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MODELLING THE THERMAL PERFORMANCE OF EARTH-TO-AIR HEAT EXCHANGERS

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Abstract—A new complete numerical model for the prediction of thermal performance of the earth-to-air heat exchangers is presented. The model describes the simultaneous heat and mass transfer inside the tube and into the soil accounting for the soil natural thermal stratification. The model is validated against an extensive set of experimental data and it is found accurate. The proposed algorithms are suitable for the calculation of the temperature and humidity variation of the circulating air and for the temperature and humidity distribution inside the ground. The presented model was developed within the TRNSYS environment and can be easily coupled with building or greenhouse simulation codes in order to describe the impact of the earth-to-air heat exchangers to indoor environments.

1. INTRODUCTION

The use of earth-to-air heat exchangers for heating and cooling of buildings and agricultural greenhouses has gained ground during the last few years, (3,14). The design of such a system involves optimisation of its geometrical characteristics as well as of the air-flow levels and the depth of the buried pipes. In this respect various simplified and detailed models have been proposed for this purpose. Recent evaluation (21) of seven different simplified models, (5,6,8,13,15,16,18), leads to the conclusion, that almost all the proposed algorithms can predict with sufficient accuracy the temperature of the outgoing air from the tubes. However, these models are characterised by a limited applicability as they do not take into account the latent heat transfer phenomena between the air and the pipe and they can not predict the humidity of the circulating air. Lack of knowledge on the moisture content of the circulated air does not allow for a complete evaluation of the comfort conditions in the building.

Furthermore, most of the simplified models ignore heat transfer phenomena (sensible and latent) in the ground and, therefore, the distribution and the variation of the ground can not be predicted. The ground temperature is of importance especially for agricultural greenhouse applications where this parameter plays a basic role.

Detailed simulation models of the thermal performance of the earth-to-air heat exchangers are mainly based on algorithms describing the simultaneous transfer of heat and mass in soils with a temperature gradient (1,2,10,11,12,17,20). However, most of the models consider an axially symmetric heat flow into the ground. This heat flow does not take into account the natural thermal stratification in the soil which alters this symmetry.

The objective of this paper is to present a more accurate and validated, transient, implicit numerical model based on coupled and simultaneous transfer of heat and mass into the soil and the pipe. The model includes a complete mathematical description of moisture migration through a soil with a temperature and, therefore, a moisture gradient which tends to redistribute the moisture content. The two interrelated phenomena are also described and since the final outcome depends on both the magnitude of the temperature and the moisture gradients. The natural thermal stratification in the soil is also considered while the soil boundary conditions are applied at the ground surface (9).

The model was validated against a long set of experimental data. The present model can accurately predict the temperature and the humidity of the circulating air as well as the distribution of the temperature and humidity into the soil. The proposed model is developed inside the TRNSYS programme environment, in order to be easily coupled with algorithms describing the dynamic performance of buildings as well as any other algorithm developed inside this code.

2. MODELLING OF EARTH-TO-AIR HEAT EXCHANGERS

The transient earth-to-air heat exchanger axisymmetric system has been expressed in polar coordinates with three independent variables (r, y, t) and two dependent variables (T, h). A typical earth tube system is shown in Fig. 1.

The equation of energy balance, as it has been proposed by Ahmed (1980) and Puri (1986), can be written as follows:

$$\rho c_p \frac{\vartheta T}{\vartheta t} = \frac{1}{r}\frac{\vartheta}{\vartheta r}\left(kr\frac{\vartheta T}{\vartheta r}\right) + \frac{\vartheta}{\vartheta y}\left(k\frac{\vartheta T}{\vartheta y}\right) - l_g \rho_m \frac{1}{r}\frac{\vartheta}{\vartheta r} \times \left(D_{u,vap} r \frac{\vartheta h}{\vartheta r}\right) - l_g \rho_m \frac{\vartheta}{\vartheta y}\left(D_{u,vap}\frac{\vartheta h}{\vartheta y}\right) \quad (1)$$

