Simple error reduction in tracer-gas field-measurements of air handling units

Peter G. SCHILD[1]

[2] SINTEF Building & Infrastructure, Oslo, Norway. Peter.Schild@sintef.no

Abstract

Tracer gas measurements are an unparalleled means of measuring air recirculation, leakage, and air flow rates in air handling systems [1-5]. *However, such measurements are subject to significant measurement uncertainty in field conditions. A common problem is imperfect mixing of tracer gas.*

One method of error minimization is to use a constrained regression model based upon mass conservation equations. This technique is widely used in multizone tracer gas studies. The method reported in this paper uses robust regression (least trimmed weighted squares) to solve the mass flow rates in air handling units (AHU) for balanced ventilation, including any recirculation/leakage in the AHU, external short-circuiting, and ex-/infiltration caused by imbalance. The method is simple enough to implement in spreadsheet software, and works even when measurement data from one or more of the sampling points, or one of the two dosing points, has to be rejected as erroneous.

Keywords: Tracer gas, Measurement, Air flow, Leakage, Recirculation, Infiltration, Ventilation, Building

1. Introduction

1.1 Background for this study

The method presented in this paper was developed to analyse tracer gas data in an IAQ audit of AHUs in schools [6]. The measurement challenges included:

- Short ducts between dosing points and sampling points, in particular, fresh air ducts.
- Many ducts connected directly to an AHU via a plenum box, each with a different flow rate. This makes it difficult to achieve a homogeneous tracer concentration in the air flowing into the AHU using just one tracer gas injection pump.
- Uneven distribution of tracer gas in air leaving the AHU, due to incomplete mixing of recirculated air inside the AHU. If a plenum box distributes this air to multiple ducts, the ducts will have different concentrations of tracer gas. It is practically impossible to measure the volume-averaged concentration accurately with just one sampling tube and no knowledge of the flow rates in the different ducts connected to the plenum box.

The above challenges made it difficult to accurately estimate mass flow rates and recirculation fractions using conventional simple methods, as data from important tracer gas sampling points occasionally had to be rejected.

1.2 Error minimization in earlier studies

Some studies have focused on ways to maximize mixing of tracer gas at injection and sampling points in ducted ventilation systems [3],[10],[11]. Another means of error reduction is to use numerical techniques to estimate the mass flows more accurately, which is the main focus of this paper. Conventional tracer gas measurements in air handling units (AHU), for example described in [7],[8], use simple equations to estimate recirculation fractions, typically equations 1 & 3 below. Such calculations do not exploit the assumption of mass conservation of the whole system. Roulet *et al.* have analyzed errors for different ways of estimating AHU recirculation fractions [9]. In multizone tracer gas studies the air flows are generally resolved by matrix operations of mass balance equations; see Chapter III in [12]. Measurement uncertainties can lead to calculating fictitious negative flows or flows between non-connected zones, This problem has been solved with constrained least squares (enforcing positive flows) [12],[14],[15] or Bayesian methods [12],[13].

1.3 Definitions of recirculation fractions

The relations here are valid for measurements of AHUs with negligible casing leakage (Fig.1).

- Recirculation in the AHU of extract air to supply air:

$$R_{32,AHU} \equiv \frac{\dot{m}_{32,AHU}}{\dot{m}_2} = \frac{\dot{m}_{32,AHU}}{\dot{m}_{12} + \dot{m}_{32,AHU}} = \frac{c_2 - c_{1b}}{c_{3b} - c_{1b}} \qquad \text{(Eq. 1)}$$

- Total recirculation of extract air to supply air (i.e. combination of recirculation in AHU and short-circuiting outside the building, from exhaust to fresh air inlet):

$$R_{32,tot} \equiv \frac{\dot{m}_{32,tot}}{\dot{m}_2} = \frac{\dot{m}_{32,AHU} + \dot{m}_{32,out}}{\dot{m}_{12} + \dot{m}_{32,AHU}} = \frac{c_2 - c_o}{c_{3b} - c_o} \quad \text{(Eq. 2)}$$

- Recirculation in the AHU from fresh air intake to exhaust air outlet. This can be significant in AHUs with rotary heat exchanger with a purge sector.

$$R_{14,AHU} \equiv \frac{\dot{m}_{14,AHU}}{\dot{m}_4} = \frac{\dot{m}_{14,AHU}}{\dot{m}_{34} + \dot{m}_{14,AHU}} = \frac{c_4 - c_{3b}}{c_{1b} - c_{3b}} \quad \text{(Eq. 3)}$$

