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# Sound Attenuation in Long Enclosures

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*An intensive review indicates that among the existing formulae on the sound attenuation in long enclosures, only the geometrical reflection model seems relatively practical. Computations with this model show the following for rectangular long enclosures: with a larger cross-sectional size the relative attenuation from a given section is less but the absolute attenuation with reference to the source power is greater; the efficiency of absorbers is higher when there is less absorption; and to obtain a higher attenuation, the absorbers should be evenly arranged in a section. In conclusion, it is still necessary to develop a more practical prediction method. Copyright © 1996 Elsevier Science Ltd.*

## NOMENCLATURE

| | |
|---|---|
| $a$ | cross-sectional height |
| $a_0$ | radius |
| $a_1$ | reference distance, 1 m |
| $a_{ghq}$ | distance between the receiver and the image source $(g,h,q)$ |
| $b$ | cross-sectional width |
| $B$ | attenuation coefficient, 0.14 |
| $c$ | sound speed in air |
| $c_b$ | sound speed in a boundary |
| CCSD | spatial complex cross-spectral density |
| $d$ | distance from source to the end wall |
| $d_b$ | boundary thickness |
| $D$ | sound attenuation ratio, dB/m |
| $E_R$ | sound energy density from the image sources |
| $f$ | frequency |
| $g,h,q$ | numbers of reflections on the ceiling, floor and walls |
| $i$ | boundary number, 1–4 |
| $I(\theta)$ | sound energy intensity in the direction $\theta$ |
| $I(v,w)$ | plane-wave weighting function |
| $I(x,y,z)$ | sound energy intensity at the receiver $(x,y,z)$ |
| $k$ | wave number vectors |
| $K$ | coefficient in relation to $\bar{\alpha}$ |
| $l_1$ | width of the boundary $i$ |
| $l_2$ | width of the boundary which is vertical to the boundary $i$ |
| $l_3$ | distance between the receiver and the cross-section with $P_{IN}$ |
| $l$ | doorway height |
| $L$ | tunnel length |
| $m,n$ | number of reflections (order of image source) |
| $P$ | sound power of the source |
| $P'$ | sound power per unit length of the line source |
| $P_{IN}$ | input power |
| $P_{SO}$ | power flow at the receiver |
| $P_{SO,i}$ | power flow at the receiver caused by the boundary $i$ |
| PWL | sound power level |
| $r$ | $= \mathbf{x} - \mathbf{x}'$, $\mathbf{x}$ and $\mathbf{x}'$ are position vectors |
| $R$ | reflection coefficient of the end wall |
| $S$ | cross-sectional area |
| SPL | sound pressure level |
| $SPL_z$ | SPL at the receiver with a distance of $z$ from the source |
| $SPL_{ref}$ | reference SPL |
| $u$ | overall sound energy density |
| $U$ | cross-sectional perimeter |
| $w,v$ | azimuthal and polar spherical angles |
| $W(v,w), W(v)$ | modal power function |
| $x_0$ | horizontal distance between the receiver and a tunnel end |
| $z$ | source–receiver distance |
| $y$ | $= \alpha_i\, l_3/2l_2$ |
| $\alpha_c,\alpha_f,\alpha_w$ | absorption coefficients of the ceiling, floor and walls |
| $\alpha_i$ | absorption coefficient of the boundary $i$ |
| $\bar{\alpha}$ | mean absorption coefficient |
| $\alpha_n$ | normal absorption coefficient |
| $\chi(\xi)$ | $= \xi K_1(\xi)$, $K_1$ is the modified Hankel function |
| $\gamma$ | rate of the attenuation along a corridor outside the direct field, $1.4U\bar{\alpha}/S$ |
| $\lambda$ | wavelength |
| $\theta$ | angle between the boundary normal and a reflection |
| $\rho$ | mean reflection coefficient |
| $\rho_\theta$ | angle dependent reflection coefficient |
| $\rho_0$ | density of air |
| $\rho_b$ | density of a boundary |
| $\rho_c,\rho_f,\rho_{w1},\rho_{w2}$ | reflection coefficients of the ceiling, floor and walls |
| si($y$), ci($y$) | $-\int_y^\infty (\sin t/t)\,dt$, $-\int_y^\infty (\cos t/t)\,dt$ |

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## INTRODUCTION

THE subject of this paper is concerned with long enclosures, such as underground stations, corridors, street tunnels and so on, where one dimension is much greater than the other two. It is noted that the other two are still relatively large compared to the acoustical wavelength.

The sound attenuation in long enclosures has been investigated for several decades. An outstanding feature of long enclosures is that the sound field is not diffuse and thus the classic room acoustical theory is not applicable [1]. Based on various assumptions and by using various methods, a number of formulae have been established.

In this paper the investigations on the sound attenuation in long enclosures are intensively reviewed and the usefulness of various formulae is compared. Through computation with some of the formulae, the basic characteristics of the sound attenuation in long enclosures are systematically analysed.

## REVIEW

### *Geometrical reflection*

As a typical case, geometrical reflection is assumed in most of the existing formulae. Diffusion is ignored, and the absorption coefficient is considered to be independent of the incident angle.

