How Accurate is our Leakage Extrapolation?
Modeling Building Leakage Using the Darcy-Weisbach Equation

Steven Rogers[1]

1 The Energy Conservatory
2801 21st Avenue South
Suite 160
Minneapolis, MN, USA 55407

ABSTRACT

This study used a mathematical model to explore the accuracy of extrapolating multi-point blower door test results down to lower pressures at which building infiltration usually occurs naturally. The mathematical model was applied to leaks of five different widths. The leakage of the five different widths was then combined in different distributions to simulate total building leakage. The calculated total building leakage was then compared to an extrapolation from the test pressures using a power law curve fit. The results showed that depending on the distribution of the leaks from widest to narrowest, extrapolation of the power law fit may significantly over-estimate or under-estimate the building leakage. At a building pressure of 1 Pa, one simulation the power law fit under-estimated the leakage by 19%, while another over-estimated the leakage by 78%. At pressures below 1Pa, the deviations were even larger.

KEYWORDS

Leakage modelling, natural infiltration, blower door test, laminar and turbulent leakage

1. INTRODUCTION

Since the earliest days of blower door tests in the 1970's, multi-point tests have been commonly used to characterize building leakage over a range of test pressures. Soon thereafter, a power law fit ($Q_v = C*\Delta p^n$) of the pressure vs. flow rate points was employed to estimate the leakage at pressures other than those tested. Such a fit is useful since it allows an estimate of the leakage behavior by extrapolating from test pressures, which are usually 15 – 100 Pa, to pressures that are most common under natural leakage conditions, about 0 – 20 Pa.

However, the accuracy of such extrapolations is not well understood. Accuracy is important because one of the primary motivations for air tightness testing is provide data for energy loss models of buildings. Since buildings lose energy from air infiltration and exfiltration due to the pressure difference created by wind and the stack effect, the test data from higher pressures must be extrapolated to the lower pressures at which air infiltration and exfiltration occur naturally.

It should be acknowledged that in addition to the extrapolation of the power law fit, there are other known problems in estimating natural infiltration. The distribution of leaks vertically in the building and the distribution of pressure differences due to wind are two other important sources of error in estimating natural building infiltration. This paper will only discuss errors due to the extrapolation of the power law fit.

2. METHOD

The Darcy-Weisbach equation is used due to its simplicity and because it characterizes the pressure vs. flow behavior in both the laminar and turbulent flow regimes, which are known to exist in building leakage. The Darcy-Weisbach equation was developed for cylindrical pipes, and may not be regarded as applicable to long, narrow cracks with corners like those found in a leaky building. However, in this study, it will be shown that the model does not need to predict the magnitude of the leakage, but only how it changes with pressure and Reynolds number. For this purpose, the Darcy-Weisbach equation and the Darcy friction factor are useful and instructive.

The leakage model consists of cracks with five different widths: 16, 7, 5, 2.2, and 1.6 mm, where width is defined as shown in Figure 1. What is most important about the choice of these dimensions is the flow regime that will occur under conditions of test pressures and conditions of natural infiltration. For long, thin cracks the flow regime (laminar or turbulent) is determined by this width, and to a lesser degree the roughness of the building materials. In the case of the largest width, 16mm, flow is turbulent at all pressures where testing would occur (above about 3 Pa) and is approximately the largest width of a crack that can be expected to frequently occur in residential construction. With the smallest width, 1.6 mm, flow is laminar at all pressures of interest, up to 100 Pa. So this smallest leak can represent all leaks 1.6mm or smaller, since they will all remain laminar at all pressures of interest.

