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Room thermal response using transfer coefficients and the rad-air model

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A short series of wall transfer coefficients b_k, c_k and d_k enables the heat flow q_i into or out of a room through the wall concerned to be evaluated in terms of temperatures at time i (= 'now') and times i-1, i-2, . . hours previously, (ie, T_{i-1} , T_{i-2} , . .) together with a similar set of values of earlier values of heat flow, q_{i-1} , q_{i-2} , . . . The technique, advanced since 1977 by ASHRAE, appears to be as flexible as finite difference methods, but is more efficient. The coefficients have been evaluated hithertoo using frequency domain methods (Laplace and Z transforms); it is shown that they can in fact be found using quite elementary mathematical methods based on time-domain solutions alone.

For applications it is appropriate use b,c,d values based on an outer film and the solid construction (of arbitrary one dimensional form) but without the inner film which is handled explicitly using the rad-air model for room radiant and convective exchange. (In this model, firstly, the surface to surface exchange of heat by radiation is replaced by a surface to star point network using an optimum matching procedure (based upon a least sums of products of differences between the two networks). Secondly, the radiant star point is explicitly taken as proxy for the volume-averaged radiant temperature in the room. Thirdly, the radiant and convective exchanges are merged using an exact thermal network transformation.)

An example calculation of room response is given in which wall transfer coefficients are used to compute wall conduction in both internal and external elements, and the rad-air model handles room exchange and estimates comfort temperature (dry resultant temperature). The drivers will be taken to be hourly values of ambient temperature, direct solar radiation on external walls and through a window, together with an hourly choice of ventilation rate. Casual gains can be included. If instead comfort temperature is chosen the heating or cooling load can be found. Calculations of this kind have been conducted now for many years, but the present method using transfer coefficients as defined above together with the rad-air formulation for room exchange provides an efficient and transparent method of performing them.

1 Introduction

One dimensional heat flow through walls is subject to the Fourier continuity equation,

$$\lambda \frac{\partial^2 T(x,t)}{\partial x^2} = \rho c_p \frac{\partial T(x,t)}{\partial t} \qquad 1$$

There are two classes of methods to examine the temperature response of a wall when excited in some way: one can divide the wall into a suitable number of slices and relate the behaviour of some particular layer to its immediate neighbours through a finite difference form of eq 1. Proceeding step by step, information is obtained about temperature everywhere within the wall. Alternatively, we can as it were stand outside the wall and express its behaviour by certain numerical quantities. The simplest and most important of these is the familiar U value, the flow through the wall in steady conditions. If ambient temperature varies sinusoidally by ±1K per day, the heat flow into a room, fU, is termed in the UK the dynamic or cyclic transmittance, [CIBSE, 1986, Section A3]; f has a magnitude (<1) and a phase lag and f depends upon the λ, ρ, c_p, and X (thickness) of each layer, together with a period of 24 hours. Y similarly describes the flow driven by unit variation of the room internal temperature. Calculations performed with such constants are much faster than finite difference calculations, but the constants can only be used with specified design driving temperatures or heat flows: they cannot be used with hourly information on meteorological conditions and patterns of usage.

In 1967 Mitalas and Stephenson demonstrated the evaluation of *response factors* for a wall that made this possible. The heat flow q_{n0} into a room at zero degrees at time i (in hours) due to values of ambient temperature T_0 at times i, i-1, i-2, . .could be expressed as

$$q_{n0,i} = \phi_{n0,0}T_{0,i} + \phi_{n0,1}T_{0,i-1} + \phi_{n0,2}T_{0,i-2} + . . \qquad 2$$

The first subscript to ϕ denotes the position of the heat flow - beyond the nth and final layer in the wall. The second denotes the position of the driving temperature, ambient in this case, and the third denotes the time lag. $\phi_{n0,k}$ is the heat flow into the room due to a triangular pulse in ambient temperature of height 1K and base 2 hours, imposed k hours previously.

For a thick wall, the $\phi_{n0,k}$ series can be very long - more than a week - so that a long series of driving temperatures has to be processed for each hourly value of $q_{n0,i}$. This is inefficient and in 1971 the same authors described an alternative series of *transfer coefficients* to find $q_{n0,i}$

$$q_{n0,i} = b_0T_{0,i} + b_1T_{0,i-1} + b_2T_{0,i-2} + . . - d_1q_{n0,i-1} - d_2q_{n0,i-2} - . . \qquad 3$$

By expressing the heat flow now in terms of recent values, a quite short series of b_k,d_k values is sufficient to find the current value. The 1993 ASHRAE Handbook of Fundamentals lists values for 41 wall constructions, the thickest and heaviest of which only require values of up to b_6,d_6. The transfer approach thus promises to be a fast method.

b_k and d_k are clearly time-domain parameters. Mitalas and Stephenson however demonstrated their evaluation by using a frequency-domain solution to eq 1, and subjected it to a Laplace or Z transform, finally converting back to the time domain. All subsequent work has seemingly followed this route, in which it is hard not to lose sight of the fundamentally quite simple heat flow process . It is to be shown here that b_k and d_k can be found using time-domain solutions throughout. An example of their use in a building context will be sketched later.