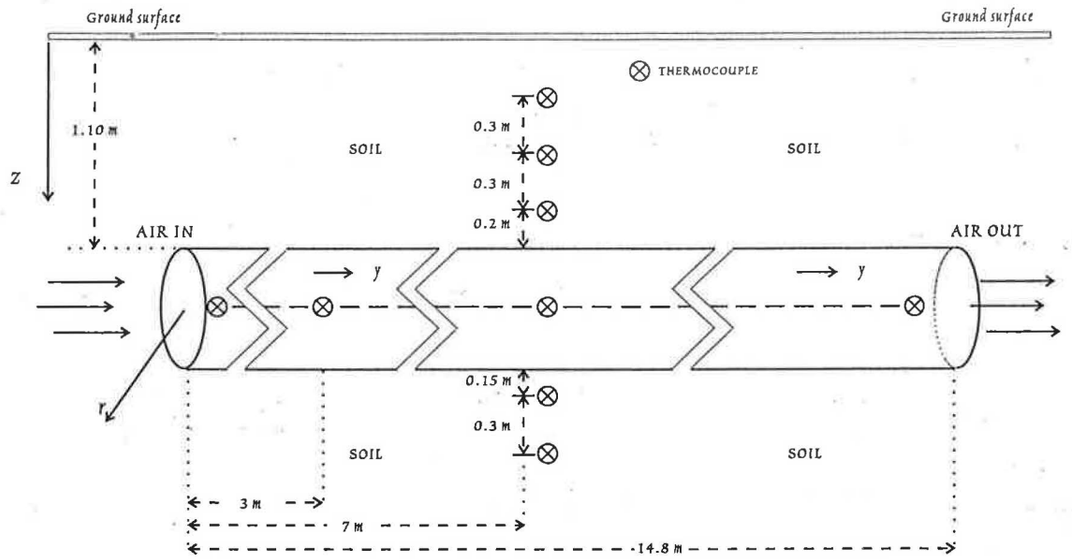

Fig. 1. A drawing of the experimental set up.

While the mass transfer equation can be given by:

$$\frac{\vartheta h}{\vartheta t} = \frac{1}{r}\frac{\vartheta}{\vartheta r}\left(D_T r \frac{\vartheta T}{\vartheta r}\right) + \frac{\vartheta}{\vartheta y}\left(D_T \frac{\vartheta T}{\vartheta y}\right) + \frac{1}{r}\frac{\vartheta}{\vartheta r}\left(D_u r \frac{\vartheta h}{\vartheta r}\right) + \frac{\vartheta}{\vartheta y}\left(D_u \frac{\vartheta h}{\vartheta y}\right) \quad (2)$$

where

$$\frac{1}{r}\frac{\vartheta}{\vartheta r}\left(D_T r \frac{\vartheta T}{\vartheta r}\right) + \frac{\vartheta}{\vartheta y}\left(D_T \frac{\vartheta T}{\vartheta y}\right)$$

is the component of moisture flux due to temperature gradient and

$$\frac{1}{r}\frac{\vartheta}{\vartheta r}\left(D_u r \frac{\vartheta h}{\vartheta r}\right) + \frac{\vartheta}{\vartheta y}\left(D_u \frac{\vartheta h}{\vartheta y}\right)$$

is the component of moisture flux due to the moisture gradient.

The initial conditions are the following:

$$T(r, y, t = 0) = T_0(r) \quad (3)$$

$$h(r, y, t = 0) = h_0(r) \quad (4)$$

The boundary conditions are:

1. co-ordinate r.

(a) at $r = R_L$.

$$T(R_L, y, t) = T(R_L) \quad (5)$$

$$h(R_L, y, t) = h(R_L) \quad (6)$$

where R_L is a large radial distance in the soil, which has been taken to equal 59 m. At this distance (R_L), the temperature and moisture distributions are not influenced by the coupled and simultaneous transfer of heat and mass into the soil caused by the presence of the earth-to-air heat exchanger system. This far-field boundary for temperature and moisture profiles in the soil has been used by Schiller (1982), Puri (1986), and Ahmed (1980). Thus, the temperature at $r = R_L$ is the undisturbed soil temperature. The moisture profile was determined using an extensive set of moisture measurements and predictions for dry and very dry soils presented in Ahmed and Shapiro (1975).

(b) at $r = R_p$.

The calculated heat transfer in the soil is equal to the amount of heat losses as air flows along the pipe.

$$G(T_a(y) - T(R_p, y, t) = -m_a C_a[dT_a(y)/dy] \quad (7)$$

where G is the overall thermal conductance of the whole earth-to-air heat exchanger system including air, pipe, and soil. Thus, G can be expressed as:

$$G = 2\pi l/\{(1/r_{in}h_c) + [ln(R_p/r_{in})/K_p]\}$$

and $T_a(0)$ equals the ambient air temperature

At $r = R_p$ the moisture content is caused by the temperature gradient since the pipe is impervious. Thus, the component of moisture flux due to moisture gradient is equal to zero.

$$\partial h/\partial r(R_p, y, t) = 0 \quad (8)$$

2. co-ordinate y.

