Uncertainties in airtightness measurements: regression methods and pressure sequences

Martin Prignon*[1], Arnaud Dawans[2] and Geoffrey van Moeseke[1]

1 Université catholique de Louvain
Place de l'université 1,
B-1348 Louvain-la-Neuve, Belgium
* *martin.prignon@uclouvain.be*

2 Entreprises Jacques Delens S.A.,
Avenue du col-vert, 1,
1170 Brussels, Belgium

ABSTRACT

This paper presents the results of a series of 30 fan pressurization tests in reproducibility conditions performed within a period of 10 days in October 2017. The tested unit is a newly constructed unoccupied apartment in Brussels. These results make possible to compare different regression methods and evaluate the impact of pressure stations chosen for these regressions. The comparison of three different methods shows that the Weighted Line of Organic Correlation and the Iterative Weighted Least Square reduce the airflow rate standard deviation and give a better uncertainty calculation than Ordinary Least Square. Further work should focus on the generalization of these results in applying these methods to other sets of reproducibility tests. The comparison of different pressure sequences shows that the European standards are coherent in the imposed requirements and the given recommendations. In addition, when performing a regression method, one should avoid low-pressure stations as much as possible in order to reduce the airflow rate standard deviation at low- and high-pressure. Even if this work shows and confirms ways to improve the reliability of calculated combined standard uncertainties, there is still an important difference between these and the observed standard deviation. The investigation of such a difference should be a priority for further researchers focusing on airtightness uncertainties.

KEYWORDS

Airtightness measurement, uncertainty calculation, regression methods, pressure sequence

1 INTRODUCTION

Although building professionals have used the fan pressurization method for years, the uncertainties related to airtightness measurements is still an important research topic. Recently, authors focus on the comparison between calculated uncertainty and standard deviation using new regression methods (Delmotte, 2017) and on the investigation of specific sources of uncertainties as such as wind (Carrié & Leprince, 2016). An important issue highlighted by researchers is the irrelevant use of Ordinary Least Square (OLS) regression method. Okuyama (Okuyama & Onishi, 2012) suggested the use of Iterative Weighted Least Square (IWLS) regression method in 2012, while Delmotte (Delmotte, 2017) suggested the use of the Weighted Line or Organic Correlation (WLOC) in 2017.

This paper deeply investigates the issue of regression methods by presenting the results of a reproducibility study. In addition, it uses the results of this study to evaluate the impact of pressure stations (i.e. the choice of pressure – airflow couples during a fan pressurization test) on the airflow rate.

2 METHODOLOGY

This paper presents the results of a series of 30 fan pressurization tests in reproducibility conditions performed within a period of 10 days in October 2017. Airtightness measurements were carried out on a newly constructed (2017) unoccupied apartment in Brussels. The apartment is a masonry construction of 228 m³ with a floor area of 90 m² located on the second floor of a three-story building. There were no airtightness requirements for the apartment itself, but the whole building (i.e. all apartment pressurized simultaneously) had to reach an air change rate lower than 0.6 h^{-1} at 50 Pa.

The operator recorded outside climatic data with a meteorological station placed on the roof above the apartment. Table 1 shows the minimum (min), maximum (max), average (μ) and standard deviation (σ) of climatic data recorded during the series of tests. Mean wind speed is computed with 10-minutes averages.

Table 1: Minimum, maximum, average and standard deviation of main climatic data (temperatures and wind speed) recorded during the testing period.

	min	**max**	μ	σ
Outside temperature [°C]	8.8	24.3	14.2	2.8
Inside temperature [°C]	20.5	22.0	21.3	0.6
Mean wind speed [m/s]	0.0	3.8	1.3	0.8
Maximum wind speed [m/s]	0.3	8.3	3.3	1.6

The same operator performed all the tests, with the same equipment and on the same apartment prepared according to method 1 of ISO 9972:2015 standards (ISO-9972, 2015). Since the climatic conditions varied within the 10-days period, this series of test is in reproducibility conditions instead of repeatability conditions (Novak, 2015). Indeed, repeatability conditions consider successive measurements carried out under the same conditions while reproducibility conditions refer to successive measurements performed under changed conditions of measurements (JCGM, 2008). However, it is important to differentiate the reproducibility conditions in this paper with the reproducibility conditions presented in airtightness literature. Indeed, in previous researches, reproducibility conditions refer to a change in operator performing the test while in this paper the same operator performs the whole series of 30 tests (Delmotte & Laverge, 2011; Novak, 2015).