2 The d_k values for a single slab

The triangular pulse referred to above can be synthesised from superposition of three temperature ramps:

from t = $-\delta$ and continuing indefinitely, $T_0 = \theta(t+\delta)$,

from t = 0 and continuing indefinitely, $T_0 = -2\theta t$

from t = $+\delta$ and continuing indefinitely, $T_0 = \theta(t-\delta)$

(δ will be taken here to be 1 hour.) Thus we have to find the temperature response of a wall, initially at zero degrees, when subjected to ramp excitation: its pulse response then follows immediately by superposition of the three corresponding solutions.

The ramp solution can be decomposed into the sum of two independent solutions - the response of the wall when subjected to a steady rise in temperature of indefinite duration (the 'steady-progressive' solution, and the solution describing the return to zero after some disturbed condition at t = 0, the 'transient solution'.

The d_k values in fact depend upon the transient solution alone and this will be shown here for the case of a single slab, thickness X, resistance $r = X/\lambda$. The equation

$$T(x,t,j) = q_j(r/j\pi)\sin(j\pi x/X)\exp(-t/z_j) \qquad 4$$

(i) satisfies the Fourier continuity equation, (ii) describes a temperature profile such that T = 0 at x = 0 and x = X, (iii) has a heat flow q_j at x = X and t = 0, and (iv) decays to 1/e of its t = 0 value in the decay time z_j, where

$$z_j = \rho c_p X^2/j^2\pi^2\lambda \qquad 5$$

z_1 for a brick wall of thickness 0.2m is around 1.8 hours and z_2 and z_3 are 1/4 and 1/9 of this value. We define $\beta_j = \exp(-\delta/z_j)$ 6

For a very thick slab, β_1 tends to 1; in any case, as j increases, β_j tends to zero. It can be shown (Davies 1995, 1996) that

$$d_1 = -(\beta_1 + \beta_2 + \beta_3 + \beta_4 \ldots)$$
$$d_2 = +(\beta_1\beta_2 + \beta_1\beta_3 + \beta_1\beta_4 + \ldots + \beta_2\beta_3 + \beta_2\beta_4 + \ldots + \beta_3\beta_4 + \ldots) \qquad 7$$
$$d_3 = -(\beta_1\beta_2\beta_3 + \beta_1\beta_2\beta_4 + \beta_1\beta_3\beta_4 + \beta_2\beta_3\beta_4 + \ldots)$$
$$d_4 = +(\beta_1\beta_2\beta_3\beta_4 + \ldots)$$

etc. These terms prove to be coefficients in the expansion of

$$J = (1 - \beta_1)(1 - \beta_2)(1 - \beta_3)(1 - \beta_4) \ldots \qquad 8$$

The value of J is easily determined and of course

$$J = \Sigma d_k \qquad 9$$

Now $\Sigma b_k/\Sigma d_k$ is exactly equal to the wall U value and it is essential that transfer coefficient calculations should yield unbiassed steady-state conditions. Certain requirement follow:

(i) There is in principle an infinite series of d_k values, but since β_j decreases rapidly with j, we may terminate the series at k = N, where $d_N = \beta_1\beta_2\beta_3 \ldots \beta_N$ is some acceptable fraction ε (10^{-3} say) of J.

(ii) To evaluate d_1 however to this accuracy, we must use the longer series N′ where N′ is determined by the condition that $\beta_{N'} < \varepsilon J$. (Values of d_k for k>N are of course effectively zero.)

(iii) Defining a non-dimensional slab thickness as

$$V = \sqrt{(\rho c_p/4\pi\delta\lambda)}X \qquad 10$$

N varies approximately as 2V+2, but N′ increases more rapidly. Further, the largest value of d_k in the series is roughly equal to 2^{V-2}, while J decreases dramatically with V:

$$J \simeq 5\times10^{-V} \qquad 11$$

This means that with increasing slab thickness, the precision with which the d_k values have to be reported increases and with V=12, it effectively exceeds machine precision. This limit would be reached for a concrete floor slab of thickness 0.28m, and with a time interval of 1 minute. This is not a practical limitation however: minute by minute values at one surface of a 280mm concrete slab due to changes at the other are not of interest. We may be concerned with minute by minute changes due excitation at the *same* surface, but these could be modelled in the short term by assuming a thinner slab.

3 Response factors

To find the wall response to a ramp excitation and so its response factors, we have to combine the transient solution with the steady-progressive solution, $T^{SP}(x,t)$. Suppose the slab discussed above were held isothermal at x = X, ($T^{SP}(X,t) = 0$) but that its x=0 surface was raised at a rate of θ K/s. If the time origin is chosen so that $T^{SP}(0,0) = 0$, then

$$T^{SP}(x,t) = \theta\{(\rho c_p X^2/\lambda)(-\tfrac{1}{3}x/X + \tfrac{1}{2}x^2/X^2 - \tfrac{1}{6}x^3/X^3) + t(1 - x/X)\} \qquad 12$$

We suppose that a suitable combination of transient terms can be taken such that at t = 0, the combined steady-progressive and transient solutions ensure that the net temperature is zero everywhere in the slab, that is

$$\Sigma T(x,0,j) = -T^{SP}(x,0), \quad \text{summing j from 1 to } \infty \qquad 13$$