(a) at $y = y_A$

$$T(r, y_A, t) = T_s(r) \quad (9)$$

$$h(r, y_A, t) = h_s(r) \quad (10)$$

(b) at $y = y_B$

$$T(r, y_B, t) = T_s(r) \quad (11)$$

$$h(r, y_B, t) = h_s(r) \quad (12)$$

The diffusivity values (D_T, D_u, $D_{u,vap}$) used in this study were obtained from Puri (1986) and Gee (1966).

The soil temperature at a point in the pipe vicinity is estimated by superposition of the temperature field due to the pipe system $T(r, y, t)$ and the undisturbed temperature field $T_u(z, t)$ due to the ground surface temperature. Undisturbed soil temperature can accurately be modelled by the following one dimensional, transient, heat conduction equation:

$$\partial^2 T_u(z, t)/\partial z^2 = 1/a * \partial T_u(z, t)/\partial t \quad (13)$$

where a is the soil thermal diffusivity and z is the depth below ground surface. The corresponding boundary conditions at $z = 0$ were the following:

$$T_u = T_{surface} = T_m - A_s * \cos[w*(t - t_0)] \quad (14)$$

$$\text{for } z \to \infty \quad T_u \text{ is finite} \quad (15)$$

An analytical solution of this equation is reported in Carslaw and Jaeger (1959). According to this analytical solution, the temperature at any depth z and time t can be found by the expression:

$$T_u(z, t) = T_m - A_s * \exp(-\lambda z) \times \cos[w(t - t_0) - \lambda z] \quad (16)$$

where A_s is the amplitude of temperature wave at the ground surface, T_m is the mean annual soil temperature, w is the frequency of annual temperature wave, and λ can be defined by the expression: $\lambda = (w/2a)^{1/2}$.

The transient earth-tube system described in this paper incorporates three independent (r, y, t) and two dependent variables (T, h). The method of control-volume formulation is used to discretise the two differential eqns (1) and (2). This method, which can be regarded as a special and new version of the method of weighted residuals, looks like the finite-difference method but it employs many ideas that are typical of the finite element methodology. The basic idea of the control-volume formulation is easy to understand and lends itself to direct physical interpretation.

The first step in the control-volume formulation is to divide the calculation domain into a finite number of non-overlapping control-volumes in such a way that there is one control-volume surrounding each area. The control-volumes are concentric cylindrical rings. The grid points are not actually points, but are cylindrical shells, assumed to be located at the mid-radius of each control-volume. Their height is half the height of the corresponding control-volume. The most attractive feature of the control-volume formulation is that the resulting solution would imply the integral conservation of quantities, such as energy, mass, and momentum, is exactly over any group of control volumes and, of course, over the whole calculation domain.

The discretization equations were derived by integrating the differential eqns (1) and (2) over each control-volume and over the time interval from t to $t + dt$. The order of this integration was chosen according to the nature of the term. For the representation of the terms $\bar{\Delta}T/\bar{\Delta}t$ and $\bar{\Delta}h/\bar{\Delta}t$ was assumed that the grid point values of T and h prevail through the control-volume. As it regards the terms $\bar{\Delta}T/\bar{\Delta}r$, $\bar{\Delta}T/\bar{\Delta}y$, $\bar{\Delta}h/\bar{\Delta}r$, and $\bar{\Delta}h/\bar{\Delta}y$, it was necessary to make a profile assumption or an interpolation formula. The simplest possibility was to assume that the value of T or h at a grid point prevails over the control-volume surrounding it. For this profile, the slope $\bar{\Delta}T/\bar{\Delta}r$ or $\bar{\Delta}h/\bar{\Delta}r$ was not defined at the control volume faces. A profile that does not suffer from this difficulty is the piecewise-linear profile. In this study, linear interpolation functions were used between the grid points.

The time dependency was best handled using implicit integration techniques. The fully implicit scheme satisfies the requirements of simplicity and of physically satisfactory behaviour. It should be noted that, while constructing the discretization equations, it was not assumed that a particular method would be used for their solution. It was useful to consider the derivation of the equations and their solution as two distinct operations, and was not necessary for the choices in one to influence the other.

In this study the algebraic equations were solved using the Gauss–Seidel iterative method. According to this method the values of the variable were calculated using each grid point in a certain order.