2.1 Uncertainties in pressure and airflow measurements

When performing a linear regression method, the uncertainties of regression parameters (i.e. the slope n and the intercept c_e) depend on the uncertainties of the response variable (i.e. $\ln(Q_e)$) and of the explanatory variable (i.e. $\ln(\Delta P_e)$).

Regarding airflow uncertainty ($u(\ln(Q_e))$), Delmotte developed equation to compute them based on the characteristics of the equipment (Delmotte, 2013). In the study presented in this paper, the maximum permissible error for the airflow measurement is the greatest between 4 % of the airflow measured and 1.7 m³/h. In addition, the maximum permissible error for the temperature measurement is 0.5 °C and the resolution of temperature measurement is 0.1 °C. Since this paper uses exactly the same equations to compute airflow rate uncertainties, they are not developed here.

Regarding envelope pressure uncertainty ($u(\ln(\Delta P_e))$), the equations used in this study are slightly different from Delmotte equations. First, this study considers the uncertainty given by the manufacturer for pressure measurement as a 95% confidence interval and not as a maximum permissible error. Therefore, equation 1 gives the uncertainty in pressure measurement ($u(\Delta p_m)$) for an equipment with a resolution of 0.1 Pa and a 95% confidence interval in the pressure measurement of the greatest between 1% of the reading and twice the resolution.

$$u(\Delta p_m) = \max\left(\sqrt{\left(\frac{0.01*\Delta p_m}{2}\right)^2 + \left(\frac{0.1}{\sqrt{12}}\right)^2} ; \sqrt{\left(\frac{0.20}{2}\right)^2 + \left(\frac{0.1}{\sqrt{12}}\right)^2}\right) \quad (1)$$

Second, the uncertainty in zero-flow pressure has three terms (Equation 2). The two firsts are for the zero-flow pressure measured before and after the test ($u(\Delta P_{0,1})$ and $u(\Delta P_{0,2})$). Equation 1 allows computing the values of these terms since the same manometer measures the pressure during the test and the zero-flow pressure. The third term is the zero-flow pressure approximation ($u(\Delta P_{0,a})$). In a real fan pressurization test, the measurement of zero-flow pressure is not possible when the fan is working. Therefore, the operator measures the zero-flow pressure before and after the test. In calculations, the zero-flow pressure during the test is equal to the average of zero-flow pressure measured before and after the test. However, the real zero-flow pressure is constantly varying and the hypothesis of constant value is a source of uncertainty.

$$u(\Delta P_0) = \sqrt{\frac{u^2(\Delta P_{0,1})}{4} + \frac{u^2(\Delta P_{0,2})}{4} + u^2(\Delta P_{0,a})} \quad (2)$$

The evaluation of zero-flow pressure approximation uncertainty is difficult because it is not a type A evaluation of uncertainty. Type A evaluation method uses repeated measurement to compute uncertainty. In this case, the real zero-flow pressure is constantly varying and it is not possible to analyze its variation by performing a series of repeated measurements. One has then to evaluate the uncertainty by scientific judgement based on available information. This is the type B evaluation of uncertainty (JCGM, 2008).

The use of zero-flow pressure tests allows evaluating the uncertainty of zero-flow pressure approximation. A zero-flow pressure test is the measurement of zero-flow pressure every seconds during 14 minutes divided in three successive periods. The first and last periods (2 minutes each) are respectively the first and second approximation periods while the second period (i.e. the middle period of 10 minutes) is the “fictitious period”. The analyze of zero-flow pressure test consists in comparing the zero-flow pressure approximation with the zero-flow pressure measured during the fictitious period (i.e. the period where, in a real test, the zero-flow pressure is not measurable). The result of the zero-flow analysis for this apartment shows an uncertainty in zero-flow pressure approximation between 0.9 and 2.5 Pa, depending on the wind speed.

Then, uncertainties in pressure measurements and in zero-flow pressure are used to compute uncertainties in envelope pressure (Equation 3) and in explanatory variable (Equation 4).

$$u(\Delta P_e) = \sqrt{u^2(\Delta P_0) + u^2(\Delta p_m)} \quad (3)$$

$$u(\ln(\Delta P_e)) = \sqrt{\left(\frac{u(\Delta p_e)}{\Delta p_e}\right)^2} \quad (4)$$

Figure 2 shows, as an example, the uncertainties in explanatory and response variables as a function of the measured pressure for the first test of the series in depressurization.