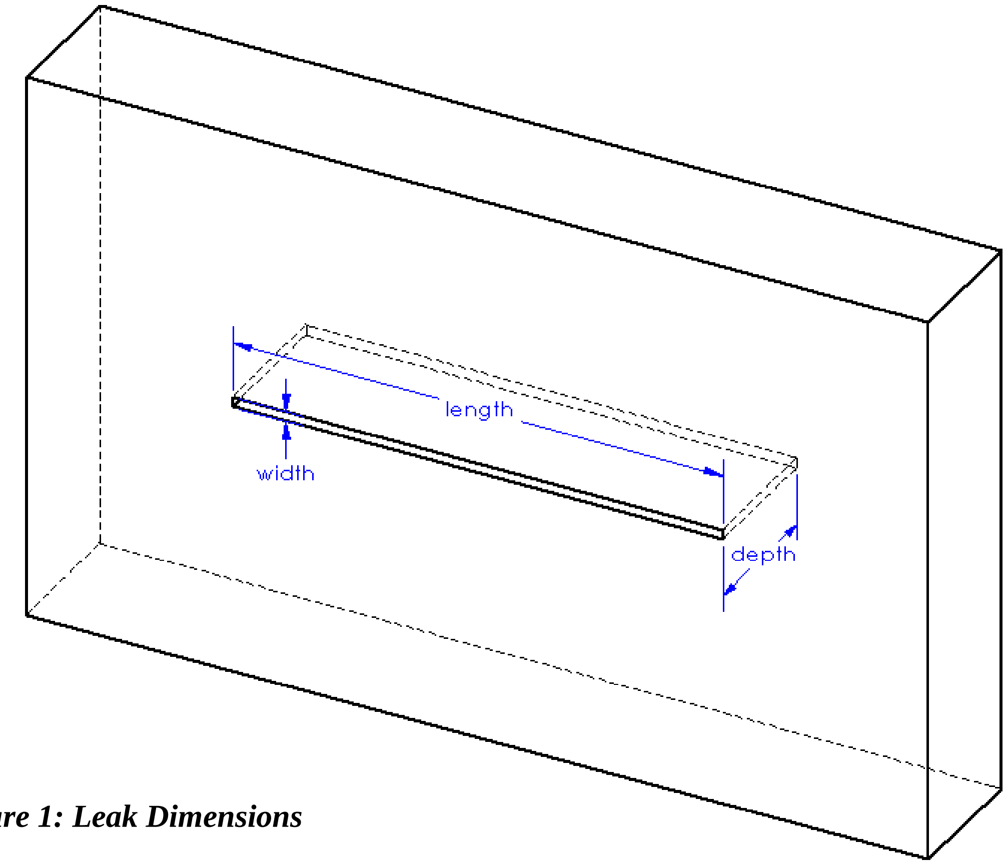

Figure 1: Leak Dimensions

The flow through each of the cracks is modeled over a range of 0.1 Pa to 100 Pa using the Darcy-Weisbach equation and Darcy friction factor. Most of the cracks change from laminar flow at low pressure differences through transitional flow and into the turbulent flow regime at higher pressure differences. Next, an arbitrary number of leaks (or leakage area) may be assigned to each of the five characteristic lengths, such that a total volume of air leakage is divided up into the five characteristic lengths. Finally, the sum of all leaks is computed at each pressure difference to calculate the total building leakage.

This approach allows the comparison of buildings with similar total leakage due to a smaller number of larger sized leaks, or a larger number of smaller sized leaks. By distributing the leaks carefully, one can even create two very different leakage distributions that result in exactly the same power law fit.

2.1 Five Leak Sizes

Initially, each of the five leaks is assumed to have an equal cross sectional area of 2500 mm^2, but each one has a different length to width aspect ratio, ranging from 10:1 to 1000:1. The widest leak is 16 x 158 mm, and the narrowest leak is 1.6 x 1580 mm. Each leak is assumed to be 100 mm in depth, where depth is the distance the air travels through the thickness of the wall (see figure 1). However, this depth has no impact on the results, since the total pressure loss is proportional to this depth and each leak will be multiplied by an arbitrary area. Again, this investigation is interested in relative differences of one leak width to another as they change over a range of pressures.

The roughness of all leaks is required to calculate a Darcy Friction Factor in the turbulent region. The roughness, ε, of the three wider leaks was assumed to be 0.25 mm. The 0.25 mm value is given from several sources as the roughness of cast concrete pipe, and was assumed to be similar to many building materials. The two narrowest leaks were assumed to be smoother at 0.135 mm and 0.111 mm.

2.2 Darcy Friction Factor

Using the roughness values above, the Darcy Friction Factor was calculated over the range of Reynolds Numbers of interest. The width, as previously defined, is the characteristic linear dimension used in the calculation of Reynold number. In the laminar flow region for Re < 2300, the Darcy friction factor is known to be equal to 64/Re. In the turbulent flow region, Re > 4000, the friction factor becomes dependent on both relative roughness and Reynolds number, and eventually becomes nearly constant at high Reynolds numbers. A numerical approximation called 'Serghides's solution' is used to calculate the turbulent friction factor.

In the transition flow region, 2300 > Re > 4000 the flow is unsteady; it varies grossly with time and space within the flowing fluid. For simplicity, the friction factor has been modeled as a straight line connecting the friction factor from the laminar to the turbulent flow regime. In reality, there is considerable uncertainty in this Reynolds number range and it is indicated by the shaded region of Figures 2 and 3.