*Image source method.* The image source method is the most conventional way to treat the geometrical reflection. By using this method the sound energy density in an infinite rectangular long enclosure with geometrically reflecting boundaries can easily be derived [2–4]. The simplest case is that all the boundaries have the same absorption coefficient, and a point source and a receiver are positioned at the centre of the cross-section

$$u=\frac{P}{4\pi c}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{\rho^{|m|+|n|}}{(ma)^2+(nb)^2+z^2} \tag{1}$$

$$u\approx\frac{P}{4\pi cz^2}\left[1+\frac{4\rho}{(1-\rho)^2}\right](z\gg a,b). \tag{2}$$

The calculation using equation (1), however, is somewhat complicated. Hence Kuno *et al.* [5] developed a method to simplify the summation in equation (1)

$$I(x,y,z)=-\frac{P}{2S}[\cos a_2z\mathrm{ci}(a_2z)+\sin a_2z\mathrm{si}(a_2z)], \tag{3}$$

where the source and receiver were at $(x_0,y_0,0)$ and $(x,y,z)$ respectively, and

$$a_2=\frac{U}{\pi S}\lg\rho.$$

Computations showed that the results of equations (1) and (3) were quite similar. The differences of the two equations were within $\pm 1$ dB in the near field and became less with the increase of source–receiver distance. Equation (3) was validated by the measurements in a corridor of 2 m by 2.6 m by 57 m with an accuracy of around $\pm 1.5$ dB, where the measurement frequencies were from 500 to 4 kHz in octave. The measurements, however, were limited within a relatively low boundary absorption, 0.04–0.07.

Both equations (1) and (3) are unable to consider the difference of absorption coefficient between various boundaries, and thus the results could be inaccurate when this difference is great, such as a hard-walled corridor with a strongly absorbent ceiling. Yamamoto [6], therefore, deduced a formula to consider the difference of absorption between the four boundaries in infinite rectangular long enclosures, that is

$$\begin{aligned}\mathrm{SPL}=\mathrm{PWL}-11-10\lg\Bigg[&\frac{1}{z^2}\\
&+\sum_{m=0}\sum_{n=0}\frac{(\rho_f\rho_c)^n(\rho_{w1}\rho_{w2})^m}{z^2+(2n+1)^2a^2+(2m+1)^2b^2}\\
&\times(\rho_c\rho_{w1}+\rho_f\rho_{w1}+\rho_c\rho_{w2}+\rho_f\rho_{w2})\\
&+\sum_{m=0}\sum_{n=1}2\frac{(\rho_f\rho_c)^n(\rho_{w1}\rho_{w2})^m}{z^2+(2n)^2a^2+(2m+1)2b^2}(\rho_{w1}+\rho_{w2})\\
&+\sum_{m=1}\sum_{n=0}2\frac{(\rho_f\rho_c)^n(\rho_{w1}\rho_{w2})^m}{z^2+(2n+1)^2a^2+(2m)^2b^2}(\rho_f+\rho_c)\\
&+\sum_{m=1}\sum_{n=1}4\frac{(\rho_f\rho_c)^n(\rho_{w1}\rho_{w2})^m}{z^2+(2n)^2a^2+(2m)^2b^2}\\
&+\sum_{n=1}2\frac{(\rho_f\rho_c)^n}{z^2+(2n)^2a^2}\\
&+\sum_{n=0}\frac{(\rho_f\rho_c)^n}{z^2+(2n+1)^2a^2}(\rho_f+\rho_c)\\
&+\sum_{m=1}2\frac{(\rho_{w1}\rho_{w2})^m}{z^2+(2m)^2b^2}\\
&+\sum_{m=0}\frac{(\rho_{w1}\rho_{w2})^m}{z^2+(2m+1)^2b^2}(\rho_{w1}+\rho_{w2})\Bigg].\end{aligned} \tag{4}$$

Corresponding to equation (1), equation (4) was also deduced by assuming geometrical reflection and using the image source method. The source and receiver were still at the centre of the cross-section. Consequently, the results of both the equations will be the same if all the boundaries have a uniform absorption coefficient.

The calculations using equation (4) showed good agreement with the measurements in a corridor of cross-section 1.76 m by 2.74 m, and absorption coefficients $\alpha_c=0.34\ldots0.63$, $\alpha_w=0.31\ldots0.74$ and $\alpha_f=0.03$. The accuracy was within about $\pm 2$ dB from 150 to 4.8 kHz. This indicated that equation (4) could be valid until a quite low frequency. Moreover, the above results demonstrated that equation (4) could also give an effective prediction even if the boundary absorption was relatively high. Unfortunately, the measurements were made with a maximum source–receiver distance of 18 m, and thus the conclusion was limited.

This limitation, in fact, has been noticed by Redmore [7]. Based on the assumption of the geometrical reflection above, Redmore used a computer model with the ray image theory to predict the sound attenuation in rectangular corridors, where two absorption coefficients, namely the average of ceiling and floor, and the average of two side walls, were considered. This model, similar to equations (1) and (4), gave good agreement with both site and scale model measurements in hard-walled corridors. However, it tended to overestimate the sound levels for corridors containing more highly absorbent material on the floor and ceiling as the distance from the source increased. This appears to be an important extension of Yamamoto's above conclusion, as in their measurements the cross-section and absorption materials (non-resonant absorbers) were similar but Redmore's corridor was around 20 m longer. A possible explanation for this overestimation could be that for highly absorbent boundaries the exclusion of diffusion and angle dependent absorption was unreasonable. In other words, the assumption of the geometrical reflection might be inapplicable in this case.