The programme has been developed inside the TRNSYS environment. TRNSYS (22) is a transient system simulation programme with a modular structure which facilitates the addition to the programme of mathematical models not included in the standard TRNSYS library.

3. MODEL VALIDATION

The collection and analysis of a comprehensive set of experimental data is considered an essential part of

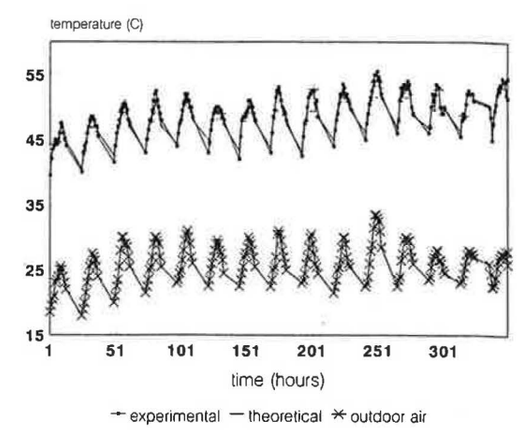

Fig. 2. Measured and predicted values of the air temperature at the outlet of the pipe during the experiment.

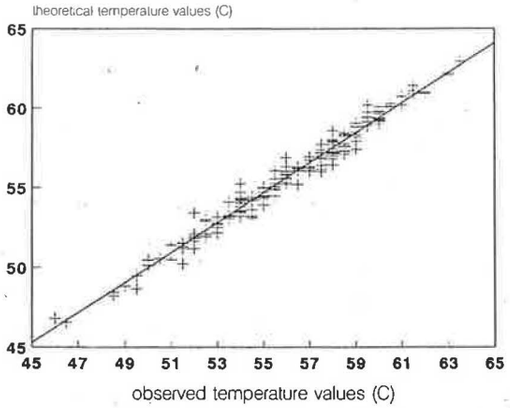

Fig. 3. Comparison of measured and predicted values of the air temperature (°C) inside the pipe at a distance of 3 m from the pipe's inlet.

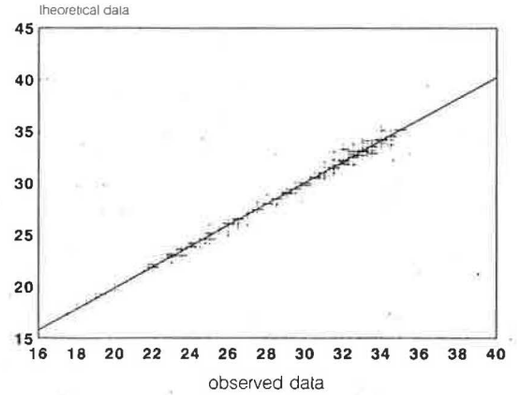

Fig. 5. Comparison of the measured and predicted soil temperature (°C) at 30 cm distance from the soil surface, 80 cm above the pipe, and 7 m y-distance from the pipe inlet.

this research in order to provide a basis for the verification of the accuracy of the mathematical model. For this purpose, a plastic pipe of 0.150 m in diameter and 14.8 m in length was buried in the soil at about 1.10 m in depth. The air velocity in the pipe was 10.5 m/sec. The temperature of the air and soil were monitored at different depths below and above the buried pipe. It was not possible to take moisture measurements for this experiment but the soil was very dry. A drawing of this experimental set up can be found in Fig. 1.

The experiments were performed during the summer and lasted 15 days. Data of the air temperature as well as the temperature of the soil below and above the pipe were recorded at 10 minute intervals throughout the experiment. In addition, hourly outdoor air temperature measurements were taken every day during the experiment.

The results of the experiment are compared with the theoretical calculations. Figure 2 shows the variation of the measured and calculated air temperature at the outlet of the pipe as well as the fluctuation of the outdoor temperature during the experiment. As shown in this figure, there is an excellent agreement between the theoretical and the measured data while the outdoor air temperature has been fluctuated between 17 and 33.5°C. Also, from the same figure, it can be seen that the fluctuation of the air temperature at the pipe outlet follows the fluctuation of the outdoor temperature. In order to show that the variation of the air temperature inside the pipe is accurately predicted, Fig. 3 compares the observed with the calculated values of the air temperature at a distance of 3 m from the pipe entry. As shown, there is an excellent agreement between the observed and the predicted values.