- Total short-circuiting outside, from exhaust air to fresh air intake:

$$R_{41,out} \equiv \frac{\dot{m}_{41}}{\dot{m}_1} = \frac{\dot{m}_{41}}{\dot{m}_{12} + \dot{m}_{14,AHU}} = \frac{c_{1a} - c_o}{c_4 - c_o} \quad \text{(Eq. 4)}$$

1.4 Conventional method of estimating mass flow rates in AHUs

The equations below can be used to estimate all the mass flow rates based upon the tracer gas concentrations and dosing rates (d_1, d_3), and the recirculation fractions calculated above. The same approach is used by Roulet & Vandaele (Section V.3 in [12]), and by Manz *et al.* [16], though both of these assume slightly different flow networks or fan locations, and dosing/sampling points compared to this study (Fig.1). The flow network for Fig.1 is given here:

$$\dot{m}_1 = \dot{d}_1 / (c_{1b} - c_{1a}) \quad \text{(Eq. 5)}$$

$$\dot{m}_3 = \dot{d}_3 / (c_{3b} - c_{3a}) \quad \text{(Eq. 6)}$$

$$\dot{m}_2 = \frac{\dot{m}_1 - R_{14,AHU}(\dot{m}_1 + \dot{m}_3)}{1 - R_{32,AHU} - R_{14,AHU}} \quad \text{(Eq. 7)}$$

$$\dot{m}_4 = \dot{m}_1 + \dot{m}_3 - \dot{m}_2 \quad \text{(Eq. 8)}$$

$$\dot{m}_{32,AHU} = R_{32,AHU}\ \dot{m}_2 \quad \text{(Eq. 9)}$$

$$\dot{m}_{12} = \dot{m}_2 - \dot{m}_{32,AHU} \quad \text{(Eq. 10)}$$

$$\dot{m}_{14,AHU} = R_{14,AHU}\ \dot{m}_4 \quad \text{(Eq. 11)}$$

$$\dot{m}_{34} = \dot{m}_4 - \dot{m}_{14,AHU} \quad \text{(Eq. 12)}$$

$$\dot{m}_{41} = R_{41,out}(\dot{m}_{12} + \dot{m}_{14,AHU}) \quad \text{(Eq. 13)}$$

- Net (useful) air exchange rate, corrected for internal recirculation inside AHU, i.e. independent of external short-circuiting:

$$\dot{m}_{net,AHU} = \max\{\dot{m}_{12}, \dot{m}_{34}\} \quad \text{(Eq. 14)}$$

- Net useful air exchange rate for the whole ventilation system (corrected for both internal recirculation and external short-circuiting):

$$\dot{m}_{net,syst} = \max\{\dot{m}_{12}, \dot{m}_{34}\} - \dot{m}_{32,out} \quad \text{(Eq. 15)}$$

- ...where $m_{32,out}$ is the part of the external short-circuiting (m_{41}) that leads extract air back to the supply air duct:

$$\dot{m}_{32,out} = \frac{\dot{m}_{41}}{1 + \frac{\dot{m}_{14,AHU}}{\dot{m}_{12}\,\dot{m}_{34}}\left(\dot{m}_{12} + \dot{m}_{34} - \dot{m}_{41} + \dot{m}_{14,AHU}\right)} \quad \text{(Eq. 16)}$$

- Resultant infiltration rate (exfiltration if negative) due to imbalance of flows in AHU.

$$\dot{m}_{\text{infil}} = \dot{m}_{34} - \dot{m}_{12} \quad \text{(Eq. 17)}$$

2. New method

2.1 Regression model description

The main objective of this study was to use constrained robust regression to provide a Maximum Likelihood estimate of the mass flows, and automatically omit suspect readings (outliers). The regression takes advantage of the fact that the system of mass conservation equations is over-determined, such that data from individual sampling or dosing points can be omitted if they are found to be too uncertain.

Fig.1 illustrates the flow paths in the studied ventilation systems.

2.2 Assumptions and preconditions

- AHU casing leakage is assumed negligible in relation to the recirculation flow rates.
- Duct leakage is assumed negligible in the ducts between the tracer gas dosing points and the AHU.