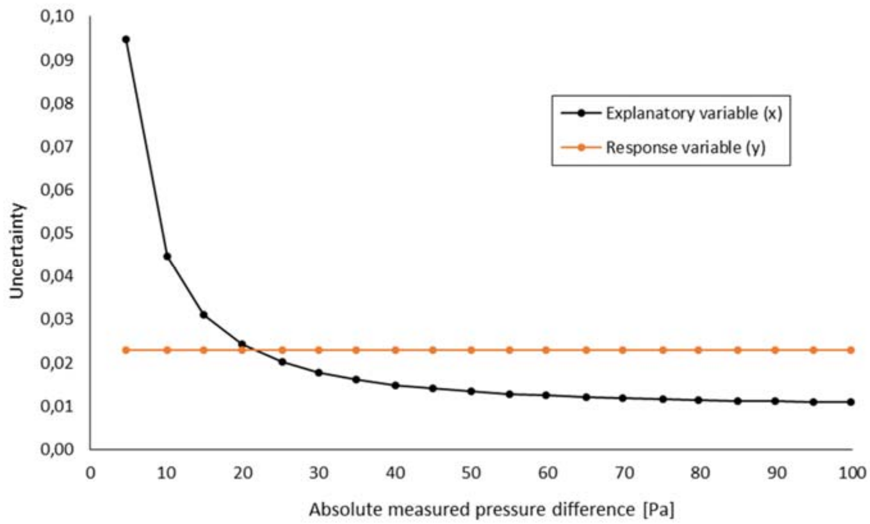

Figure 1: Combined standard uncertainty in explanatory (x_i) and response (y_i) variables as a function of the measure pressure difference for the first test of the series in depressurization case.

2.2 Regression methods

The uncertainty in regression parameters (i.e. the slope n and the intercept c_e), and therefore in the building airtightness, depends on the uncertainties in explanatory and response variables, but also on the regression method. In practice, operators mainly use the Ordinary Least Square (OLS) regression method to determine the regression parameters. However, the OLS method is appropriate only when the uncertainties on the response variable are constant for all the points used for the regression and if the uncertainties on the explanatory variable are negligible (Delmotte, 2013). Helsel and Hirsch explained that for cases that do not meet these assumptions, the uncertainty on regression parameters computed with OLS were not reliable (Helsel & Hirsch, 2002). Since in the current case both variables have the same order of magnitude, Figure 1 shows that the uncertainty on the explanatory variable is non-negligible and OLS method is irrelevant. This paper investigates two alternative methods suggested in literature: the Iterative Weighted Least Square method (IWLS) and the Weighted Line of Organic Correlation method (WLOC) (Delmotte, 2017; Okuyama & Onishi, 2012).

Iterative Weighted Least Square (IWLS)

Multiple sets of equations exist to solve the iterative approach required to perform Iterative Weighted Least Square (IWLS) method. This section describes the unified equations developed by York (York, Evensen, Martınez, & Delgado, 2004). IWLS method consists in assuming a starting value for the slope and then re-compute this value based on uncertainties in variables.

This method has three main steps that repeats: (i) computation of weights (equations 5 to 7) (ii) computation of residuals (equations 8 to 11) and (iii) computation of the new slope (equations 12 and 13). The iteration process stops when the difference between two consecutive slopes (flow exponent) is lower than a chosen tolerance. In this study, the tolerance was set to 10^{-10}. Once this the iteration stops, a fourth step (iv) gives the second regression parameter as a function of the slope and the means obtained from last iteration (Equation 14)

