Abstract The paper is a study of the effect of air infiltration on temperature and water vapour pressure distributions within a porous layer. A theoretical formula describing these is proposed. The paper concludes with some new recommendations to the designer.

## Moisture within a porous layer in a building envelope

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#### List of symbols

- t Temperature (°C)
- e Water vapour pressure (Pa)
- $\lambda$  Thermal conductivity (W m<sup>-1</sup>K<sup>-1</sup>)
- $\mu_{\rm m}$  Vapour permeability (g m<sup>-1</sup>h<sup>-1</sup>Pa<sup>-1</sup>)
- $c_p$  Specific heat capacity at constant pressure (W h kg<sup>-1</sup>K<sup>-1</sup>);  $c_p = 0.28$  W h kg<sup>-1</sup>K<sup>-1</sup>
- $c_{\rm m}$  Specific moisture content of air;  $c_{\rm m} = 0.621$
- p Air pressure (Pa)
- $q_{a}$  Rate of air infiltration (g m<sup>-2</sup> h<sup>-1</sup>)
- k Air infiltration coefficient (g  $m^{-1}h^{-1}Pa^{-1}$ )
- $\rho_e$  Vapour densit y (kg m<sup>-3</sup>)
- $\gamma_{\rm w}$  Density of liquid water (kg m<sup>-3</sup>);  $g_{\rm w} = 1000$  kg m<sup>-3</sup>
- $\gamma_{\rm d}$  Dry material density (kg m<sup>-3</sup>)
- $\mu$  Molecular mass of water (g mol<sup>-1</sup>);  $\mu = 18$  g mol<sup>-1</sup>
- R Universal gas constant (J mol<sup>-1</sup>K<sup>-1</sup>); R = 8.3144 J mol<sup>-1</sup>K<sup>-1</sup>
- T Thermodynamic temperature (K)
- $\varphi$  Relative humidity of air
- $\rho_{\rm g}$  Saturation vapour density (kg m<sup>-3</sup>)
- *u* Moisture content (%)

#### 1 Introduction

In China, as elsewhere, moisture causes problems in building envelopes, most seriously in cold storage facilities. This can eventually lead to increased energy loss, surface condensation and possibly even irreversible structural damage.

Remedial measures for these problems are based on experience and judgement. Appropriate measures can be devised once the moisture sources and their associated transport mechanisms have been established. The usual measures are based on the theory of water vapour diffusion. Moisture-proof building envelopes are designed with a water vapour barrier at the side with the higher temperature and higher water vapour pressure<sup>(1)</sup>. However this does not been completely control the moisture within the porous layer, because there is a complex interaction between physical processes. The fundamental physical processes are: water vapour diffusion, air infiltration, phase changes and opposite movement of the two water phases. These physical processes are in dynamic equilibrium within a porous layer<sup>(2,3)</sup>.

Given a steady state, this paper addresses the effect of air infiltration on the temperature and the water vapour pressure distributions within a porous layer. Finally some new recommendations are made for the designer.

# 2 Effect of air infiltration on temperature and water vapour pressure

Under the steady state in one dimension, the thermal and moisture balance equations for a porous material layer with air infiltration are described in Reference 2 as follows:

$$d^2t/dx^2 + (c_p q_q/\lambda) dt/dx = 0$$
(1)

$$\mu_{\rm m} \, {\rm d}^2 e/{\rm d}x^2 - c_{\rm m} q_{\rm a} ({\rm d}/{\rm d}x)(e/p) = 0 \tag{2}$$

where

$$q_{a} = -k \, \mathrm{d}p/\mathrm{d}x = k(p_{0} - pd)/d \tag{3}$$

Equations 1 and 2 are easily solved as follows:

$$t = t_0 + (t_d - t_0)(1 - \exp(c_p q_a x/\lambda)/(1 - \exp(c_p q_a d/\lambda))$$
(4)

$$= \{(e_{0}p_{d}/p_{0} - e_{d})(p/p_{0})^{-kc_{m}}/\mu_{m} + [e_{d} - e_{0}(p_{d}/p_{0})^{-kc_{m}}/\mu_{m}]p/p_{0}\} \\ \div [p_{d}/p_{0} - (p_{d}/p_{0})^{-kc_{m}}/\mu_{m}]$$
(5)

When  $q_a \rightarrow 0$ , t and e will tend to :

$$t = t_0 + (t_d - t_0)x/d$$
 (6)

$$e = e_0 + (e_d - e_0)x/d$$
(7)

which are the temperature and water vapour pressure distributions often used by designers.

Analysing the solutions (4) and (5), it can be concluded that the effect of air infiltration on the temperature is similar to that on the water vapour pressure.

#### 2 Fundamental principle

In a porous material the moisture content at any point is the sum of three parts: water vapour (gas), water (liquid) and ice (solid), that is

$$u = u_1 + u_2 + u_3 \tag{8}$$

where  $u_1, u_2, u_3$  are the moisture content, the vapour content, the water content and the ice content respectively. Above the ice point equation 8 becomes

$$u = u_1 + u_2 \tag{9}$$

or

$$u_1/u + u_2/u = 1$$
 (10)

which holds as the law of mass transfer at any point in the porous material. Taking the gradien of equation 10:

$$\nabla(u_1/u) + \nabla(u_2/u) = 0 \tag{11}$$

Chen Qigao and Tang Mingfang

$$\nabla(u_{n}/u) = -\nabla(u_{n}/u) \tag{12}$$

which shows that the gradients of the vapour and liquid components are equal in magnitude but opposite in direction. The vapour in the material moves towards the lower temperature and the liquid to the higher ('moisture self-balance').