- Steady air flow rate (CAV). VAV systems should be manually overridden during the test.
- Perfect mixing of tracer gas (uniform concentration across over cross section) at the sampling points used in the regression analysis, and at points of entry into the AHU.
- It is assumed that all flow paths have a negligible time delay, except for the flow though the building from the supply duct to the extract duct. Quasi-steady concentration conditions are therefore assumed during each sampling interval at all sampling points.
- Because of the short measurement period, as mentioned above, we do not attempt to estimate the air exchange efficiency or total exchange rate in the building (i.e. resolve both flows Q_{03} and Q_{30} in [12]), however, infiltration/exfiltration caused by imbalance in the AHU (i.e. the difference between supply & exhaust flow) is calculated.

2.4 Algorithm for reconstructing concentrations and estimating flow rates

The equations below are used iteratively to find the optimum value for the five mass flows m_{12}, $m_{32,AHU}$, m_{34}, $m_{14,AHU}$, and m_{41} shown in Fig.1. . Equations 18-26 below estimate the correct concentrations at all measuring points (also the ones where we lack good measurements of the response variables). An iterative process finds the mass flows that result in the smallest deviation between estimated and measured values of tracer gas concentration

The explanatory variables (boundary conditions) are concentrations c_o (background) and c_{3a} (extract duct), and dosing rates d_1 and d_3, all of which must be measured accurately.

$$R_{41,out} = \frac{\dot{m}_{41}}{\dot{m}_{12} + \dot{m}_{14,AHU}} \quad \text{(Eq. 18)}$$

$$R_{14,AHU} = \frac{\dot{m}_{14,AHU}}{\dot{m}_{34} + \dot{m}_{14,AHU}} \quad \text{(Eq. 19)}$$

The following calculations are done for each logging interval (*i*) in the time series to estimate the likely tracer gas concentrations:

$$\hat{c}_{3b,i} = c_{3a,i} + \frac{\dot{d}_{3,i}}{\dot{m}_{34} + \dot{m}_{32,AHU}} \quad \text{where } \dot{d}_{3,i} \text{ is zero when no dosing} \quad \text{(Eq. 20)}$$

$$a_i = \frac{\dot{m}_{34}\,\hat{c}_{3b,i} + \dot{m}_{41}\,\dot{d}_{1,i}\,/(\dot{m}_{12} + \dot{m}_{14,AHU})}{\dot{m}_{34} + \dot{m}_{14,AHU}} \quad \text{where } \dot{d}_{1,i} \text{ is zero when no dosing} \quad \text{(Eq. 21)}$$

$$b_i = c_{o,i}\left(\frac{\dot{m}_{12} + \dot{m}_{14,AHU} - \dot{m}_{41}}{\dot{m}_{12} + \dot{m}_{14,AHU}}\right) \quad \text{(Eq. 22)}$$

$$\hat{c}_{1a,i} = \frac{a_i\,R_{41,out} + b_i}{1 - R_{41,out}\,R_{14,AHU}} \quad \text{(Eq. 23)}$$

$$\hat{c}_{1b,i} = \hat{c}_{1a,i} + \frac{\dot{d}_{1,i}}{\dot{m}_{12} + \dot{m}_{14,AHU}} \quad \text{where } \dot{d}_{1,i} \text{ is zero when no dosing} \quad \text{(Eq. 24)}$$

$$\hat{c}_{2,i} = \frac{\dot{m}_{12}\,\hat{c}_{1b,i} + \dot{m}_{32,AHU}\,\hat{c}_{3b,i}}{\dot{m}_{12} + \dot{m}_{32,AHU}} \quad \text{(Eq. 25)}$$

$$\hat{c}_{41,i} = \frac{\dot{m}_{34}\,\hat{c}_{3b,i} + \dot{m}_{14,AHU}\,\hat{c}_{1b,i}}{\dot{m}_{34} + \dot{m}_{14,AHU}} \quad \text{(Eq. 26)}$$

The sampled concentration data in thus study had a normal distribution, i.e. with zero kurtosis and skewness. However, it is heteroscedastic, i.e. the standard deviation is not constant but increases with magnitude of tracer gas concentration, with an exponent of approx. 0.7; see Fig.2. This may partly be explained by the fact that higher concentrations are associated with lower volume flow rate, lower turbulence, and thus poorer mixing. Therefore a weighted least-squares estimator has been used, where the weighting function is the reciprocal of the variance of the difference between the estimated and measured concentrations. The error residuals are therefore all normalized.