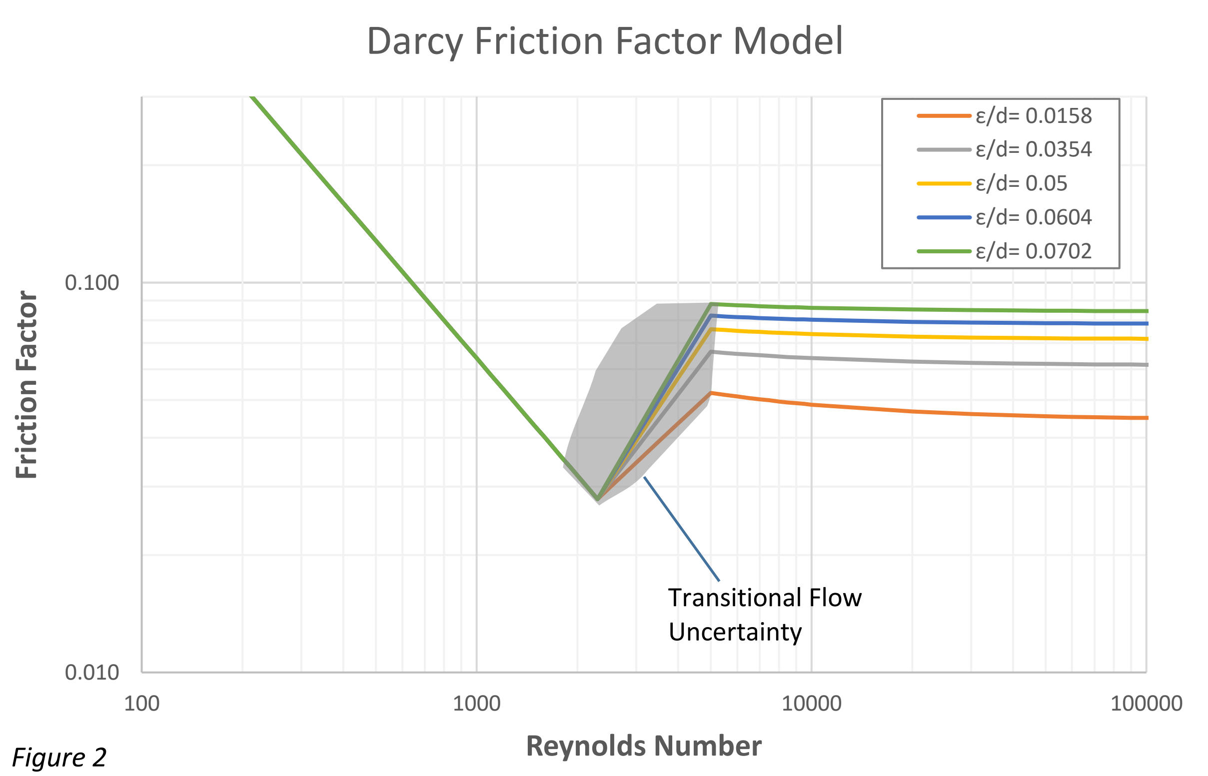

Figure 2

2.3 Flow vs. Pressure Difference

Once we have friction factors, the air velocity through each leak size can be calculated using the Darcy-Weisbach Equation, shown here solved for velocity.

$$v = \sqrt{\frac{2 * D_h * \Delta p}{\rho * f_D * L}} \qquad (1)$$

Where:

v = the average velocity of air through the leak

D_h = the hydraulic diameter of the leak

Δp = the pressure difference inside to outside the building

ρ = the density of air

f_D = the Darcy friction factor

L = the depth of the leak (as shown in Figure 1)

Since the Darcy friction factor depends on the Reynolds number, which depends on the velocity, the solution is iterative. Once the average air velocity through each leak is calculated, the volumetric flow rate is obtained by multiplying velocity by the cross-sectional area of the leak.

Figure 3 shows the relationship between the pressure difference and the volumetric flow through a leak. The 7mm leak was chosen since it passes through all three flow regimes between 1 and 100 Pa pressure difference. Notice that in the laminar region where flow rate is proportional to pressure (n=1), the slope is the steepest, and in the turbulent region the slope is flatter where (n=0.5). The figure also shows the range of pressures at which blower door tests are usually run (between 15 and 100 Pa), and the pressures at which natural infiltration usually occurs (under 20 Pa).