The angle dependence of the absorption coefficient was considered by Sergeev [8, 9] when he derived a series of formulae for long enclosures in a similar manner as the above. This consideration, however, was limited to relatively hard boundaries:

$$\rho_\theta = e^{-\psi/\theta} \quad (\pi/3 \le \theta \le \pi/2)$$
$$= \text{const} \quad (0 \le \theta \le \pi/3), \tag{5}$$

where

$\psi = 4\text{Re}(1/p)$ (boundary with infinite thickness)

$= -4|p|^2\text{Im}[p \operatorname{ctg}(2\pi\sqrt{N^2-1}d_b/\lambda)]$ (boundary with finite thickness)

$$p = \frac{M}{\sqrt{N^2-1}}$$

$M = \rho_b/\rho_0$

$N = c/c_b$.

Unfortunately, no experimental result was given to demonstrate that the above consideration of the angle dependent absorption could enable better predictions.

*Wave theory*. In contrast to the specular, single plane wave reflection theory of geometrical acoustics as used above, Davies [10], in a manner of plane wave decomposition, derived a series of formulae to estimate the sound attenuation in rectangular corridors. The estimates, however, were still based on following the propagation of plane waves on a form of geometrical acoustics. The analyses were at high-frequency, at which the modal summations could be replaced by suitable integrals. Attention was limited to the total acoustic power flow, and not to the details of the cross-sectional variations of the sound pressure field.

The diffraction was ignored. In other words, the uniform impedance condition was assumed. As the diffraction effects might accumulate after several reflections, only two cases were considered: either highly absorbent materials, where all the energy of a plane wave was absorbed effectively after two reflections, or relatively hard boundary materials, where the true sound field involved only a small perturbation of the rigid boundary case.

Two idealised sound sources were assumed. One of them was the equal energy source with which the propagating modes had equal energy. This could be described as the sound field resulting from a noise source in a reverberate room at the input end of the corridor. The other source was a simple source with which there was relatively more energy in the higher order modes.

Based on the above assumptions, the formulae for calculating the receiver/input energy ratio were given

$$\frac{P_{SO}}{P_{IN}} = \left(1+\sum_{i=1}^{4}\frac{P_{ABS,i}}{P_{IN}-P_{ABS,i}}\right)^{-1}, \tag{6}$$

where

$$P_{ABS,i} = P_{IN} - P_{SO,i}. \tag{7}$$

For the low absorbent boundaries, the simple source and large values of $l_3/l_1$

$$\frac{P_{SO,i}}{P_{IN}} = \frac{2}{\pi}[\text{ci}(y)\sin y - \text{si}(y)\cos y]. \tag{8}$$

For the low absorbent boundaries, the equal energy source and large values of $l_3/l_1$

$$\frac{P_{SO,i}}{P_{IN}} = -\frac{2}{\pi}[\text{ci}(y)(y\cos y - \sin y) + \text{si}(y)(y\sin y + \cos y)]. \tag{9}$$

For the high absorbent boundaries and the simple source

$$\frac{P_{SO,i}}{P_{IN}} = 1 - 2\alpha_i\left(1-\frac{2}{\pi}\tan^{-1}\frac{4l_2}{l_3}\right) + \alpha_i^2 \left(1+\frac{2}{\pi}\tan^{-1}\frac{2l_2}{l_3} - \frac{4}{\pi}\tan^{-1}\frac{4l_2}{l_3}\right) - \frac{\alpha_i l_3}{4l_1}(1-\alpha_i)\ln\left(\frac{l_3^2}{16l_2^2+l_3^2}\right) - \frac{\alpha_i^2 l_3}{4l_1}\ln\left(\frac{l_3^2}{4l_2^2+l_3^2}\right). \tag{10}$$

For the high absorbent boundaries and the equal energy source

$$\frac{P_{SO,i}}{P_{IN}} = 1 - 2\alpha_i\left(1-\frac{2}{\pi}\tan^{-1}\frac{4l_2}{l_3}\right) + \alpha_i^2 \left(1+\frac{2}{\pi}\tan^{-1}\frac{2l_2}{l_3} - \frac{4}{\pi}\tan^{-1}\frac{4l_2}{l_3}\right). \tag{11}$$

When $\alpha_i = 1$, equation (11) can be used to calculate the sound attenuation in open corridors or the corridors with doorways

$$\frac{P_{SO,i}}{P_{IN}} = 1 - \left(1-\frac{2}{\pi}\tan^{-1}\frac{2l_2}{l_3}\right)\frac{l}{l_1}. \tag{12}$$

The theoretical estimates were compared to the measurements in two corridors in a 1/3 octave band centred at 2 kHz. The equal energy source was simulated by a loudspeaker placed in a large, hard-walled stairwell at one end of a corridor. The simple source was simulated by a loudspeaker placed at a corner of the other corridor. The agreement between calculations and measurements, however, was not as good as that of equations (1)–(4). The calculations tended to underestimate the actual attenuation. Possibly this was because the assumptions of Davies' method were too strict to be practically achieved, even for a designed measurement.