Least Trimmed Sum of Squares (LTS) robust regression is basically ordinary least squares regression in which the largest residuals are omitted [17]. Equation 27 shows the LTS function, where the *j* is the j^{th} order of all the normalized residuals ranked in order of magnitude, and the trimmed subsample size (h) must satisfy $5 \times N/2 < h \leq 5 \times N$ where N is the number of intervals in the time series (i) and 5 is the number of response concentrations (k).

$$LTS = \sum_{j=1}^{h} \left[r_{[j]}^{2} \right] \text{ where residual } r_{[j]} = r_{k,i} = \frac{c_{k,i} - \hat{c}_{k,i}}{\hat{\sigma}_{k,i}} \text{ and std.dev. } \hat{\sigma}_{k,i} \propto \hat{c}_{k,i}^{0.7} \qquad \text{(Eq. 27)}$$

Where the normalized residuals for the five reconstructed concentrations (response variables) are estimated thus:

For c_{1a}, c_{1b}, c_2: $$r_{k,i} = \frac{\left| c_{k,i} - \hat{c}_{k,i} \right|}{\hat{\sigma}_{k,i}} \quad \text{where} \quad \hat{\sigma}_{k,i} \propto \hat{c}_{k,i}^{0.7} \qquad \text{(Eq. 28)}$$

For c_{3b}: $$r_{3b,i} = \frac{\left| c_{3b,i} - \hat{c}_{3b,i} \right|}{\hat{\sigma}_{3b,i}} \quad \text{where} \quad \hat{\sigma}_{3b,i} \propto \hat{c}_{3b,i}^{0.7} \cdot \sqrt{2 - \frac{\hat{c}_{3b} - \hat{c}_{3a}}{\hat{c}_{3b}}} \qquad \text{(Eq. 29)}$$

For c_4: $$r_{4,i} = \frac{\left|c_{4,i} - \hat{c}_{4,i}\right|}{\hat{\sigma}_{4,i}} \quad \text{where } \hat{\sigma}_{4,i} \propto \hat{c}_{4,i}^{0.7} \cdot \sqrt{1 + \min\left[1, \frac{\max\left\{\hat{c}_{1\text{b},i}, \hat{c}_{3\text{b},i}\right\}}{\hat{c}_{4,i}}\right]} \quad \text{(Eq. 30)}$$

LTS was chosen due to its resistance to outliers in both the explanatory and response variables [18]. The amount of trimming (h) can be adjusted manually after studying a normalized plot of the residuals (Fig.3). The objective is to reject only sample data that is obviously erroneous. Anywhere from 80% to 95% are a typical values, but 100% may be used if there are no suspect data, in which case the method is identical to ordinary least squares.

Since it is impractical to solve LTS by 'brute-force', i.e. searching through all possible combinations of subsamples, efficient specialized search algorithms are used to provide near optimal solutions efficiently [18]. The search method used in this study was generic multivariate optimization (e.g. "Solver" tool in popular spreadsheet software), to minimize the error function (Eq. 28). The optimization must constrain the mass flows to be non-negative.

The iteration requires realistic initial guesses for the estimated parameters (m_{12}, $m_{32,\text{AHU}}$, m_{34}, $m_{14,\text{AHU}}$, and m_{41}). Equations 9 to 13 above are used to estimate these five mass flow rates based on the supply and exhaust fan rated/commissioned flow rates (m_2 and m_4 respectively), together with estimated recirculation fractions estimated with equations 1, 3 and 4.

2.5 Instrumentation and measurement procedure

An example of suitable instrumentation is a photoacoustic multi-gas monitor with continuous (multiplexed) monitoring in 6 channels. Sampling points should not be located inside the AHU, but rather be in the ductwork leading to or from the AHU, with sufficient distances to allow for good mixing [10]. The background concentration c_o is not measured on its own channel, but is ascertained by measuring in all 6 channels for a period before dosing starts.

This method can be carried out with continuous injection of one tracer gas (dosing at one location at a time, say 1 hour at each), or simultaneous injection of two different tracer gases at locations d_1 and d_3. In the case of using only one tracer gas (e.g. SF_6), one should inject at d_1 before d_3. When dosing has ceased at d_3, sampling should continue on all 6 channels for some length of time while the tracer gas concentration in the building decays. This step-down period can be used in the regression. A special feature of this decay period is that the tracer gas in the return duct(s) is better mixed compared to injection at d_3. The multiplexed samples must be interpolated in time (assuming an exponential curve during step-down), to obtain simultaneous values for each of the 6 channels, i.e. for each sampling point every 6^{th} value is a measurement while the other 5 are interpolated.