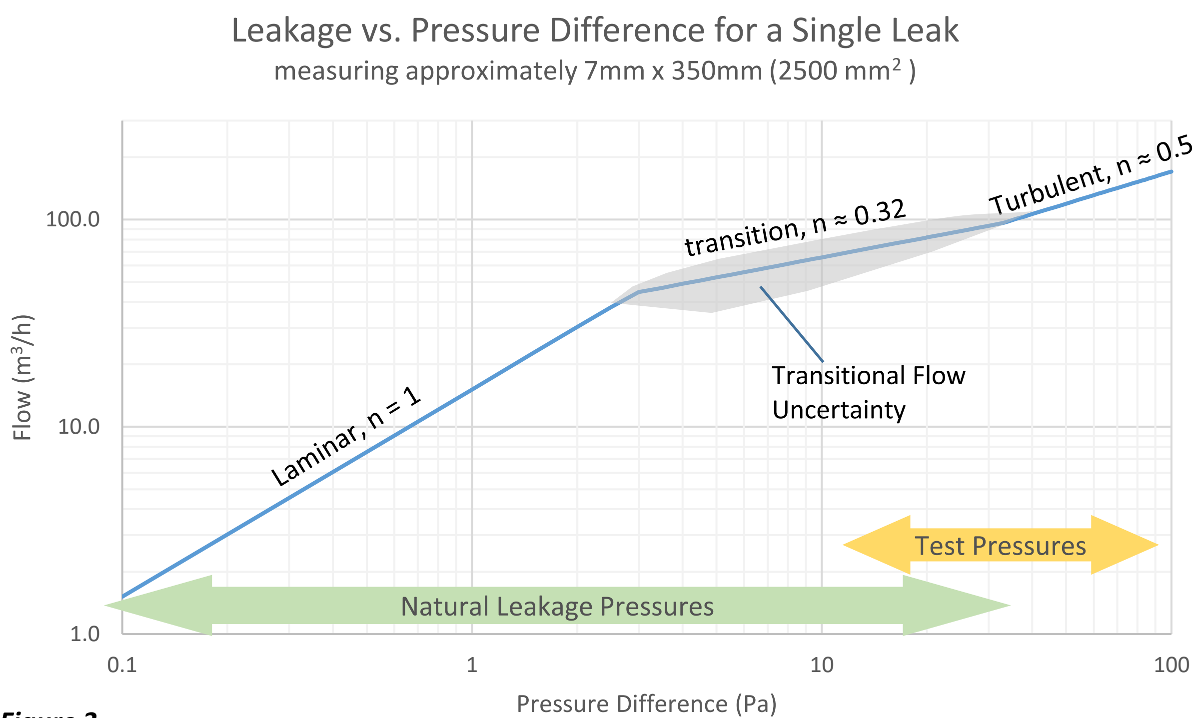

Figure 3

Figure 4 shows the leak rate vs. pressure difference behavior for all the leak sizes modeled. Notice that in some cases a narrower leak may have the same or more flow than a wider leak.

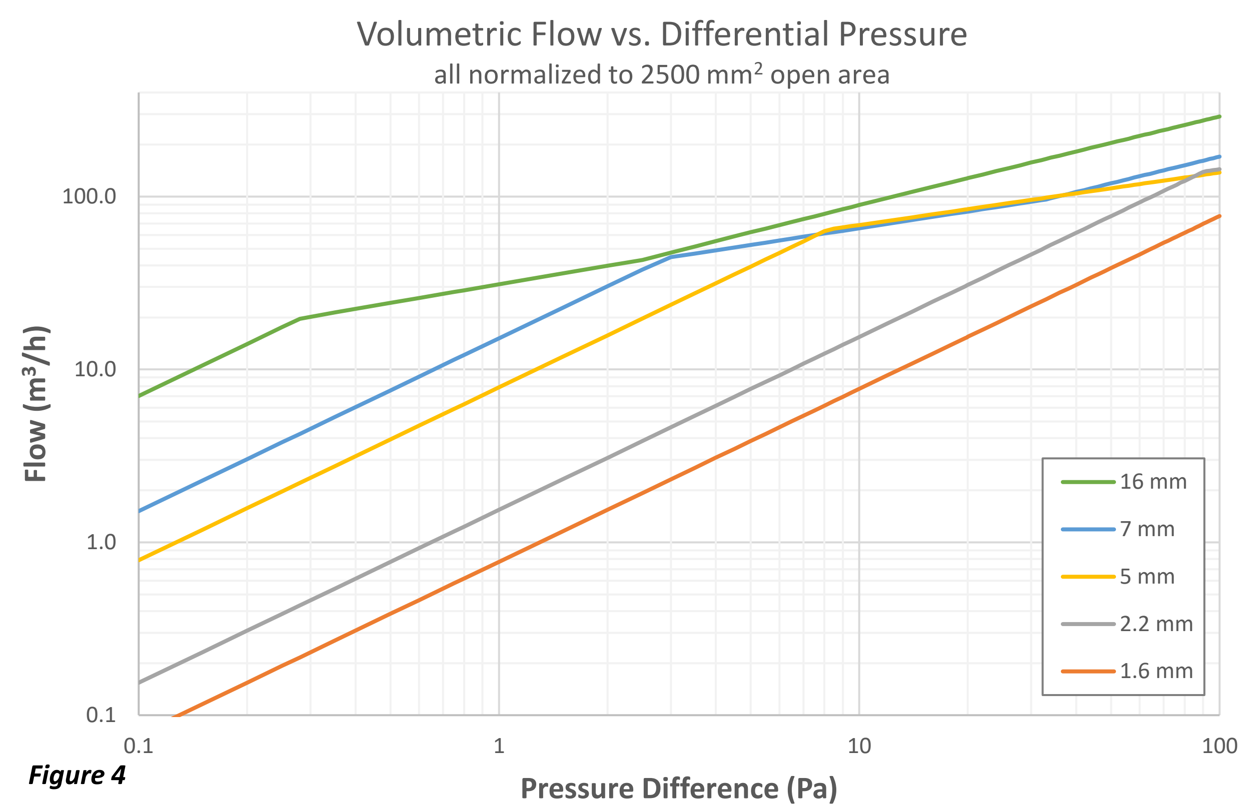

Figure 4

2.4 Combining Individual Leaks into Total Leakage

Once the pressure difference vs. leakage behavior is characterized, multiple sizes of leaks can be combined into a total. Originally, all the leaks were normalized to an open area of 2500 mm^2. Now we replace these areas with new areas that are somewhat arbitrary. The objective is to iteratively adjust these areas until the calculated leakage results are similar to what we find by field testing buildings.

