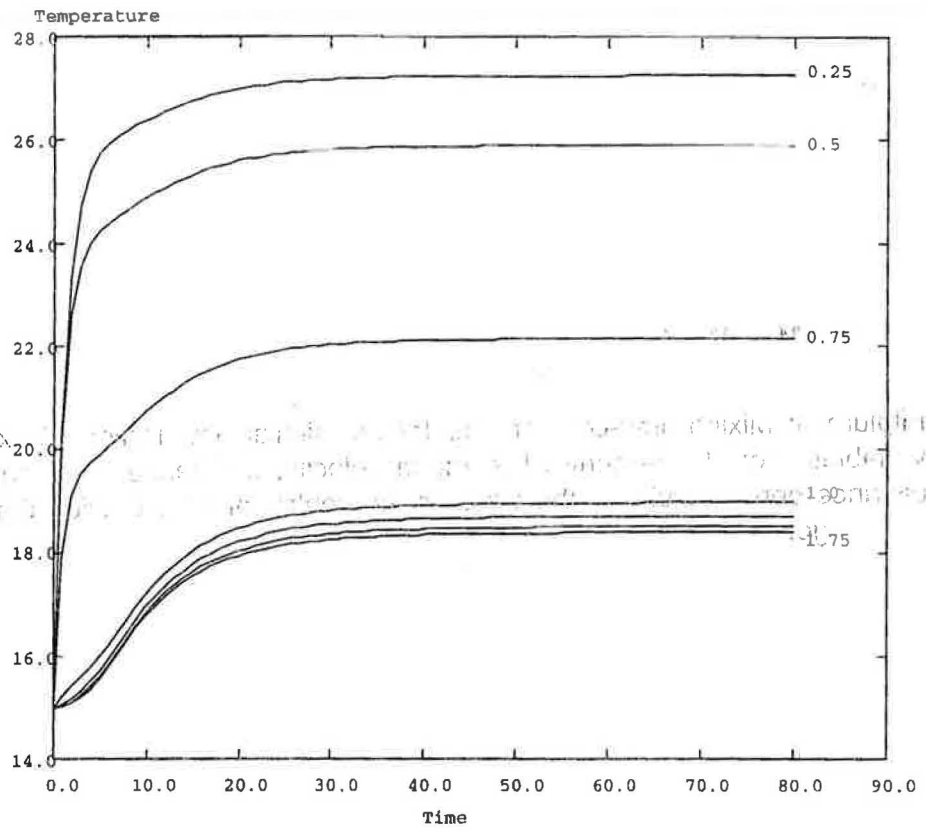


Figure 4: Temperature histories calculated at a distance of 2 m in front of the inlet, at 0.25 m, 0.5 m, 0.75 m, 1 m, 1.25 m, 1.5 m and 1.75 m above ground level.