2.7 Limitations

Concentrations c_o and c_{3a} should not have systematic errors, as they are explanatory variables! The latter variable (c_{3a}) poses a challenge when dosing at d_3 if there are multiple extract ducts collected via a short plenum chamber, each with a different concentration. One solution is to close all ducts but one, or dose in all of the connecting ducts, neither of which are easy to do. Alternatively, if c_{3a} is erroneous, then the samples from a step-down period will be particularly valuable, or one can conduct a full regression (i.e. no trimming) to improve the estimates of the main flow rates but at the cost of the accuracy of the recirculation fractions. Measurement of c_2 must be accurate for at least half its samples; this is because there is no unique solution if c_2 is not used in the regression.

3. Results and discussion

Tests using synthetic measurement data containing deliberate outliers show that the method manages to trace these and eliminate them from the analysis, and thus perfectly recreate the mass flows, within the limitations mentioned above.

Further work: The next step is to extend the method to estimate variance in the mass flows and recirculation fractions. This should also take into account uncertainty in the explanatory variables as well as instrumentation uncertainty. There is also a need to assess the uncertainty due to ignoring casing leakage. This can be done either analytically or by appropriate bootstrapping method for time series data.

Estimation accuracy can be further improved by taking covariance into account, for example correlation in time for the time series data during periods with steady-state conditions [13]. The method could also be combined with a priori information on measured flow rates and their variance, for example venturi-type fan inlets, such as described in [13].

4. Conclusions and further work

This study has demonstrated the application of robust regression using least trimmed weighted squares to improve the estimates of the various mass flows in air handling units for balanced ventilation, including recirculation, leakage and infiltration due to imbalance. The method provides constrained estimates of the mass flows (non-negative and conserved), and at the same time omits outliers from tracer gas measurements. The method can equally be used for measurements of CO_2, or water vapour recirculation in AHUs. The model can be applied to analyze air handling units with any degree of recirculation. Tests using synthetic measurement data show that the method provides better estimates than conventional simple methods for analysing tracer gas data in the presence of outlier data. It is simple enough to be implemented on spreadsheet software. Improvements to the method have been suggested.

Nomenclature

a_i , b_i	Intermediate parameters used in mass flow reconstruction algorithm [mg/kg]
c	Tracer gas concentration (gravimetric) [mg/kg]
$\dot{d}$	Dosing rate [mg/h]
$\dot{m}$	Mass flow rate [kg/h]
R	Recirculation ratio [-]

Subscripts:

1,2,3,4	Fresh, Supply, Return (extract), Exhaust air ducts
a,b	Location upstream or downstream of dosing, respectively
i	Sample number, for tracer gas concentration that is logged at regular intervals
k	Channel number of tracer gas sampling

Acknowledgements

The study was financed by Undervisningsbygg (UBF) and Utdannelsesetaten (UDE) in Oslo. The Norwegian energy agency Enova supported authorship of this paper.

References

[1] Roulet, C-A.; Foradini, F.; Cretton, P. “Use of tracer gas for disgnosis of ventilation systems”, *Proceedings, Healthy Buildings 1994*, Budapest, Vol.2, pp 521-526

[2] Roulet, C-A.; Foradini, F.; Deschamps, L. “Measurement of air flow rates and ventilation efficiency in air handling units”. *Proceedings, AIVC 20th Conference and Indoor Air 99*. Vol.5, 1999

[3] Cheong, K.W. “Airflow measurements for balancing of air distribution system — tracer-gas technique as an alternative?” *Building and Environment*, Vol.36 (2001), pp.955–964

[4] Roulet, C-A.; Heidt, F.D.; Foradini, F.; Pibiri, M-C. ”Real heat recovery with air handling units”. *Energy and Buildings*, Vol.33 (2001), pp.495-50

[5] McWilliams, J. *Annotated Bibliography 12, Review of Airflow Measurement Techniques*. Belgium: AIVC – Air Infiltration and Ventilation Centre. 2002 [also doc. LBNL-49747]

[6] Petersen, A.J.; Bryn, I.H.; Schild, P.G.; Haugen, E.N.; Nilsson, G.; Høydahl, K. *Roterende varmegjenvinnere i skoler*. Oslo: Erichsen & Horgen AS, Rapport 002. 2008

[7] Standard EN 308:1997, *Heat exchangers. Test procedures for establishing the performance of air to air and flue gases heat recovery devices*.