In the steady state in one dimension, we have the differential equation:

$$\nabla(u_1/u) = -\nabla(u_2/u) = f(x) \tag{13}$$

The solutions of which are:

$$u_1/u = (u_1/u)_0 + [(u_1/u)_d - (u_1/u)_0]F(x)$$
(14)

1 - 12

$$u_2/u = (u_2/u)_0 - [(u_1/u)_d - (u_1/u)_0]F(x)$$
(15)

where

$$F(x) = \frac{\int_{0}^{x} f(x) dx}{\int_{0}^{d} f(x) dx}$$
(16)

Obviously F(x) can be defined through f(x). We can choose f(x) empirically. For example, as choosing f(x) = 1 or f(x) = x, the distributions of the two-phase moisture ratios are linear or parobolic respectively.

#### 3 Moisture content formula

Suppose a porous material layer does not expand when it absorbs moisture. Then the vapour moisture  $u_1$ , liquid content  $u_2$  and porosity PO are related through:

$$u_1/\rho_e + u_2/\gamma_w = PO/\gamma_d \tag{17}$$

To obtain u, a plausible form of  $u_1/u$  is assumed, even if it deviates from the actual solution. For the given problem, the simplest form of  $u_1/u$  that can be assumed is linear in x, i.e.

$$u_1/u = g(x) = A + Bx \tag{18}$$

From equations 8 and 16 the following is obtained:

$$u = \operatorname{PO}\gamma_{\rm w}/\gamma_{\rm d}(1 + g(x)(\gamma_{\rm w}/\rho_{\rm e} - 1))$$
<sup>(19)</sup>

Because the equation of state of an ideal gas is:

$$\rho_{e} = \mu e/RT \tag{20}$$

equation 19 can be written as:

$$u = PO\gamma_u / \gamma_d (1 + g(x)(\gamma_u RT/\mu e - 1))$$
(21)

Substituting temperature T, water vapour pressure e and vapour moisture ratio g(x) into equation 21, we can obtain the moisture distribution function of the porous layer.

Substituting 
$$\rho_{\rm e} = \varphi \rho_{\rm E}$$

into equation 19 we obtain:

$$u = (PO\gamma_w r_E / \gamma_d) \varphi / [g(x)\gamma_w - \rho_E(g(x) - 1)\varphi]$$
(22)

Equation 22 is similar to the empirical formula 23 for moisture balance in an isothermal process:

$$u = a\varphi/(b - \varphi) \tag{23}$$

#### 4 Example

In the summer of 1983 a field study was made of the Tian Jin fruit cold storage facility. Under the roof there was a 0.6 m thick rice hull layer in an insulation attic. The layer was on the ceiling and exposed to the air at one side. The measured moisture content within the layer and the surface temperature on both sides of the layer are shown in Table 1.

| $\frac{\text{Table 1}}{x(m)}$ | Measured data |     |     |     |     |     |      |  |
|-------------------------------|---------------|-----|-----|-----|-----|-----|------|--|
|                               | 0.0           | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6  |  |
| u (%)                         | 65            | 13  | 12  | 12  | 11  | 9   | 9    |  |
| t(°C                          | 3.6           |     |     |     |     |     | 31.3 |  |

Because the measured data are insufficient to calculate the moisture content within the rice hull layer by using equation 21, we assume that the temperature and water vapour pressure within the rice hull layer are linear in x. The water vapour pressure on both sides of the layer is calculated by assuming relative humidities of 100% and 80% respectively. The vapour moisture ratio on each side of the layer is calculated by using the respective measured moisture contents.

Figure 1 shows the contrast between the two results. The moisture content calculated from formula 21 approximates to the measured results.



#### 5 Conclusion

- (a) Under air infiltration the temperature and water vapour pressure within a porous layer are not linear in x, where x is the normal coordinate. When air infiltrates from the higher temperature and higher water vapour pressure side to the lower temperature and lower water vapour pressure side, the temperature and water vapour pressure within the layer are slightly higher than those without air infiltration, otherwise they are slightly lower.
- (b) Moisture phases move in opposite directions. Vapour moves from the higher temperature side to the lower temperature side driven by the vapour pressure gradient, and liquid moves from the lower temperature side to the higher temperature side by capillarity. Thus a water vapour barrier at the higher temperature side is not enough to keep the insulation dry, because the liquid can move into the layer from the lower temperature side. The optimum moisture control measure is an air gap at the

Building Services Engineering Research and Technology

lower temperature side and an air barrier at the higher temperature side. These can prevent moisture transfer into the layer from both sides.

(c) According to formula 21, the moisture content depends on the temperature, water vapour pressure, vapour moisture ratio and the coefficients describing the propertis of the material.

### References

- 1 Chen Qigao The theory on heat and mass transfer calculation and design for building envelopes (Chongqing: Institute of Architecture and Engineering Press) (1990)
- 2 Chen Qigao The fundamentals of thermophysics for buildings (Xian Jiaotong University Press) (1991)