*Line source*. Corresponding to the above investigations on a single source, line sources have also been investigated. Similar to equation (1), Kuttruff [2] gave a theoretical formula to calculate the sound energy density in infinite long enclosures with a line source along the width and in the middle of the cross-section, where the geometrical reflection was also assumed and the receiver was at the centre of the cross-section

$$u = \frac{P}{4c}\sum_{n=-\infty}^{\infty}\frac{\rho^{|n|}}{\sqrt{(na)^2+z^2}} \tag{13}$$

$$u \approx \frac{P}{4cz}\left(1+\frac{2\rho}{1-\rho}\right)(z \gg a). \tag{14}$$

Different from Kuttruff, Said [11, 12] derived a formula to consider the different absorption coefficients for the floor, ceiling and walls in rectangular street tunnels with a line source along the centre of the cross-section and with the same length as the tunnel, where geometrical reflection was still assumed

$$E_R = \frac{P'}{4a_1c}\sum_{m,n,q}\frac{(1-\alpha_c)^g(1-\alpha_f)^h(1-\alpha_w)^q a_1}{\pi a_{ghq}} \times\left(\arctan\frac{x_0}{a_{ghq}}+\arctan\frac{L-x_0}{a_{ghq}}\right). \quad (15)$$

To simplify equation (15), Said developed a statistical method, where the average absorption coefficient of all the boundaries was used

$$E_R = \frac{-P'\pi}{2cU\ln(1-\bar{\alpha})}\left\{2-\exp^2\left[\frac{\ln(1-\bar{\alpha})\pi S x_0}{U}\right] -\exp^2\left[\frac{\ln(1-\bar{\alpha})\pi S(L-x_0)}{U}\right]\right\}. \quad (16)$$

It was demonstrated that equation (16) was valid for $\bar{\alpha} < 0.4$. In addition, calculations in a tunnel (height: 5 m; width: 15 m; length: 100 ... 500 m) showed that the sound absorption materials on the ceiling were very effective for noise reduction. With an absorbent ceiling the effect of the absorbent walls was not significant. Said, however, did not validate his theory by measurement.

*Consideration of diffusion*

As mentioned above, the assumption of geometrical reflection could be unreasonable when the boundaries are highly absorbent. Moreover, acoustically hard but rough boundaries often exist in long enclosures. Hence it is necessary to consider the diffuse reflection.

Yamamoto [6] gave a theoretical formula for the semi-diffuse field, namely, that the sound density was the same in a cross-section

$$\mathrm{SPL}_z = \mathrm{SPL}_{z=0} - 2.17\frac{\bar{\alpha}U}{S}z. \quad (17)$$

Calculations showed that in this case the sound attenuation was greater than in the sound field formed by the geometrically reflecting boundaries assumed above. However, in comparison with the measurements for validating equation (4), equation (17) was far from accurate. Possibly this was because the assumption of the semi-diffuse field was not applicable in this corridor.

Ollendorff [13] developed a theoretical method to calculate the noise level and reverberation time in rectangular street tunnels by using the partial differential equation of the diffusion of phonons. As an example, calculations were made for a street tunnel with a section of 12 m by 4 m, where it was assumed that the floor was totally reflective and the other boundaries were totally absorbent. Ollendorff's method, however, seems too complicated to be used practically.

Kuttruff [2] proved theoretically that with diffusely reflecting boundaries the sound attenuation along the length was greater than that of the geometrically reflecting boundaries. It was explained that with diffusely reflecting boundaries the sound rays had more chances of impinging upon the boundaries. According to the Lambert cosine diffuse rule, $I(\theta) \sim \cos\theta$, two formulae for calculating the sound energy density at the centre of the cross-section were given, where the angle independent absorption coefficient was still assumed.

(1) For a point source at the centre of a circle cross-section

$$u = \frac{P}{4\pi cz^2} + \frac{2\rho P}{\pi^2 a_0^2 c}\int_0^\infty \frac{[\chi(\xi)]^2\cos\left(\xi\frac{z}{a_0}\right)}{1-\rho\lambda(\xi)}\mathrm{d}\xi, \quad (18)$$

where

$$\lambda(\xi) \approx \frac{1}{1+\frac{4}{3}\xi^2}. \quad (19)$$

(2) For a line source along the width and in the middle of a rectangular cross-section with geometrically reflecting walls and diffusely reflecting ceiling and floor

$$u = \frac{P}{4cz} + \frac{2\rho P}{\pi ac}\int_0^\infty \frac{e^{-\xi}\cos\left(\xi\frac{z}{a}\right)}{1-\rho\chi(\xi)}\mathrm{d}\xi. \quad (20)$$

Kuttruff's work is of great theoretical importance. However, due to the assumption of the simple diffusion and the lack of solutions for more general cases, the formulae seem less practical.

Leschnik [14] investigated the sound distribution in rectangular street tunnels by using a computer model with the Monte-Carlo method. The vehicles acted as sound sources as well as diffusers. It was found that the sound attenuation along the tunnel with a single source was greater when there were more vehicles in the tunnel. The computer model was validated by measurements in a 1 : 20 scale model (5 m × 6.5 m × 40 m, full-size) and site measurements in two tunnels (4.5 m × 6 m × 45 m and 5 m × 10 m × 1000 m). The importance of Leschnik's work was that the effectiveness of diffusers on the sound attenuation in long enclosures was experimentally demonstrated. However, in his model, the tunnel boundaries were still geometrically reflective.

Consequently, the effect of diffusely reflecting boundaries on the sound attenuation in long enclosures was experimentally demonstrated by Kang [15]. The experiments were carried out in a 1 : 16 scale model of an underground station in London. It was found that with ribbed diffusers the sound attenuation could be significantly increased. For example, with a source–receiver distance of 50 m, the attenuation became 22 dB from 16 dB.