The values of q_j (eq 4) can be found by Fourier analysis. Both sides of eq 13 are multiplied by $\sin(k\pi x/X)$ and integrated from x = 0 to x = X. Cross product terms (terms in j and k) on the left side are zero and only the term involving $\sin^2(j\pi x/X)$ is finite. Most of the terms on the right side are either zero or cancel and we find

$$q_j = 2\theta\rho c_p X/(j\pi)^2 = 2\theta c/(j\pi)^2 \qquad 14$$

where c is the slab thermal capacity, $\rho c_p X$ [J/m²K]. Thus if at t = 0, the temperature is everywhere zero, that T(X,t) is always zero but that for t > 0, T(0,t) = θt, the temperature is given by

$$T(x,t) = \Sigma T(x,t_j) + T^{SP}(x,t)$$
$$= \Sigma\theta cr(j\pi)^{-3}\sin(j\pi x/X)\exp(-t/z_j) + \theta\{(cr)(-\tfrac{1}{3}x/X + \tfrac{1}{2}x^2/X^2 - \tfrac{1}{6}x^3/X^3) + t(1 - x/X)\} \qquad 15$$

Since $q(x,t) = -\lambda\partial T(x,t)/\partial x$, the surface fluxes are

$$q(0,t)/\theta = t/r + c[\,^1/_3 - 2\Sigma(j\pi)^{-2}\exp(-t/z_j)] \qquad 16a$$
$$q(X,t)/\theta = t/r + c[-^1/_6 - 2(-1)^j\Sigma(j\pi)^{-2}\exp(-t/z_j)] \qquad 16b$$

Similar expressions were given by Mitalas and Stephenson, based on the expression for the temperature given by Churchill who deduced it using a Laplace transformation. It will be seen however that it can be found by routine Fourier analysis in the time domain.

The set of response factors ϕ_{10} describes the heat flow from the wall remote from the point of excitation. The first factor $\phi_{10,0}$ is simply the value of $q(X,t)/\theta$ at $t = 0$ due to a ramp imposed at $t = -\delta$, that is, $q(X,\delta)/\theta$ given by eq 16b. The kth value (k>0) is given as

$$\phi_{10,k} = q(X,k\delta+\delta)/\theta - 2\,q(X,k\delta-0)/\theta + q(X,k\delta-\delta)/\theta$$
$$= (2c/\pi^2)\Sigma(-1)^{j+1}j^{-2}\exp(-k\delta/z_j)[\exp(+\delta/z_j) - 2 + \exp(-\delta/z_j)],\ j = 1 \text{ to } \infty \qquad 17$$

$\phi_{10,k}$ is a series of numbers rising from a small value at k=0 to some weak maximum and tending to zero. ϕ_{00} describes the flow into the wall at the point of excitation and its values are found from eq 16a. $\phi_{00,0}$ is a comparatively large positive quantity, $\phi_{00,1}$ has a not-so-large negative value and subsequent values are negative and tend to zero.

4 Generalisation of the model

Real walls may consist of several layers of materials and consideration may have to be taken of outside and inside films representing convective and radiative transfer, and similar transfer across a cavity. Although steady-progresive and transient solutions in each layer can be written down, any direct attempt to impose continuity of temperature and heat flow at their interfaces leads to intractible algebra. This can be avoided by expressing transfer properties of any layer by its transmission matrix, one for steady-progressive excitation and another for transient flow. The overall behaviour of the wall can then be found by multiplication of the transmission matrices. This is described in Davies (1995, 1996).

The temperature distibution at t=0 in layer L of the wall can be written

$$T^{SP}_L(x,0) = A_L + B_L x/X_L + C_L x^2/X_L^2 + D_L x^3/X_L^3 \qquad 18$$

and matrix multiplication of the n layers provides these values. The form of the wall transient transmission matrix is of form

$$\begin{bmatrix} T_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} \begin{bmatrix} T_n \\ q_n \end{bmatrix} \qquad 19$$

Values of the decay times are given when both T_0 and T_n are zero, that is when $e_{12} = 0$. Unlike the case of the single slab where z_j was given by an explicit formula, in the general case a search has to be made for the values of z that make e_{12} zero and this is, computationally speaking, the most time-consuming part of the evaluation of the transfer coefficients.

A further generalisation relates to the Fourier analysis needed to find the q_j values. Here, multiplication of a term of type $\sin(j\pi x/X_L)$ by $\sin(k\pi x/X_L)$ requires that this product be multiplied by $\rho_L c_{pL}$ for the layer in question, and that the integral, so evaluated for layer L, be summed over all n layers of the wall. This follows from an orthogonality theorem which can be proved analytically for two slabs, and can be shown computationally to be true for any number of layers (Davies 1997).

The scheme outlined above for a single slab however remains the same. Once the wall decay times z_j are known, the d_k values follow. Similarly, the wall ramp response and so the response factors $\phi_{n0,k}$ and $\phi_{nn,k}$. The series $\phi_{0n,k}$ is identical with $\phi_{n0,k}$ and $\phi_{00,k}$, the heat flow into a wall external surface due to external excitation is not normally of interest, though it may be for an unsymmetrical internal wall.

5 The coefficients b_k and c_k

The derivation of the d_k coefficients of eq 3 has already been presented. It can be shown that the b_k coefficients are given as

$b_0 = d_0\phi_{n0,0}$

$b_1 = d_0\phi_{n0,1} + d_1\phi_{n0,0}$

$b_2 = d_0\phi_{n0,2} + d_1\phi_{n0,1} + d_2\phi_{n0,0}$ 20

$b_3 = d_0\phi_{n0,3} + d_1\phi_{n0,2} + d_2\phi_{n0,1} + d_3\phi_{n0,0}$

The c_k coefficients are similarly given, using values of $\phi_{nh,k}$.