[8] “*Ventilation measurements and other tracer-gas applications”*. Ballerup, Demnark: Innova AirTech Instruments AS (now: LumaSense Technologies AS), booklet

[9] Roulet, C-A.; Zuraimi, M.S.; Sekhar, S.C.; Tham, K.W. “Tracer gas measurement of airflow rates in spaces with several air-handling units, recirculation, or large time constants”. *HVAC and R Research*, Vol.12 (2006), No.3, pp 477-496

[10] Offermann, F.J.; Robertson, J.P.; Webster, T. “The impact of tracer gas mixing on airflow rate measurements in large commercial fan systems”. *Proceedings: Indoor Air 2002*. pp 320-325

[11] Silva, A.R.; Afonso, C.F. “Tracer gas dispersion in ducts—study of a new compact device using arrays of sonic micro jets”, *Energy and Buildings*, Vol.36 (2004), pp.1131–1138

[12] Roulet, C-A.; Vandaele, L. “*Airflow Patterns Within Buildings: Measurement Techniques*”. AIVC Technical Note 34. Air Infiltration and Ventilation Centre, 1991

[13] Sherman, M.H. “On the estimation of multizone ventilation rates from tracer gas measurements”. *Building and Environment*, Vol.24 (1989), No.4, pp.355–362 (also doc. LBL-25772)

[14] Okuyama, H.; Onishi, Y.; Tanabe, S.; Kashihara, S. “Statistical data analysis method for multi-zonal airflow measurement using multiple kinds of perfluorocarbon tracer gas”. *Building and Environment*. Vol.44 (2009), No.3, pp 546-557

[15] Miller, S.L.; Leiserson, K.; Nazaroff, W.W. “Nonlinear least-squares minimization applied to tracer gas decay for determining airflow rates in a two-zone building”. *Indoor Air*, Vol.7 (2004), No.1, pp.64-75. [DOI 10.1111/j.1600-0668.1997.t01-1-00008.x]

[16] Manz, H.; Huber, H.; Helfenfinger, D. “Impact of air leakages and short circuits in ventilation units with heat recovery on ventilation ef® ciency and energy requirements for heating”. *Energy and Buildings*. Vol.33 (2001), pp. 133-139