Step (i)

$$w(x_i) = \frac{1}{u^2(x_i)} \quad (5) \qquad w(y_i) = \frac{1}{u^2(y_i)} \quad (6)$$

$$W_i = \frac{w(x_i)w(y_i)}{w(x_i)+n^2w(y_i)} \quad (7)$$

Step (ii)

$$\bar{x} = \frac{\sum_{i=1}^{N} W_i x_i}{\sum_{i=1}^{N} W_i} \quad (8) \qquad \bar{y} = \frac{\sum_{i=1}^{N} W_i y_i}{\sum_{i=1}^{N} W_i} \quad (9)$$

$$U_i = x_i - \bar{x} \quad (10) \qquad V_i = y_i - \bar{y} \quad (11)$$

Step (iii)

$$\beta_i = W_i \left[\frac{U_i}{w(y_i)} + \frac{mV_i}{w(x_i)}\right] \quad (12) \qquad n = \frac{\sum_{i=1}^{N} W_i \beta_i V_i}{\sum_{i=1}^{N} W_i \beta_i U_i} \quad (13)$$

Step (iv)

$$c_e = \bar{y} - n\bar{x} \quad (14)$$

Equations 12 and 13 give the uncertainties in both regression parameters. These equations are only valid if the uncertainties in explanatory and response variables are uncorrelated. This case meet this assumption since the uncertainty in explanatory variable is constant (Figure 1).

$$u(c_e) = \sqrt{\frac{1}{\sum_{i=1}^{N} W_i} + \left(\bar{x} + \bar{\beta}\right)^2 u^2(n)} \quad (15) \qquad u(n) = \sqrt{\frac{1}{\sum_{i=1}^{N} W_i\left(\beta_i - \bar{\beta}\right)^2}} \quad (16)$$

Weighted Line of Organic Correlation

Another method for linear regression without iterative procedure is by minimizing the sum of the products of the horizontal and vertical distances between the observations and the fitting line. This method is the Line of Organic Correlation (Helsel & Hirsch, 2002), also called the geometric mean regression (GM) or the standardized major axis (MA) (Saqr & Khan, 2014). If uncertainties in both explanatory variables are constant, the regression parameters and their uncertainties are unbiased. However, this is not true in this case since the explanatory variable is not constant. To get rid of the “constant uncertainties in variables” constraint, it is possible to use a Weighted Line of Organic Correlation (WLOC) (Delmotte, 2017; Saqr & Khan, 2014). Delmotte recently developed in a conference paper the equations related to the WLOC method in the context of airtightness measurements (Delmotte, 2017).

Methods comparison: standard deviation vs. computed uncertainties

There are two ways to compute linear regression uncertainties. The first is by performing repeated measurements and by computing the standard deviation of the results. If the number of repeated measurement is enough, this is the “reference uncertainty” (i.e. the uncertainty considered as the true uncertainty). However, in practice, the operator performs only one test. In that case, the calculation of regression parameters depends on the regression method used (e.g. equations 15 and 16 for IWLS method) and on the uncertainties on variables (e.g. equations 1 to 4 for explanatory variable). These are the “calculated uncertainties”. If all the sources of uncertainties in variables are considered and if the appropriate regression method is used, the “calculated uncertainties” should be equal to the “reference uncertainties” (i.e. standard deviation observed for repeated measurements).

This paper computes and compares for each regression method (1) the average and the standard deviation of the airflow rate for the 30 tests and (2) the ability to give a reliable calculation of uncertainties (i.e. the difference between "reference uncertainty" and "calculated uncertainty").

2.3 Sequence of pressure

ISO 9972:2015 standards (ISO-9972, 2015) give some requirements and recommendations regarding the pressure sequence in the fan pressurization test. Firstly, the lowest pressure station must be lower than the greatest between 10 Pa and five times the zero-flow pressure. Secondly, there must be minimum five pressure stations with constant steps between them. Thirdly, the distance between two stations must be lower or equal to 10 Pa. Fourthly, the highest-pressure station should be minimum 50 Pa (but standard recommends a higher value). Lastly, standard recommends taking the average between pressurization and depressurization as a result.

This paper focuses on four different pressure sequences consistent with European standards. Three numbers ($a{:}b{:}c$) define each pressure sequence: the lowest-pressure station (a), the highest-pressure station (c) and the step between two stations (b). The four sequences tested are *Sequence 1 – 5:5:100*, *Sequence 2 – 10:5:100*, *Sequence 3 – 10:10:100* and *Sequence 4 – 5:5:50*. These pressure sequences allows to investigate the impact of low-pressure stations (Sequence 2), the impact of reducing the number of pressure stations (Sequence 3) and the use of low-pressure stations only (Sequence 4).