6 Transfer coefficients and internal exchange in an enclosure

To illustrate the use of these coefficients in finding the response of an enclosure to excitations of various kinds, we consider a room shown in fig 1, of floor area 4×4m^2 and height 2½m. The north and south walls are exposed, the south wall having a single glazed window of 4m^2 area. In order to illustrate the principles with a minimum of complication, the room will be supposed to be flanked above and below and to the east and west by rooms undergoing the same behaviour. Further, the floor, and ceiling, and the east and wall surfaces will be supposed to be a common temperature. (There is no difficulty representing these four surfaces separately.)

The north and south walls are supposed to be of brick of 220mm thickness, with λ = 0.84W/mK, ρ = 1700kg/m^3 and c_p = 800J/kgK. The outside film resistance is taken to be 0.06m^{2}K/W. This provides values for the transfer coefficients.

	outer wall			internal partitions	
k	b_k	c_k	d_k	c_k	d_k
0	0.003222	20.100780	1	18.031627	1
1	0.167770	-32.660349	-1.039043	-18.686151	-0.163762
2	0.335657	15.044552	0.236217	0.654978	0.000115
3	0.077515	-1.945390	-0.008542	-0.000454	-0.000000
4	0.001888	0.046551	0.000022		
5	0.000003	-0.000089	-0.000000		
sum	0.586055	0.586055	0.188654		

It will be noted that $\Sigma b_k = \Sigma c_k$ and also that $\Sigma b_k/\Sigma d_k = 3.106507$. Now the U value is $(0.06+0.22/0.84)^{-1} = 3.106509$ W/m^{2}K. Thus $\Sigma b_k/\Sigma d_k$ = U, as must be the case.

The outer wall consists of only two 'layers', the outer film and the brick itself. There is no difficulty in including insulation, plaster or a cavity; this construction was studied so as to show the attenuation of the effect of solar gains by mass rather than by resistance. The walls inside walls are supposed to be of 110mm brick and for convenience the floor and ceiling are taken to have this construction too. Since the surrounding spaces were supposed to be undergoing the same thermal history, the internal partitions are in effect adiabatic at a depth of 0.055mm. Only the c_k and d_k coefficients are relevant; they are given above. In this case, $\Sigma c_k = 0$.

In the ASHRAE listing of b_k, d_k values, account was taken of the inside film resistance which represents lumped convective and radiative exchange in the room. The above values exclude this film; its effect is taken account of separately using the rad-air model (Davies 1992). This model provides a node T_{av}, representing the volume-averaged air temperature, at which convective heat Q_a acts. It also uses the rad-air node T_{ra}, formed using an exact circuit transformation as a linear combination of T_{av} and the radiant star node T_{rs}. T_{rs} is a node through which all radiant exchange which physically takes place between surfaces is taken to pass, and at which the radiant component Q_r of an internal source of radiation acts. In forming the combination, T_{ra} in fact replaces T_{rs} and Q_r is handled as an augmented input $Q_r(1+\alpha')$ at T_{ra} with withdrawal of the the excess $Q_r\alpha'$ from T_{av}. The effect of the transformation is to merge the physically distinct convective and radiant mechanisms and all exchange between surface j at T_j and surface k is assumed to take place via T_{ra}.

The total conductance H_j between T_j and T_{ra} is

$H_j = C_j + S_j$, 21

where C_j is the convective component, equal to $A_j h_{cj}$ and h_{cj} was assumed to be $3W/m^2K$ for all surfaces. S_j is the radiative component:

$$S_j = A_j h_r \varepsilon_j / (1 - \varepsilon_j - \beta_j \varepsilon_j) \qquad 22a$$

where $\beta_j = 1 - b_j - 3.54(b_j^2 - ½b_j) + 5.03(b_j^3 - ¼b_j)$ 22b

and $b_j = A_j$/total enclosure area.

$h_r = 5.7W/m^2K$ at about 20°C. The values of H_j (W/K) so found are:

H_3 (window), 35.06; H_4 (floor, ceiling, both flanking walls), 477.85; H_5 (south wall) 52.60; H_6 (north wall) 87.66. The link H_2 between T_{ra} and T_{av} is on a different footing from that to solid surfaces; it is sufficient to report here that it was 322.72W/K. $\alpha' = \Sigma C_j/\Sigma S_j$.

7 Excitation of the enclosure.

The enclosure can in general be supposed to be excited by three independent quantities, internal heat gains of various kinds (heat/cooling appliances, cooking, occupants etc) of total value Q_t (to be ignored here), ambient air temperature T_0, and solar radiation I variously acting on the north and south facades, by absorption at the glass and the transmitted fraction being supposed absorbed at T_4. (The model can readily be extended to separate the four T_4 surfaces and so track the sun, but this complication is avoided here so as to concentrate on the main essentials.) All values are supposed known at hourly intervals. The window absorbed and transmitted fractions are found using Frenel relations, (Threlkeld, p 353). 4mm clear glass (refractive index 1.526 and extinction coefficient $6.85m^{-1}$) was assumed, so that hourly values of shortwave α and τ could be found. (Diffuse gains will be ignored, again to avoid complication.)

Hourly values of the ventilation rate V will be assumed. (V = (hourly volume air change rate/3600)×1200.)