This adjustment of leakage areas is the reason why our model does not need to accurately predict the magnitude of leakage for the various leak sizes, but only their relationship to each other and how they change over the pressure range. In this study, we are interested in understanding how the sum of many individual building leaks of various sizes changes with pressure, particularly at lower pressures. The pattern of flow as it changes from laminar to turbulent in building leaks is likely to follow a pattern very similar to the Darcy friction factor and Darcy-Weisbach equation, since they both result from the same fundamental physics described by the Navier-Stokes equations.

In a first example, we can begin by allocating the leakage areas so that the total leakage at 50 Pa is about 1200 m^3/h, and each of the five leak sizes accounts for 20% of the leakage. This gives the following results.

Table 1: Equal Leakage Distribution, Flow @ 50 Pa = 1206 m^3/h

Leak Size	16 mm	7 mm	5 mm	2.2 mm	1.6 mm
Leakage Area (mm^2)	2950	5045	5572	7834	15590
% of Total Leakage @50Pa	*20.0%*	*20.0%*	*20.0%*	*20.0%*	*20.0%*

When the leakage from all leaks is equal at 50 Pa, a power law fit of a multi-point test yields the following: $Q_v = 117.4 * \Delta p^{0.595}$. However, our experience with real-world building leakage tests tells us that the exponent *n* is usually closer to 0.65. So our leakage distribution should be adjusted to have more leakage from the smaller leaks in proportion to the larger ones, in order to better reflect our experience with real-world leakage.

2.5 Comparing Three Different Distributions

Next, we will modify the leakage areas until we have a distribution that gives us a coefficient *n* close 0.65. To achieve this we progressively increase the leakage area as the leak size gets smaller, which results in the following proportions of leakage.

Table 2: Leakage Distribution A, Flow @ 50 Pa = 1219 m^3/h

Leak Size	16 mm	7 mm	5 mm	2.2 mm	1.6 mm
Leakage Area (mm^2)	1875	3375	6075	10935	19683
% of Total Leakage @50Pa	*12.6%*	*13.2%*	*21.6%*	*27.7%*	*24.9%*

The Power Law fit is: $Q_v = 94.410 * \Delta p^{0.653}$

We can then plot the total leakage at pressures from 0.1 Pa up to 100 Pa. We can assume 9 test pressures of 12, 18, 22, 28, 36, 42, 50, 56, and 64 Pa. Figure 5 shows the leakage at all pressures in orange, and the test points in blue. The blue dotted line is the power law fit of the test points. It has an exponent of 0.653, which was the objective. Notice that total leakage is very close to the power law fit at pressures near the test points, but it deviates markedly as pressures get further from the test pressures.