*An empirical formula*

Redmore [16] established an empirical formula by performing a series of tests in a 1 : 8 scale model (height: 1.6 ... 3.2 m; width: 1.28 ... 2.48 m; length: 18.4 ... 36.8 m, full size). The source was a loudspeaker positioned behind a small hole on an end wall. The formulae were as follows:

$$\mathrm{SPL}_z = 10\lg\rho_0 c + 10\lg P + 10\lg \times\left(\frac{1}{2\pi z^2}+\frac{B}{U\bar{\alpha}}10^{-\gamma z/10}\right) - \mathrm{SPL}_{\mathrm{ref}} \quad (21)$$

$$\mathrm{SPL}_z = 10\lg\left\{\frac{1}{2\pi z^2} + \frac{B}{U\bar{\alpha}}10^{-\gamma z/10} + R\left[\frac{1}{2\pi(2d-z)^2} + \frac{B}{U\bar{\alpha}}10^{-\gamma(2d-z)/10}\right]\right\}. \quad (22)$$

Equation (21) was used for the absolute SPL and equation (22) was applied to at least two receivers to predict comparative levels outside the direct field. In equation (22) the effect of end walls was considered. The mean absolute difference between the predicted and measured values was 0.4 dB in the scale model and 1.4 dB in a corridor. However, Redmore's formulae, which were obtained from limited boundary and dimension conditions, are not necessarily applicable in a further range, although this was not clearly indicated by Redmore in his paper [16].

Redmore also demonstrated that in a side corridor the absorbent materials on the walls were more effective than on the ceiling or floor since the sound rays entering the side corridor were no longer omnidirectional.

*Duct theory*

The sound attenuation in ducts has been thoroughly investigated [17]. Although the dimension and boundary conditions are quite different, some of the theoretical principles for ducts are also useful for long enclosures, especially at low frequency. The following semi-empirical formulae for calculating the sound attenuation in ducts are most commonly used [18, 19]:

$$D = K\bar{\alpha}\frac{U}{S} \quad (23)$$

$$D = 1.5\alpha_n\frac{U}{S}. \quad (24)$$

By using equations (23) and (24) the attenuation along the length is linear, which is fundamentally different from the measurements in long enclosures [1–14]. Moreover, equations (23) and (24) are applicable at low frequency ($\lambda \gg a,b$) and become inaccurate as the frequency increases [18]. Furthermore, equations (23) and (24) are valid for porous materials, but limited for selective structures. By making a series of calculations at the resonant frequencies of the selective structures in a duct, Piazza [20] indicated that there was great difference between the above formulae and the more exact theoretical formulae given by Morse [21]. Unfortunately, Morse's formulae were only for ducts and not applicable for long enclosures.

Mechel [22] investigated the sound field in a duct with periodically arranged partition panels on an absorbent boundary. It was shown that the panels prevented the realisation of the maximum attenuation which was predicted by the theory for the homogeneous absorbent boundaries. With a comb-line, the periodicity of the boundary introduced new attenuation maxima and a fluctuating sound pressure along the duct.

Doak [23–25] derived a series of theoretical formulae for the sound field inside and outside hard-walled ducts. For a point source, Doak carried out some illustrative numerical calculations for a rectangular duct with five different source positions and the following modes: (0,0), (0,1), (1,0), (0,2), (1,1). It was demonstrated that the mean square pressure did not have any simple, universal relationship to the acoustic power.

Baxter and Morfey [26] established a method to determine the distribution of the power among the propagation modes in ducts. The sound attenuation in ducts depends on this distribution, among other factors. In other words, if this distribution is variable, a wide range of liner properties could produce the same attenuation. This method was based on a statistical description in which the power distribution was regarded as a continuous function of the modal coordinates.

The idea of this method can briefly be described as follows. In free-wave field, that is, the sound field made up of uncorrelated plane waves propagating in different directions, $I(v,w)$ can be estimated experimentally from the measurement of a finite number of CCSD, a typical second-order statistic of the field. Although the actual measurement process is very complicated, the basis of the technique seems simple

$$\mathrm{CCSD}(r) = \iint \mathrm{d}w\,\mathrm{d}v\,\sin v I(v,w)\exp(\sqrt{-1}kr). \quad (25)$$

In a rectangular hard-walled duct, if the frequency is high enough and the modal weighting smooth enough for the modal sums to be approximated by the integrals, and the modes are uncorrelated, then

$$I(v,w) = (-|k|^2\rho_0 c/2\pi^2)W(v,w). \quad (26)$$

In a hard-walled circular duct, if a further condition is satisfied, that is, the modal power distribution at any frequency is a function only of modal cut-off frequency, then

$$I(v) = (-|k|^2\rho_0 c/2\pi^2)W(v). \quad (27)$$

With equations (26) and (27) the modal power distribution can be obtained from $I(v,w)$. An experimental technique for assessing the modal power distribution in a duct (diameter: 0.6858 m; length: 2.032 m; cut-off frequency: 5 kHz) was developed.

In equations (9) and (11) it was assumed that the propagation modes carried equal energy. This can be described as

$$\begin{aligned} W(v,w) &= \text{const} \quad (0 \le v < \pi/2) \\ &= 0 \quad (\pi/2 < v \le \pi). \end{aligned} \quad (28)$$

*Comparison*

The formulae obtained by assuming the geometrical reflection and using the image source method [equations (1)–(4)] seem reasonable and practical. As there are many long enclosures with acoustically hard and smooth boundaries, these formulae have a considerable range of applicability. The corresponding formulae for line sources [equations (13)–(16)] should also be reasonable, although no comparison between measurements and calculations has been found.