The model can be evaluated in either of two modes: (i) find the pattern of daily variation of the 6 enclosure temperatures and so evaluate comfort temperature, (ii) fix comfort temperature at some value and find the heating or cooling needed to maintain it hour by hour.

Values of ambient temperature T_0 and solar radiation I were taken from the 1986 CIBSE Guide for May 22nd: Table A8.3 for ambient temperature and pA2-91 for radiation (latitude 55°).

We are not interested in outside surface temperatures and so for transmission through the walls, ambient temperature is replaced by sol-air temperature:

$$T_{sa} = T_0 + I\alpha r_0 = I\times 0.8\times 0.06. \qquad 23$$

(If these outside surface temperature were of interest, they would have to be handled in the same way as the six internal temperatures as shown below. In this case, the transfer coefficients would have to found for the wall without its outside film resistance r_o.)

8 Continuity equations in the enclosure.

Of the total internal heat input Q_t, a certain fraction p, assumed known, is input radiantly and the remainder, $Q_t(1-p)$ convectively. The convective input acts at T_{av} or T_2. The radiant input is handled as $Q_t p(1+\alpha')$ at the rad-air node T_{ra} or T_1 together with a withdrawal of $Q_t p\alpha'$ from T_2. Imposed inputs at the solid surfaces will be notated as Q_j; part of this flows to T_{ra} and the component into the wall will be notated as Q'_j. Continuity equations can then be written down for each node at time level i:

$$\begin{aligned}
T_1:&\quad (T_1 - T_2)H_2 + (T_1 - T_3)H_3 + (T_1 - T_4)H_4 + (T_1 - T_5)H_5 + (T_1 - T_6)H_6 &&= Q_t(1+\alpha') \\
T_2:&\quad (T_2 - T_1)H_2 + (T_2 - T_0)V &&= Q_t(1-p) - Q_t\alpha' \\
T_3:&\quad (T_3 - T_1)H_3 + (T_3 - T_0)A_3/r_0 &&= Q_3 \qquad 24 \\
T_4:&\quad (T_4 - T_1) + Q'_4 &&= Q_4 \\
T_5:&\quad (T_5 - T_1) + Q'_5 &&= Q_5 \\
T_6:&\quad (T_6 - T_1) + Q'_6 &&= Q_6
\end{aligned}$$

The heat flow Q'_j into storage element j has to be found from the transfer coefficients. We use the notation $B_{jk} = b_{jk}A_j$ (units W/K) and $C_{jk} = c_{jk}A_j$. Then

$$Q'_{4,i} = C_{40}T_{4,i-0} + C_{41}T_{4,i-1} + C_{42}T_{4,i-2} + \ldots - d_{41}Q_{4,i-1} - d_{42}Q_{4,i-2} - \ldots \qquad 25$$

$$\begin{aligned} Q'_{5,i} &= Q_{5,i}(\text{out}) - Q_{5,i}(\text{in}) \\ &= [C_{50}T_{5,i-0} + C_{51}T_{5,i-1} + C_{52}T_{5,i-2} + \ldots - d_{51}Q_{5,i-1} - d_{52}Q_{5,i-2} - \ldots] \\ &\quad - [B_{50}T_{sa,i-0} + B_{51}T_{sa,i-1} + B_{52}T_{sa,i-2} + \ldots - d_{51}Q_{5,i-1} - d_{52}Q_{5,i-2} - \ldots] \end{aligned} \qquad 26$$

$Q'_{6,i}$ is found similarly to $Q'_{5,i}$. In these equations, all values of T_0 and T_{sa} are known at time level i and values of T_j at time levels previous to i are to be placed on the right side of the equations. $T_{j,i}$ values however are unknown and so their coefficients C_{j0} have to be included on the left. We have a matrix equation of form **Ax = b** where **Ax** is

$$\begin{matrix} H_2+H_3+H_4+H_5+H_6 & -H_2 & -H_3 & -H_4 & -H_5 & -H_6 \\ -H_2 & H_2+V_i & & & & \\ -H_3 & & H_3+A_3/r_6 & & & \\ -H_4 & & & H_4+C_{40} & & \\ -H_5 & & & & H_5+C_{50} & \\ -H_6 & & & & & H_6+C_{60} \end{matrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} \qquad 27$$

and **b** is

$$\begin{matrix} Q_{ti}p(1+\alpha') \\ Q_{ti}(1-p) - Q_{ti}p\alpha' + V_iT_{0,i} \\ Q_{3i} + A_3r_6^{-1}T_{0,i} \\ Q_{4i} - \Sigma C_{4k}T_{4,i-k} \\ Q_{5i} + \Sigma B_{5k}T_{sa,i-k} - \Sigma C_{5k}T_{5,i-k} \\ Q_{6i} + \Sigma B_{6k}T_{sa,i-k} - \Sigma C_{6k}T_{6,i-k} \end{matrix} \qquad 28$$

C_{jk} terms are to be summed from k = 1 to N_k and B_{jk} from 0 to N_k.

These equations have to be solved for hourly time steps. The values of **x** are the 6 values of T_j. After solution, the current value of Q_j(out) must be computed for use at the next time step.