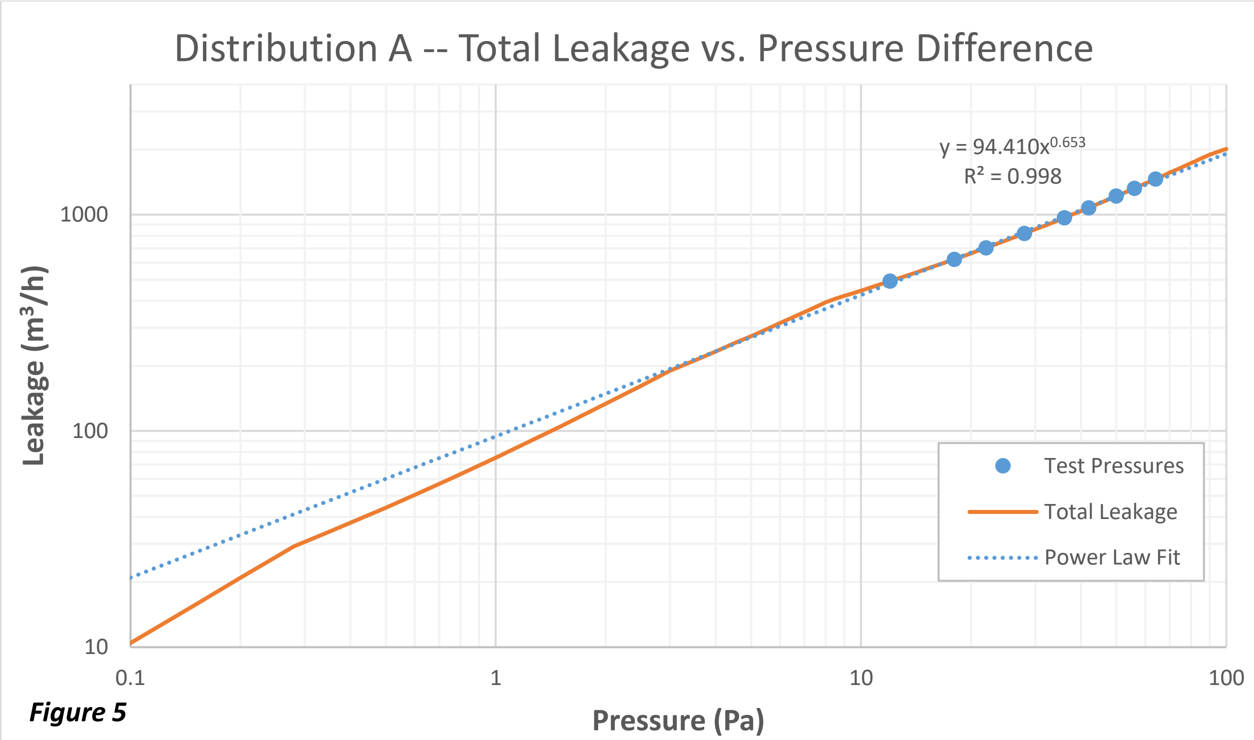

Figure 5

Next, we consider whether a significantly different distribution of leaks could also give the same resulting flow and exponent. Leakage Distribution B shows another possible distribution of leaks in which most of the leakage is 1.6 mm or 16mm with much less leakage in between. By adjusting the leakage area of small and large leaks, a total leakage can be calculated which results in a power law fit that is almost identical to Leakage Distribution B.

Table 3: Leakage Distribution B, Flow @ 50 Pa = 1217 m^3/h

Leak Size	16 mm	7 mm	5 mm	2.2 mm	1.6 mm
Leakage Area (mm^2)	7663	1120	1400	1750	27410
% of Total Leakage @50Pa	*51.4%*	*4.4%*	*5.0%*	*4.4%*	*34.8%*

The Power Law fit is: $Q_v = 94.401 * \Delta p^{0.653}$

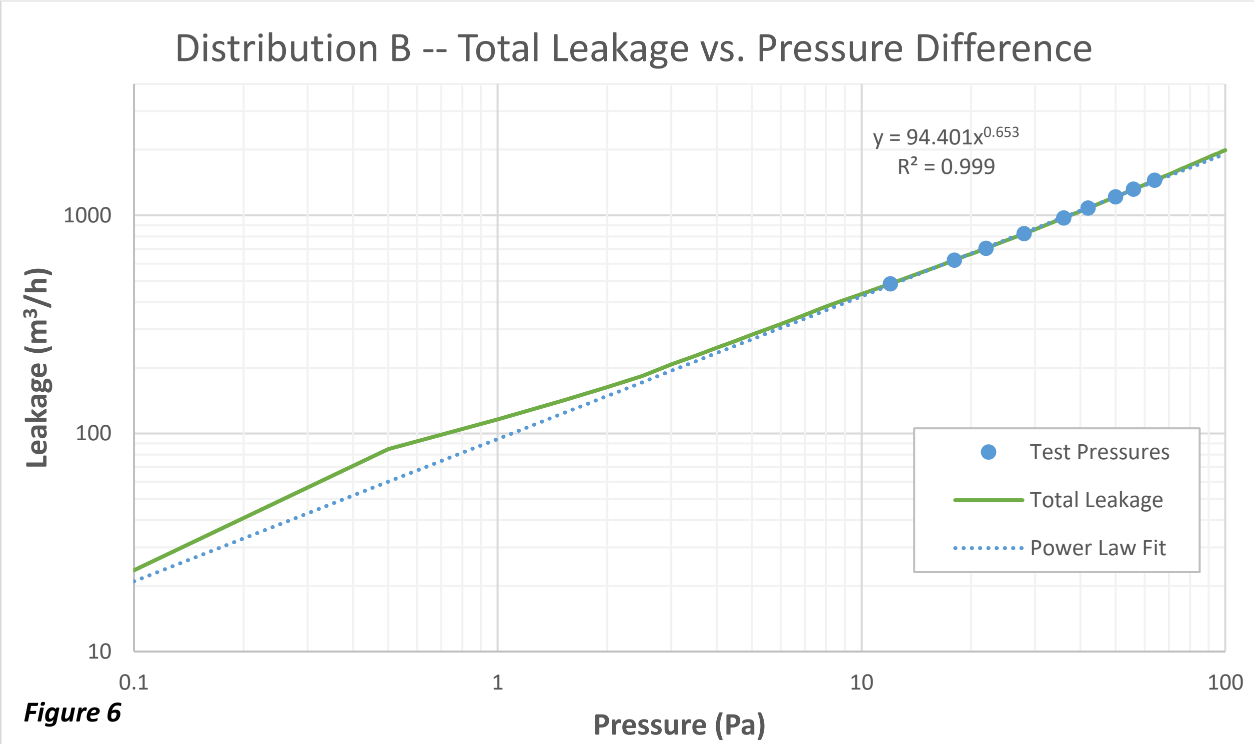

Figure 6

Just as with distribution A, the power law fit of Distribution B is very close to the calculated total leakage around the test pressures. However in this case, the Total Leakage is greater than the power law predicts, whereas with Distribution A, the Total Leakage was less than the power law predicts. So the "error" could be positive or negative.