Davies' formulae [equations (6)–(12)], obtained using the wave theory, are theoretically important. However, as the assumptions were too strict, and the comparison between measurements and predictions was not quite satisfactory, the practical application of this method seems limited.

Redmore's empirical formulae [equations (21) and

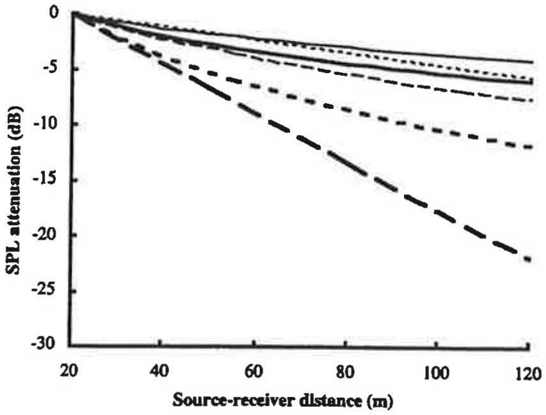


Fig. 1. SPL attenuation from 20 m calculated using equation (1) (dotted: $\bar{\alpha} = 0.05$; thick dotted: $\bar{\alpha} = 0.2$), equation (8) (line: $\bar{\alpha} = 0.05$; thick line: $\bar{\alpha} = 0.2$) and equation (22) (dashed: $\bar{\alpha} = 0.05$; thick dashed: $\bar{\alpha} = 0.2$). $S = 5$ m by 5 m.

(22)], obtained from limited experiments, might be questionable if extended to a further range. The limitation of this method is discussed in the following part of this paper.

There are obvious differences between the three methods above, although in principle they are comparative. Figure 1 shows the SPL attenuation from 20 m calculated using equations (1), (8) and (22) in the case of $\bar{\alpha} = 0.05$ and 0.2, where $S = 5$ m by 5 m. It can be seen that the attenuation calculated using equation (8) is less than that using equation (1), especially for $\bar{\alpha} = 0.2$. Possibly this is because in equation (8) the assumption of "hard boundary" tends to be unsatisfied with the increase of absorption coefficient. Conversely, the attenuation calculated using equation (22) is much greater than that using equation (1). This might be caused by the fact that the calculation range is outside the original range of equation (22).

The theoretical and experimental works on the effect of diffusers on the sound attenuation in long enclosures are of great significance in relation to the acoustic design of long enclosures. However, the corresponding formulae [equations (17)–(20)] seem less practical.

Duct theories are useful for the basic understanding of sound attenuation in long enclosures. However, due to the difference of the wavelength/dimension ratio, it seems unreasonable to directly apply duct theories in long enclosures.

In summary, up to now, the only method which seems practical is the simple geometrical reflection method, namely equations (1)–(4), although its range of applicability is limited. The other methods discussed are of theoretical significance and can be used for qualitative analyses.

## COMPUTATION

The objective of this part is to investigate the effect of some factors on the sound attenuation in long enclosures. The factors, which are considered to be important for the acoustic design of long enclosures, are cross-sectional size and form, and absorption amount and arrangement.

The calculations are carried out by assuming the geometrical reflection, point source and rectangular cross-section. Correspondingly, equations (1) and (4) are used. For comparison, Davies' method and Redmore's empirical formulae are also used. The diffusion methods are not chosen, as no formula has been found to be directly comparative with equations (1) and (4).

The range of sectional size and form is representative of actual tunnels, in particular some underground stations. It should be noted that most corridors also fall within this range. In principle, this range is within the applicable range of equations (1) and (4), as well as Davies' formulae. However, as indicated above, the calculation results with highly absorbent boundaries at long distances should be understood qualitatively rather than quantitatively. Conversely, this range is out of the original range of Redmore's empirical formulae and consequently, the unsuitability of Redmore's method in a further range is discussed.

There are two kinds of sound attenuation. One is the relative attenuation with reference to a given section and the other is the absolute attenuation with reference to PWL. In Davies' formulae, the former is in correspondence with the attenuation of the total energy in the whole section, and the latter the attenuation of the energy per unit area in a section.

*Cross-sectional size and form*

Figure 2 shows the attenuation of the total energy in the whole section [$10\lg(P_{SO}/P_{IN})$] at 120 m in six long enclosures with different sectional forms and sizes, where $\bar{\alpha} = 0.05$ and 0.2. The calculations are carried out by using equation (8) (the simple source). It can be seen that the larger the section is, the less the attenuation is. This is because the number of reflections in a given time is less for a larger section. Similarly, the calculations based on equations (1) and (22) (Fig. 3) show that the relative SPL attenuation from 20 to 120 m becomes less with increase of the sectional size, although the absolute values with the two formulae are quite different.