When the room is allowed to float, values of comfort temperature T_c (half radiant, half air temperature) rather than individual room temperatures will be reported here. The radiant star temperature T_{rs} is given as

$$T_{rs} = T_{ra}(1+\alpha') - T_{av}\alpha' = T_1(1+\alpha') - T_2\alpha' \qquad 29a$$

so $$T_c = \tfrac{1}{2}T_{rs} + \tfrac{1}{2}T_{av} = \tfrac{1}{2}T_1(1+\alpha') + \tfrac{1}{2}T_2(1-\alpha') \qquad 29b$$

When comfort temperature is to be fixed, the eq. 29b has to be included in the 6 above. That requires a 7th row and column in **A**. The final row is $[\tfrac{1}{2}(1+\alpha')\ \tfrac{1}{2}(1-\alpha')\ 0\ 0\ 0\ 0\ 0]$. If cooling proves to be necessary, it is most probably achieved by a ducted air supply acting at node 2 so that an additional term, Q_{load} has to be included in the second row of eqs 27. The final column of **A** is then $[0\ -1\ 0\ 0\ 0\ 0\ 0]^T$. The required value of T_c provides the final item in **b**. The heating or cooling load Q_{load} is then the final item in **x** (with units of W).

Suppose that the program is presented with relevant meteorological and occupational data at time level i = 0. To compute T_j at i = 1, use must be made of temperatures and heat flows for the previous N hours - up to 10 perhaps - but these are unknown and must be preset to zero. Thus the program must be run for some while before valid T_j values are returned. This period is of order 5 times the room response time, may be several days.

9 Estimates for steady-periodic excitation

The model is intended for use with hourly (or other) meteorological data and heat input and ventilation rates resulting from occupancy. To illustrate its use here, the response to the repeated action of a single day will be shown, Table 1

Table Variation of comfort temperature resulting from varying ambient temperature and solar radiation, and the heating/cooling load Q_{load} to achieve a room temperature of 20°C

				V = 0		ventilation by 5ac/h night		day	
h	T_0	I_S	I_N	T_c	T_c	T_c	T_c	Q_{load}
0	11.0	0	0	22.7	18.1	21.4	21.8	124
1	10.0	0	0	22.3	17.6	21.1	21.4	193
2	9.0	0	0	21.9	17.0	18.9	21.0	260
3	9.0	0	0	21.5	16.5	18.4*	20.7	308
4	8.5	0	25	21.1	16.0	17.7*	20.3	361
5	9.0	0	135	20.7	15.7	17.3*	19.9	393
6	9.5	0	110	20.3	15.4	16.9*	19.6	420
7	10.5	0	0	20.0	15.3	16.6*	19.3	433
8	12.0	145	0	20.0	15.5	17.6*	19.3	361
9	13.5	285	0	20.4	16.0	18.1	19.7	172
10	15.0	395	0	21.1	16.9	18.9	20.5	-91
11	16.5	470	0	22.0	18.0	19.9	21.4	-377
12	18.0	495	0	23.0	19.2	21.0	22.4	-638
13	19.0	470	0	23.9	20.2	22.0	22.7*	-824
14	19.5	395	0	24.6	20.9	22.7	23.3*	-902
15	20.0	285	0	24.9	21.3	23.1	23.5*	-869
16	19.5	145	0	24.9	21.3	23.1	23.3*	-718
17	19.0	0	0	24.7	21.1	23.0	23.0*	-552
18	18.5	0	110	24.4	20.9	22.8	22.7*	-429
19	17.5	0	135	24.2	20.6	22.7	23.1	-316
20	16.0	0	25	24.0	20.1	22.5	22.9	-209
21	15.0	0	0	23.7	19.6	22.3	22.7	-120
22	13.5	0	0	23.4	19.2	22.0	22.4	-31
23	12.0	0	0	23.0	18.7	21.7	22.1	53
24	11.0	0	0	22.7	18.1	21.4	21.8	124

* Times of 5ac/h.

The columns show the daily variation of comfort temperature under the action of a uniform ventilation rate of zero and of 5 air changes an hour. Peak temperatures are reached at around 1600 and 1530h respectively. To see the effect of imposing night or day ventilation T_c values were found when the rate was zero for 18 hours with periods of 5 ac/h during the night and during the day; there is little difference in the afternoon but night ventilation leads to lower values during the morning, as would be expected.

The values reported above result from the independent action of three excitations: ambient temperature, solar radiation either absorbed or transmitted by the window, and solar radiation absorbed on the north and south wall exteriors. Under the action of ambient temperature alone, the daily mean value T_c must equal the daily mean of T_0. The variation about the mean of T_0, T_c(V=0) and T_c(V=5) were 11.5, 1.1 and 2.6K respectively, showing the stabilising action of the relatively heavy construction.

Elevations due to window gains alone varied between 4.7 and 8.8K for zero ventilation and 1.7 and 5.2K for 5ac/h. The elevations due to wall gains alone were smaller and varied less - 1.6 to 2.1K for zero ventilation and 0.7 to 1.1K for 5ac/h.

The final column gives the heat or cooling need to maintain a temperature of 20°C. Since the ducted air system assumed would deliver slightly pressurised air, so reducing infiltration losses,

a zero ventiation rate was assumed. The high night heat need comes about because the north and south brick walls do not include insulation of any kind.

10 Discussion.

Confining its discussion to a homogeneous slab of material, this article provides a sketch of how transfer coefficients for building walls can be found. The approach via purely time domain methods appears to be new. The d_k coefficients derive from the decay times of the wall, regardless of how it is excited; the b_k and c_k values come from the further processing of heat flows due to excitation by a triangular pulse of temperature (of height 1K and normally of base 2 hours).