In Fig. 4 the variation of the soil temperature as recorded at a depth of 0.9 m below the ground surface and 0.2 m above the pipe is compared with the theoretical predictions. The y-distance from the pipe inlet was equal to 7 m. As shown, there is also an excellent agreement between the predicted and the measured values for the soil temperature. The maximum difference between the predicted and the measured values rarely exceeds 0.3°C. The same agreement between the predicted and the observed temperatures is obtained for two points located at a depth of 0.3 and 0.6 m below earth's surface and at a distance from the pipe of 0.8 and 0.5 m, respectively, while the y-distance from pipe inlet remained equal to 7 m (Figs. 5 and 6).

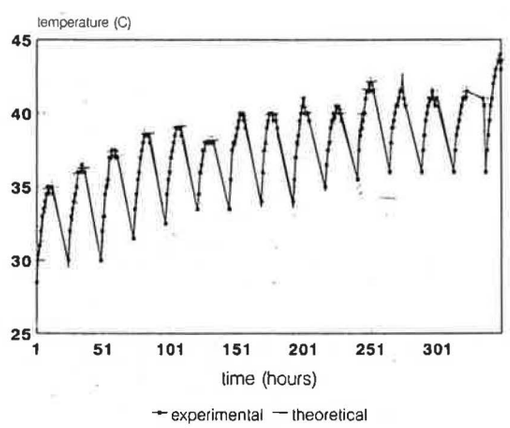

Fig. 4. Measured and predicted soil temperature (°C) at 20 cm above the pipe, 7 m y-distance from the pipe inlet, and at a depth of 90 cm during the experiment.

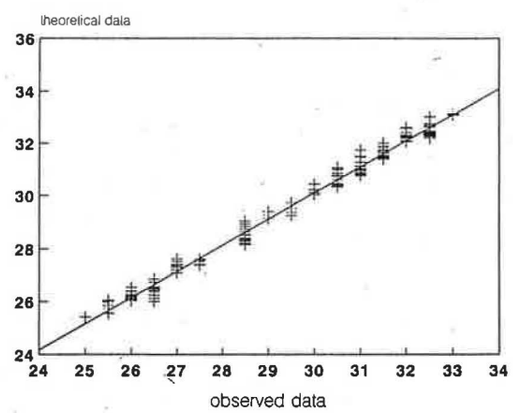

Fig. 6. Comparison of the measured and predicted soil temperature (°C) at 60 cm below the ground surface, 50 cm above the pipe, and 7 m y-distance from the pipe inlet.

4. CONCLUDING REMARKS

An accurate, transient, implicit numerical model based on coupled heat and mass transfer has been developed to describe earth-tube system thermal performance. The superposition technique was used to analyse the influence of the temperature field due to the ground surface temperature on the soil temperature at any point in the pipe vicinity.

The proposed model was validated against experimental data and was found to accurately predict the temperature of the circulated air and the temperature distribution of the ground, as well as the overall thermal performance of the earth-to-air heat exchangers.

NOMENCLATURE

- a thermal diffusivity (m^2/sec)
- $A_{\bar{s}}$ amplitude of temperature wave at the ground surface (K)
- C_a specific heat of the air (J/kgK)
- C_p specific heat capacity (J/kgK)
- $D_{u,vap}$ isothermal diffusivity of moisture in vapor form (m^2/sec)
- D_T thermal moisture diffusivity (m^2/sec per K)
- D_u isothermal moisture diffusivity (m^2/sec)
- G Overall thermal conductance of the earth to air heat exchanger system (W/m)
- h moisture content (kg of moisture/kg of dry soil)
- h_c heat transfer coefficient (W/m^2K)
- $h_s(r)$ soil moisture at a large distance from pipe axis where moisture is not influenced by the earth tube system
- k soil thermal conductivity (W/mK)
- K_p thermal conductivity of pipe (W/mK)
- l pipe length (m)
- l_g moisture heat of vaporisation (J/kg)
- $\dot{m}_a$ mass flow rate of ambient air through pipe (kg/sec)
- Q_l heat loss from the air (W)
- Q_t heat transferred in the soil (W)
- r polar co-ordinate, radial distance from the tube axis (m)
- R_L an arbitrarily large distance from the pipe axis into the soil (m)
- r_{in} inner pipe radius (m)
- R_p outer pipe radius (m)
- T soil temperature (°C)
- t time (sec)
- $T_a(y)$ temperature of the air which flows along the pipe (°C)
- $T(R_p, y, t)$ temperature of pipe (°C)
- T_m mean annual soil temperature (°C)
- t_0 phase constant
- $T_s(r)$ undisturbed soil temperature which is not influenced by the earth tube system (°C)
- $T_u(z, t)$ undisturbed soil temperature at a depth (z) below ground surface at any time (t) (°C)
- w frequency of annual temperature wave (1/sec)
- y polar co-ordinate, axial distance from the tube inlet (m)
- y_A an arbitrarily large axial distance from pipe inlet (m)
- y_B an arbitrarily large axial distance from pipe outlet (m)
- z depth below ground surface (m)