10000  
1000  
X 100

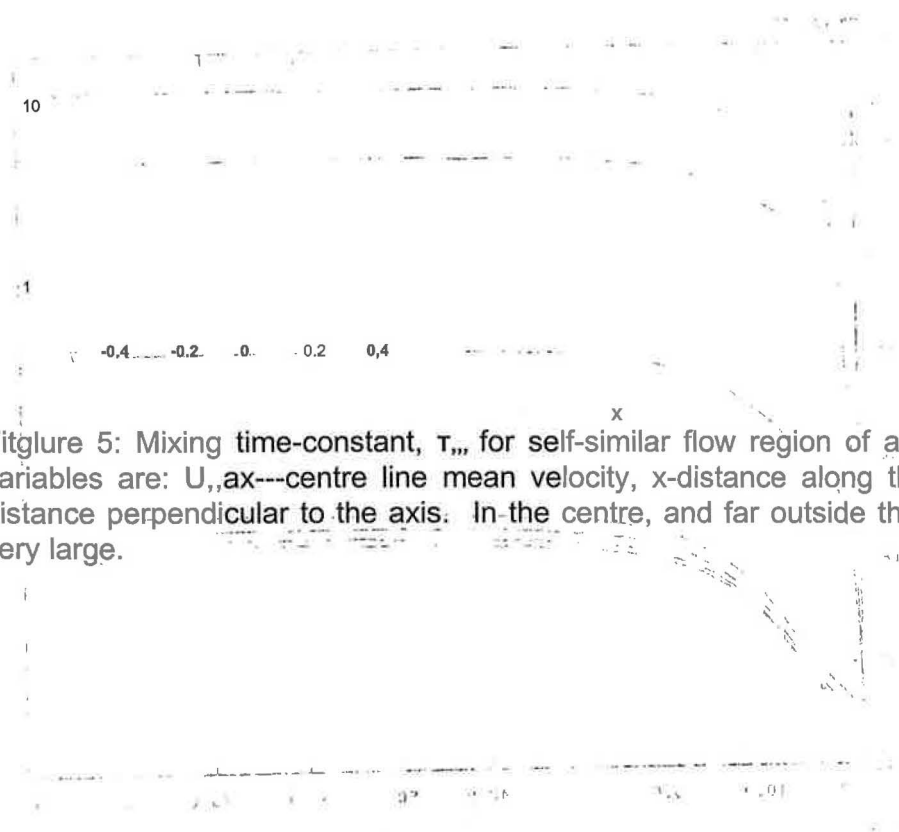


Figure 5: Mixing time-constant,  $\tau_m$ , for self-similar flow region of axially symmetric jet. The variables are:  $U_{c,ax}$ —centre line mean velocity,  $x$ —distance along the axis of the jet,  $y$ —radial distance perpendicular to the axis. In the centre, and far outside the jet, the time-constant is very large.

in the same way as in the case of a turbulent jet. The results are shown in Figure 5. The time-constant is very large in the centre and far outside the jet.

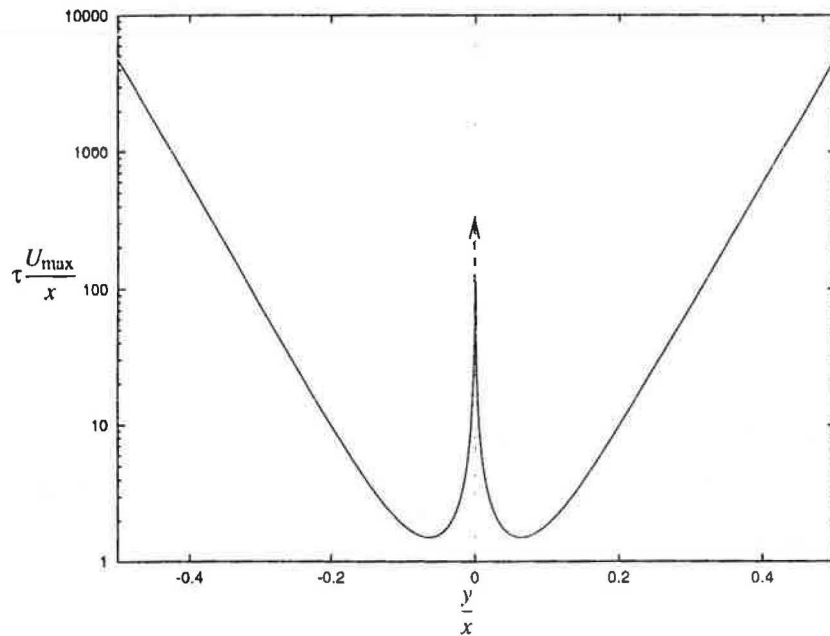


Figure 5: Mixing time-constant,  $\tau_m$ , for self-similar flow region of axially symmetric jet. The variables are:  $U_{max}$ —centre line mean velocity,  $x$ —distance along the axis of the jet,  $y$ —radial distance perpendicular to the axis. In the centre, and far outside the jet, the time-constant is very large.