A third distribution of leaks was also created which also results in the same total leakage at 50Pa and the same exponent: 0.653. In this case, most of the leakage was assumed to be from the 5 mm and 2.2 mm leaks, with very little from the smaller or larger leaks.

Table 4: Leakage Distribution C, Flow @ 50 Pa = 1219 m^3/h

Leak Size	**16 mm**	**7 mm**	**5 mm**	**2.2 mm**	**1.6 mm**
Leakage Area (mm^2)	250	250	10764.05	23175	250
% of Total Leakage @50Pa	*1.7%*	*1.0%*	*39.0%*	*58.0%*	*0.3%*

The Power Law fit is: $Q_v = 94.403 * \Delta p^{0.653}$

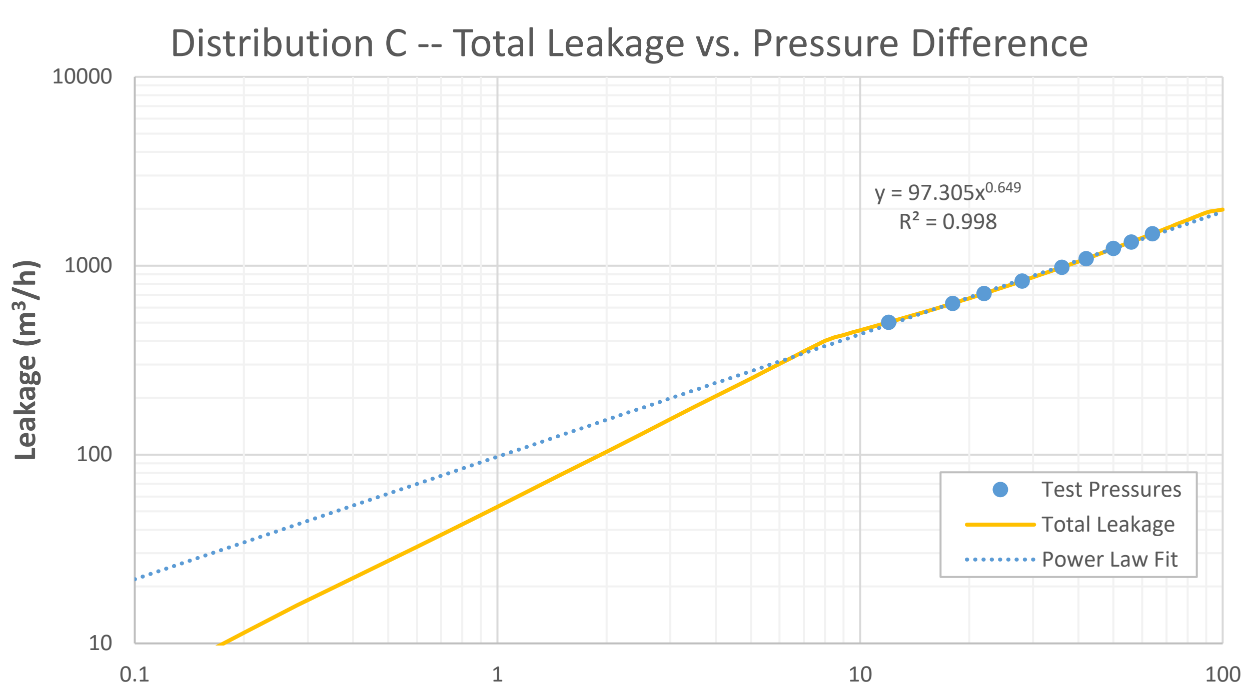

Figure 7

Like the others, distribution C deviates significantly from the power law fit model at lower pressures.

3 RESULTS

Now we return to the original question: how accurate is an extrapolation from leakage measurements taken at higher pressures to leakage occurring naturally at a few Pascals? Figures 8 shows a comparison of the results from Distributions A, B and C. Here, the vertical axis is the percent deviation of the flow estimated using the power law fit from the calculated total leakage. This makes it easier to compare the three results and understand the percent "error" that might be implied by extrapolating a power law fit from the test pressures down to lower pressures. At a pressure of 1 Pa, we have deviations of -19%, +26%, and +78% for distributions A, B, and C respectively.