On the other hand, the attenuation of the energy per unit area in a section [$10\lg(9P_{SO}/SP_{IN})$] is greater for a larger section, which is also shown in Fig. 2, where the unit area is assumed to be 3 m by 3 m. This result seems closer to the subjective loudness. Similarly, the SPL attenuation with reference to PWL at 120 m calculated

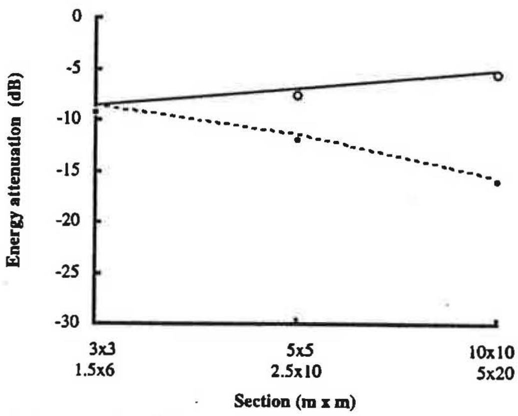


Fig. 2. Attenuation of the total energy in the whole section (line: $a = b$; circle: $a = 4b$) and the energy per unit area in a section (dotted: $a = b$; point: $a = 4b$). $\bar{\alpha} = 0.05$ and $l_3 = 120$ m. Calculated using equation (8).

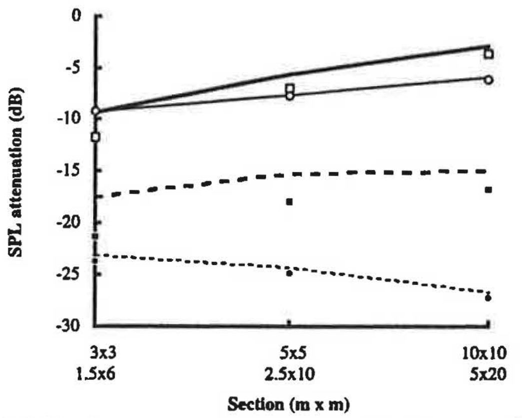


Fig. 3. SPL attenuation at 120 m from 20 m [thick line: equation (22), $a = b$; square: equation (22), $a = 4b$; line: equation (1), $a = b$; circle: equation (1), $a = 4b$] and with reference to PWL [thick dotted: equation (21), $a = b$; filled square: equation (21), $a = 4b$; dotted: equation (1), $a = b$; point: equation (1), $a = 4b$]. $\bar{\alpha} = 0.05$.

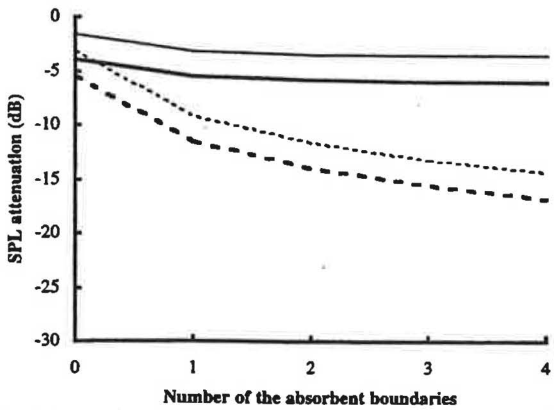


Fig. 5. SPL attenuation at 50 m (line: from 20 m; dotted: with reference to $P_{IN}$) and 120 m (thick line: from 20 m; thick dotted: with reference to $P_{IN}$) with different number of absorbent boundaries ($\alpha_i = 0.8$). $S = 3$ m by 3 m. Calculated using equations (9) and (11).

using equation (1) also increases with increasing sectional size (Fig. 3).

Conversely, by using equation (21) the SPL attenuation with reference to PWL at 120 m decreases with increasing sectional size, as shown in Fig. 3. This seems physically unreasonable, which could be one reason for using Redmore's empirical formulae in a limited range.

In summary, with a larger cross-sectional size the attenuation of the total energy in the whole section is less, but the attenuation of the energy per unit area in a section is greater. In other words, the relative attenuation from a given section is less but the absolute attenuation with reference to PWL is greater. In addition, Figs 2 and 3 show that with the same sectional area, when the width : height is 1 : 4, the sound attenuation is close to or slightly greater than that of the square section.

*Absorption amount*

Figure 4 shows the SPL attenuation (with reference to PWL and 20 m) at 50 m in a long enclosure with $\bar{\alpha} = 0.1 \ldots 0.5$ and $S = 3$ m by 3 m. By using equation (1) the attenuation, especially with reference to PWL, increases concavely with linear increase of the absorption coefficient. This phenomenon, which has also been noticed

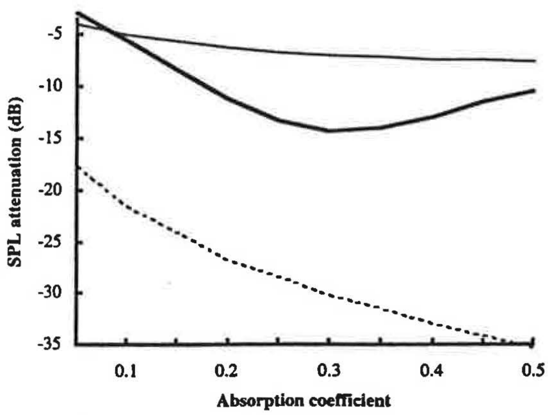


Fig. 4. SPL attenuation at 50 m with different absorption coefficients [line: equation (1), from 20 m; dotted: equation (1), with reference to PWL; thick line: equation (22), from 20 m]. $S = 3$ m by 3 m.

in normal enclosures [27], seems physically reasonable. Conversely, by using equation (22) the SPL attenuation from 20 m becomes less for a higher absorption coefficient when $\bar{\alpha} > 0.3$. This seems physically unreasonable, which might be a further reason for using Redmore's empirical formulae in a limited range.