[17] http://en.wikipedia.org/wiki/Least_trimmed_squares

[18] http://en.wikipedia.org/wiki/Robust_regression

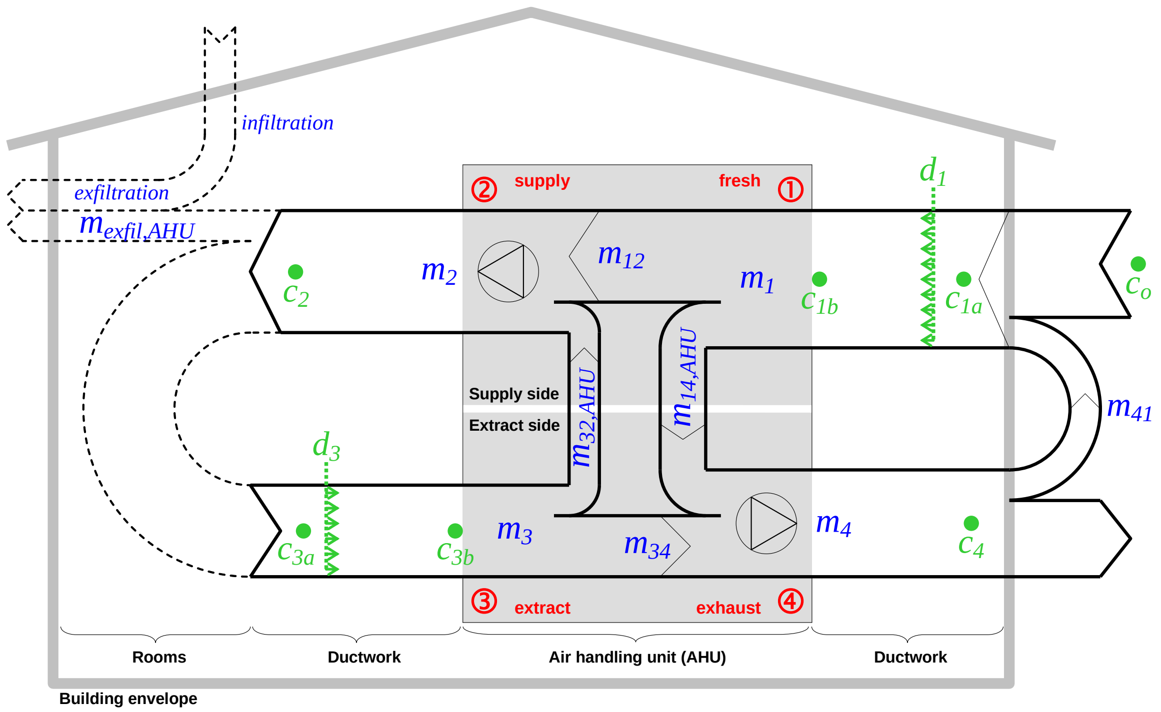

Fig. 1. Illustration of mass flow balance solved by the AHU model in this paper. Zero casing leakage is assumed. "*d*" is dosing probe, "*c*" is sampling point, "*m*" is mass flow. All mass flows are non-negative except $m_{\text{exfil,AHU}}$ which may be negative.

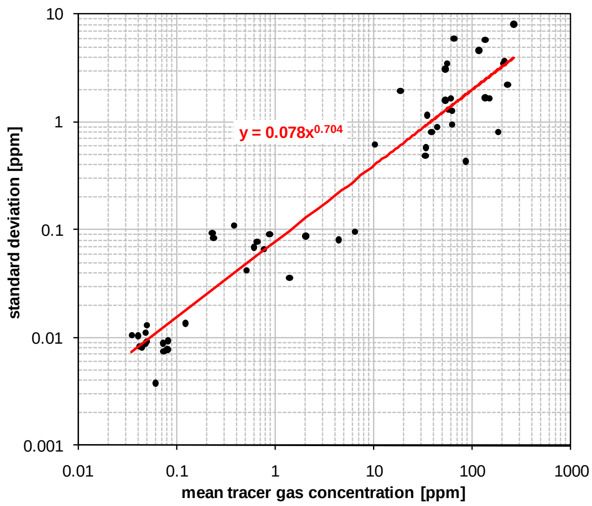

Fig. 2. Standard deviation as a function of tracer gas concentration in the field tests. This relationship is not assumed to be universally valid, but should be checked in each study.

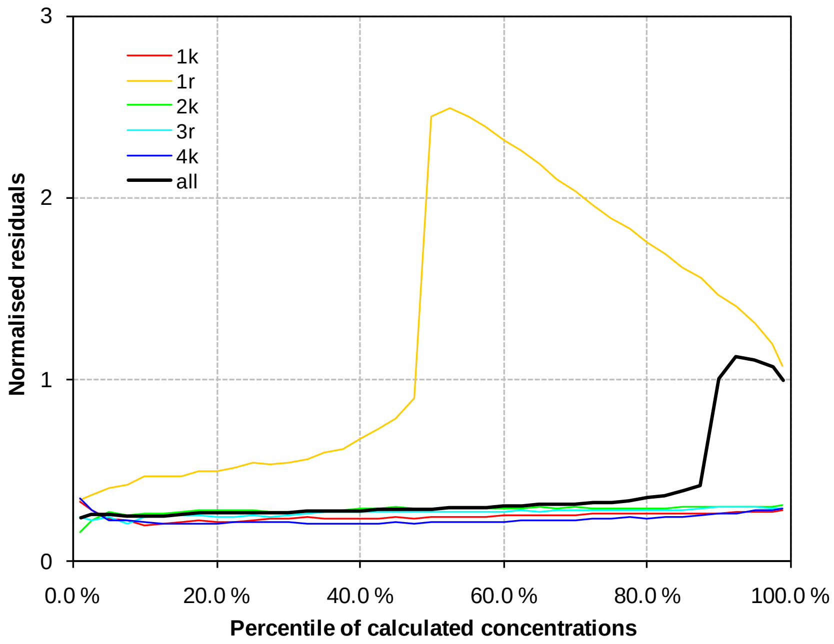

Fig. 3. Probability density of the normalized residual time series data at each sampling point, divided by an expected normal distribution. A horizontal flat line means that the data is normally distributed. Each sampling point is a different coloured line. Values above 1 are omitted (trimmed) in LTS algorithm. The thick black line is the entire data set for all sampling points.