Their use has been illustrated here by examining the response of a simple cell, four of whose surfaces are lumped, driven by ambient temperature and direct solar gains via the window and two external walls. The method has been used too to examine the CEN Validation Model (CEN 1995) in which all six surfaces were specified individually, one with a window, so that a total of 9 nodes were needed. Account was taken too of diffuse radiation and internal gains. The time taken to process one day's estimates was around 0.1s. The present approach gave values for air temperature (the required measure) whose daily mean values agreed closely with the target figures, but indicated substantially smaller variation in temperature than did the target values. (The reason for the difference is not known.) The transfer coefficients however yield values for the wall admittance Y and transmittance fU (measures of their response to sinusoidal excitation of period 24 hours) in close agreement with those found by independent and exact frequency domain analysis. This and other checks indicate that the transfer coefficients themselves are not in error. Further, an enclosure when driven by strictly sinusoidally varying ambient temperature can be examined either using Y and fU values, or transfer coefficients. In a test case, the two methods were found to give the same values for enclosure temperatures. This suggests that the current approach may be more accurate than the finite difference method used to find the CEN target values. A further, simpler and independent CEN test is concerned with the response of a wall, perfectly insulated on its inner surface and subjected to ramp excitation at its exterior: hourly values of the temperature of the adiabatic surface are required a hourly intervals. In this case three values are available: exact values found using a solution in Carslaw and Jaeger, quoted CEN values found using a finite difference method, and values found using the appropriate transfer coefficients. In all cases but one, the transfer coefficient values were closer to the exact ones than the finite difference estimates.)

The use of transfer coefficients has been recommended by ASHRAE since 1977. The scheme sketched here differs from the ASHRAE method in two respects:

(i) ASHRAE b,d values for a wall include the inner film while the present values exclude it. This enables heat flows which act at room surfaces, notably solar gains but possibly a heated floor or cooled ceiling, to be represented without difficulty. It is not clear how this can be done with ASHRAE values since it is not possible to find the $(b,d)_{\text{without film}}$ set from the $(b,d)_{\text{with film}}$ set.

(ii) ASHRAE uses its b,d set to find the heat flow into a room held at constant temperature, given hourly values of sol-air temperature. The associated cooling load is found independently using a set of v,w coefficients, which are tabulated for 12 or 14 zone types within which values are quoted for heat originating from solar, conduction, lighting etc, and people (McQuiston and Parker 1994). The present approach combines these calculations: the c,d set replaces the v,w set; all that is expected of the user is that he/she should know the appropriate radiant fraction p of any internal source of heat.

A limitation of transfer coefficients as presented here should be mentioned. They are based on ramp excitation and are well suited to representing the action of steadily varying drivers such as ambient and solar radiation. They cannot represent exactly the effect of a step driver such as an electical heat source which steps from zero to 2kW imposed at 0900h say. Such excitation has to be represented as ramp inputs having values of 0, 1kW and 2kW at 0800, 0900 and 1000h

respectively. It is possible to set up an alternative of set of transfer coefficients which represent the temperature response to a square topped heat pulse of $1 W/m^2$ and duration of 1 hour. The net response of a room would then be composed of the independent effects of ramp and step drivers.

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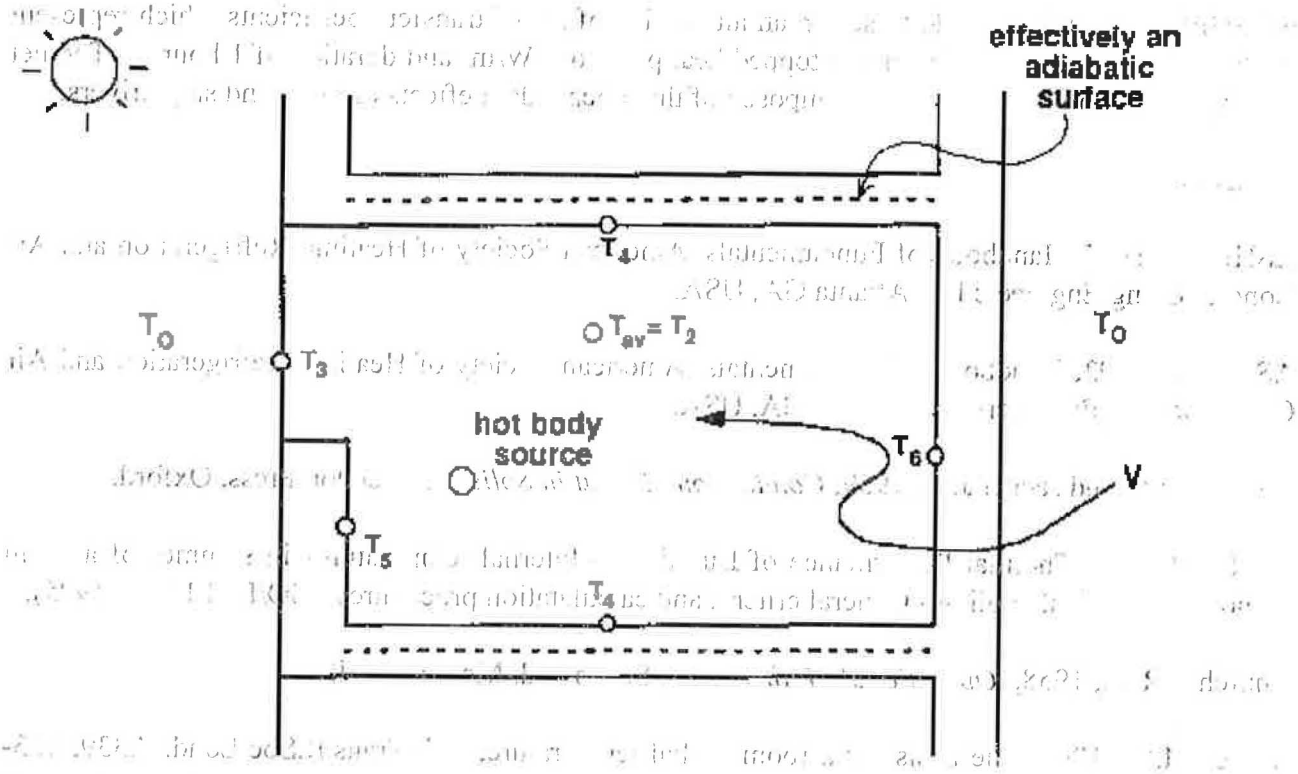