Figure 5 shows the SPL attenuation (with reference to 20 m and $P_{IN}$) at 50 and 120 m in a long enclosure with one to four absorbent boundaries ($\alpha_i = 0.8$), again for $S = 3$ m by 3 m. For the sake of convenience, the calculations are according to equations (9) and (11) (the equal energy source). The absorption coefficient of the other boundaries is assumed as 0.05. Similar to Fig. 4, when the number of absorbent boundaries increases, the attenuation increases concavely, especially the attenuation with reference to $P_{IN}$.

In short, Figs 4 and 5 illustrate that the efficiency of absorbers is greater when there are less absorbers.

*Absorption arrangement*

Figure 6 shows the SPL attenuation with reference to PWL at 120 m in a long enclosure with the same amount of absorption but different absorber arrangements. The calculations are based on equation (4), which is the only formula available considering a wide range of absorption difference between four boundaries. It is seen that the attenuation is obviously higher when the absorbers are on three or four boundaries. A possible reason for this is that with more than one hard boundary it is possible for some reflections to reach the receiver without impinging upon the absorbent boundaries. For this reason, in the case of one or two absorbent boundaries, the attenuation is significantly greater when the absorption coefficient of the hard walls becomes 0.1 from 0.01. Conversely, with the same amount of total absorption, the attenuation of three or four absorbent boundaries is nearly the same.

Generally speaking, the absorbers are more effective when they are evenly distributed. However, it should be indicated that in Fig. 6 the absorption coefficients in some cases are quite high and the exclusion of diffusion might be unreasonable. If diffusion had been included in the calculation, the attenuation differences created by the placement of absorbers could be significantly less.

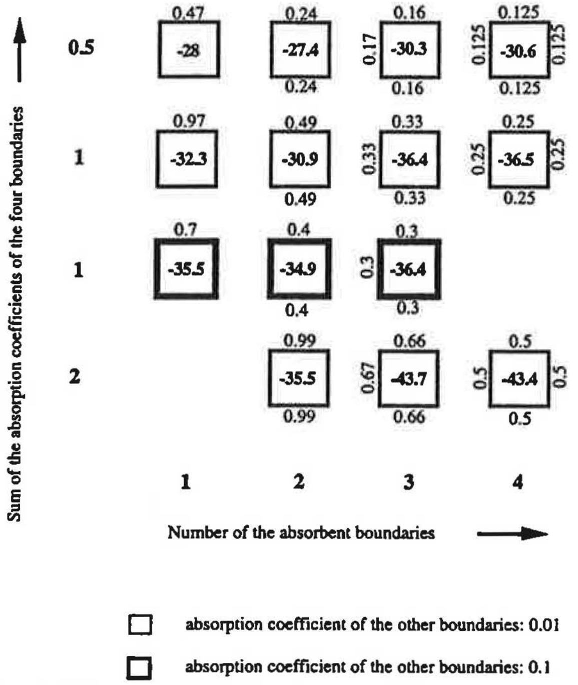


Fig. 6. SPL attenuation with reference to PWL at 120 m with different absorber arrangements. $S = 5$ m by 5 m. Calculated using equation (4).

## DISCUSSION

As indicated above, the application range of existing formulae for predicting the sound attenuation in long enclosures is limited by some assumptions, such as geometrical reflection or optimal diffusion. The sound attenuation in actual long enclosures is more complicated [28–30]. For example, using the above formulae it is difficult to predict the "plateau" phenomenon [7]; that is, at a point beyond the direct field in a corridor the SPL increases sharply by about 2.5 dB, and remains above the previous SPL for more than 8 m.

Therefore, from the viewpoint of acoustical design, a more practical method for the prediction of the sound attenuation in long enclosures is still necessary, where the angle dependent absorption coefficient and more practical diffusion should be considered. These two factors are of significant importance in long enclosures, because the boundaries are likely to have a higher diffusion coefficient than normal enclosures, and the sound propagation at a long distance may favour wave directions that are nearly tangential to the boundaries. Some further factors, such as the strategic arrangement of absorbers and diffusers along the length, the form of cross- and longitudinal section, the characteristic and position of sources, etc. should also be taken into account.

It is difficult to achieve the above requirements with a simple formula. Alternatively, computer modelling seems to be a better solution. As the basis of a computer model, the basic characteristics of sound behaviour in long enclosures should be investigated at first. For this purpose, scale modelling might be a useful tool. In addition, for some complicated cases, the results of computer modelling could be corrected by the empirical data obtained from scale model or site measurements.

## CONCLUSION

A number of formulae for the sound attenuation in long enclosures have been found from the literature. Among them, only the geometrical reflection model seems practical, although its range of applicability is limited to acoustically hard and smooth boundaries. The other formulae, especially those considering diffusion, are of theoretical importance, but they seem of a less practical nature than the geometrical reflection model.

Computations with some of the above formulae with the assumption of geometrical reflection, where diffusion and the angle dependent absorption coefficient are ignored, show that in rectangular long enclosures:

- with a larger cross-sectional size the relative attenuation from a given section is less but the absolute attenuation with reference to PWL is greater;
- with the same cross-sectional area, when the width/height ratio is 1 : 4, the sound attenuation is close to or slightly greater than that of the square section;
- the efficiency of absorbers is higher when there is less absorption;
- to obtain a higher attenuation, the absorbers should be evenly arranged in a section.

The discussion suggests that a more practical method for predicting the sound attenuation in long enclosures is still necessary. In conclusion, it would appear that the combination of computer and scale modelling is a good solution.

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