Greek characters

- ρ soil density (kg/m^3)
- ρ_m density of moisture (kg/m^3)

REFERENCES

1. A. Ahmed, Simulation of simultaneous heat and moisture transfer in soils heated by buried pipes, Ph.D. Dissert., The Ohio State University, Columbus (1980).
2. J. W. Alpert, Heat transfer from a buried pipe system under irrigation, Ph.D. Dissert., The Pennsylvania State University (1974).
3. M. Antinucci, B. Fleury, J. Lopez d' Asiain, E. Maldonado, M. Santamouris, A. Tombazis, and S. Yannas, Passive and hybrid cooling of buildings-state of the art, Vol. 11, pp. 251–272 (1992).
4. H. S. Carslaw and J. C. Jaeger, Conduction of heat in solids, Oxford Science Publishers (1959).
5. B. Chen, T. Wang, J. Maloney, and M. Newman, Measured and predicted cooling performance of earth contact cooling tubes, Proc. ASES 1983 Annual Meeting, Minneapolis, MN (1983).
6. D. Elmer and G. Schiller, A preliminary examination of the dehumidification potential of earth-to-air heat exchangers, Proc. st nat. passive cooling conference, Miami, FL, pp. 161–165 (1981).
7. G. W. Gee, Water movement in soils as influenced by temperature gradient, Ph.D. Dissert., Washington State University (1966).
8. H. J. Levit, R. Gaspar, and R. D. Piacentini, Simulation of greenhouse microclimate by earth-tube heat exchangers, *Agricultural and Forest Meteorology* **47**, 31–47 (1989).
9. G. Mihalakakou, M. Santamouris, and D. Asimakopoulos, Modelling the earth temperature using multiyear measurements, *Energy and Buildings* **19**, 1–9 (1992).
10. V. M. Puri, A finite element analysis of earth tube heat exchangers, ASAE paper No NAR84-307, ASAE, St Joseph, MI (1984).
11. V. M. Puri, Feasibility and performance curves for intermittent earth-tube heat exchangers, ASAE paper No 84-4537 (1986).
12. V. M. Puri, Earth tube heat exchanger performance correlation using boundary element method, *ASAE Trans.* **30**, 514–520 (1987).
13. E. A. Rondriguez, J. M. Cjudo, and S. Alvarez, Earth-tube systems performance, Proc. Cib meeting, air quality and air conditioning, Paris, France (1988).
14. M. Santamouris, A. Argiriou, and M. Vallindras, Design and operation of a low energy consumption passive solar agricultural greenhouse, *Solar Energy*, **52**(5), 1994.
15. M. Santamouris, and C. C. Lefas, Thermal analysis and computer control of hybrid greenhouse with subsurface heat storage, *Energy in Agriculture* **5**, 161–173 (1986).
16. G. Schiller, Earth tubes for passive cooling, the development of a transient numerical model for predicting the performance of earth-to-air heat exchangers, Msc thesis, MIT, Mechanical Engineering (1982).
17. A. R. Sepaskhah, Experimental analysis of subsurface heating and irrigation of the temperature and water content of soils, Ph.D. Dissert., Oregon State University (1974).
18. A. L. T. Seroa da Motta and A. N. Young, The predicted performance of buried pipe cooling system for hot humid climates, In: E. Bilgen and K. G. T. Hollands, Proc. INTERSOL '85, pp. 759–770 (1985).
19. H. N. Shapiro, Simultaneous heat and mass transfer in porous media with application to soil warming with power plant wast heat, Ph.D. Dissert., The Ohio State University, Columbus (1975).
20. D. L. Slegel and L. R. Davis, Transient heat and mass transfer in soils in the vicinity of heated porous pipe, *J. Heat Transfer* **99**, 541–546 (1977).
21. T. Tzaferis, D. Liparakis, M. Santamouris, and A. Argiriou, Analysis of the accuracy and sensitivity of eight models to predict the performance of earth-to-air heat exchangers, *Energy and Buildings* **18**, 35–43 (1992).
22. TRNSYS 13.1, Atransient system simulation program developed from Solar Energy laboratory, University of Wisconsin-Madison, Madison, WI (1990).