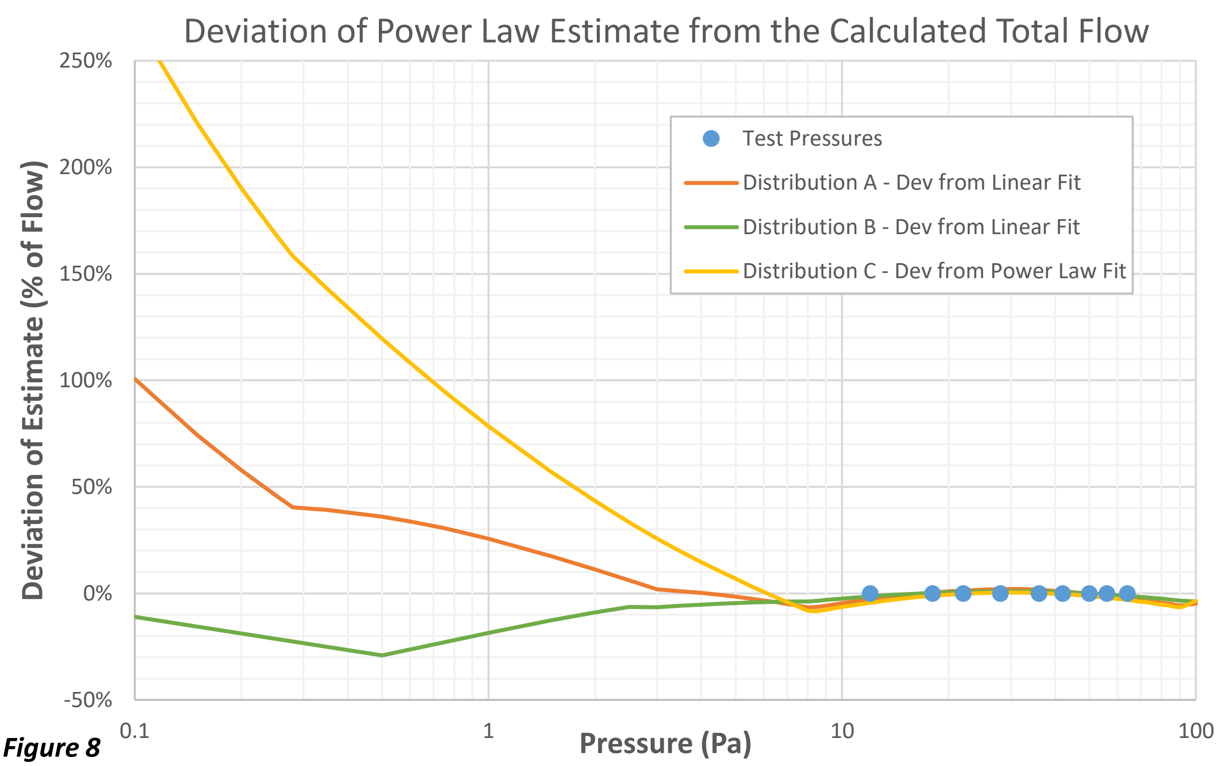

Figure 8

4 Necessary Assumptions

It is important to review the assumptions necessary for these results to be informative and useful. The first assumption is that real-world building leaks are of sizes that include both laminar and turbulent leaks. This is almost certainly true since real-world power law fits of measured leakage do not have exponents close to 1.0 or 0.5, which would be the case if all building leaks were laminar or turbulent respectively. Real world power law exponents are usually between 0.6 and 0.75.

The second assumption is that some leak sizes transition from laminar flow to turbulent flow as pressure increases within the pressure range of interest. This is also almost certainly true since the width of real-world leaks is continuously variable and the width of the leaks will determine the Reynolds number in the leak.

The third assumption is that in real-world building leaks, the relationship between pressure and flow changes in a way similar to that in round pipes. Since the Darcy-Weisbach equation was developed to describe flow in round pipes, we must consider whether it can be applied to long, narrow leakage geometries. This assumption is probably valid since the fundamental boundary layer physics that govern transitions from laminar to turbulent flow in round pipes are the same as those that govern flow through other geometries. The relationship between pressure and flow is best understood by referring to Figure 3. When viewed on a log-log plot, the change in the flow vs pressure appears as a straight line with a different slope as the flow changes from laminar to turbulent. If real-world leaks change slope in a similar way, then these results are useful irrespective of the accuracy of the calculated leakage flow rate.

To look at it another way, we can use the 7mm wide leak as an example. In our simulation, we calculated that a leak 7mm wide, having a total open area of 2500 mm^2 has a leakage of 66 m^3/hr at a building pressure of 10 Pa. But for the purposes of this study, it doesn't matter if the actual leakage from a leak this size is 33, 66, or 133 m^3/hr. What is important is only that the flow regime will transition from laminar through transitional flow to turbulent flow at pressures somewhere between 0.1 Pa and 100 Pa. When this occurs, the slope of the line will change on the log-log plot and this will introduce error in the power law fit.

5 Discussion and Conclusions

These simulations can give us some idea of the shapes and inflections we might see in the error curves of a power law fit. They can also show what order of magnitude the errors might be at various pressures. They also seem to indicate that the true errors might be either positive or negative.

What cannot be determined from these results is which of the three simulations is most similar to buildings in the real world. Further study would be required to determine this.

The well-known and widely used power-law fits accurately predict leakage near the test pressures. However, at much lower pressures the true leakage might be higher or lower than the power law fit predicts and the error associated with this extrapolation might be quite large.

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