Figure 1 Elevation through the test cell

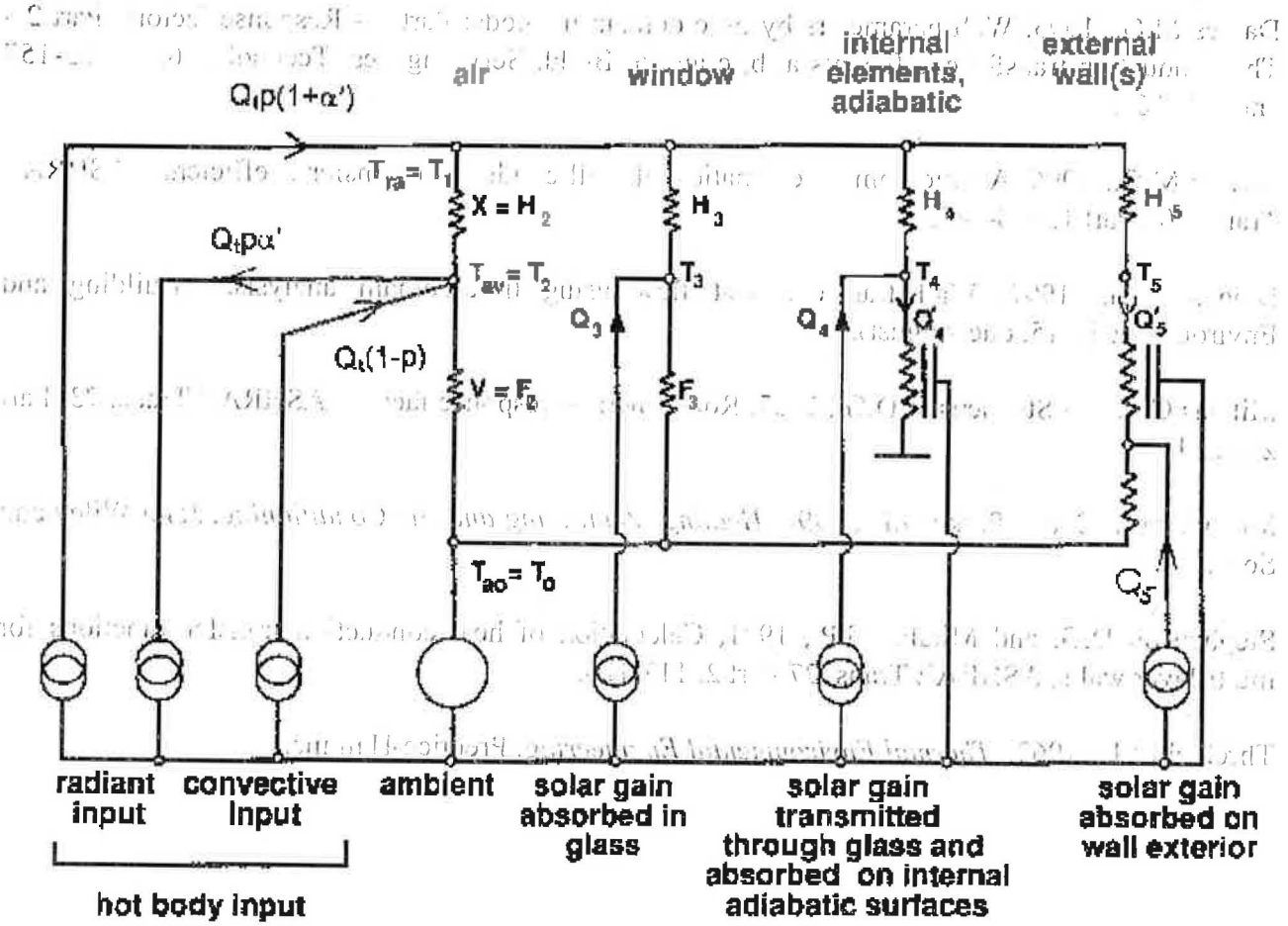

Figure 2 Thermal circuit of the enclosure(The north wall T6 is not shown. It is similar to T6)