3 RESULTS

The airflow rate at 50 Pa using OLS regression method on *10:10:100* sequence for the 30 tests are:

- Mean of 229 m^3/h and standard deviation of 5.2 m^3/h (2.3%) in depressurization,
- Mean of 277 m^3/h and standard deviation of 4.4 m^3/h (1.6%) in pressurization and
- Mean of 253 m^3/h and standard deviation of 3.1 m^3/h (1.1%) in average

Table 2 shows that the variation of results in this research is similar to the variations of previous reproducibility tests made in similar conditions.

Table 2: Results of multiple reproducibility tests previously performed in similar conditions (same operator, same equipment, same apartment and varying climatic conditions) available in literature.

	Number of tests	**Average airflow rate at 50 Pa (m^3/h)**	**Standard deviation (%)**
(Persily, 1982)	17	3860	1.1 %
(Kim & Shaw, 1986)	7	1104	1.6 %
(Delmotte & Laverge, 2011)	10	732	1.4 %
(Novak, 2015)	92	1005	1.8 %
(Bracke, Laverge, Bossche, & Janssens, 2016) (1)	58	234	1.4 %
(Bracke et al., 2016) (2)	53	132	2.3 %
This paper	30	253	1.2 %

3.1 Change in regression method

Table 3 shows that the three different regression methods give similar values for the airflow rates at multiple pressure differences when averaging the results of the 30 tests.

Table 3: Average airflow rate for the 30 tests at low-pressure (10 Pa), mid-pressure (50 Pa) and high-pressure (100 Pa) differences

	Average airflow rate at 10 Pa (m^3/h)	**Average airflow rate at 50 Pa (m^3/h)**	**Average airflow rate at 100 Pa (m^3/h)**
OLS	82.70	252.62	409.15
IWLS	82.07	252.77	410.70
WLOC	82.76	252.80	409.17

On the contrary, the standard deviation of the airflow rates for the 30 tests depend on the regression method used. Figure 2 shows that at low- and high-pressure the airflow rates computed with WLOC and IWLS methods have lower standard deviation than with OLS method. At mid-pressure (i.e. around 50 Pa), none of the three methods seems to give lower standard deviation. These results are consistent with the observations made by Delmotte (Delmotte, 2017).

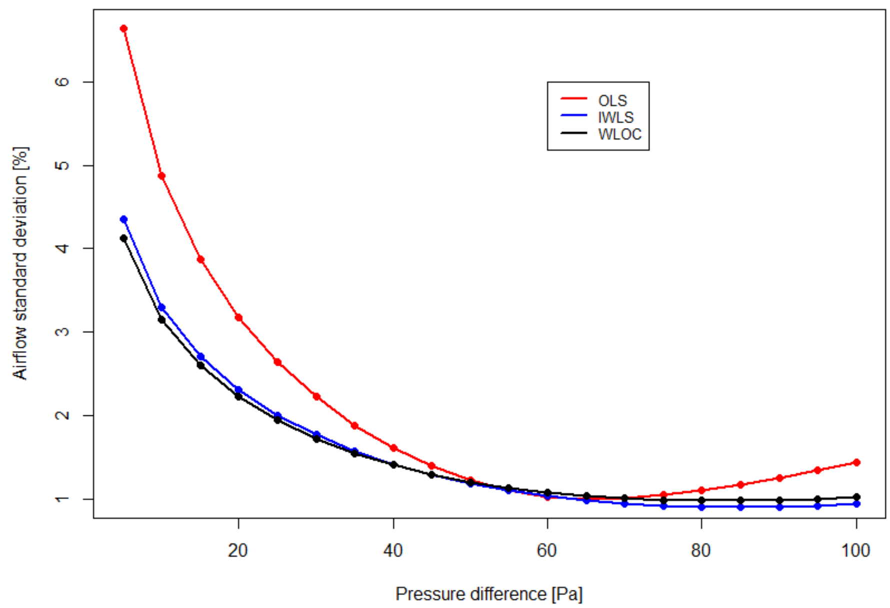

Figure 2: Standard deviation (in %) of the airflow rates computed for the 30 tests as a function of the pressure difference for the OLS, the IWLS and the WLOC regression methods.

In addition to its impact on standard deviation, the regression method also affects the calculated uncertainties. Figure 3 shows the average of 30 tests for both regression parameters (black dot), their calculated uncertainty (full blue line) and their standard deviation (dashed red line). These results firstly confirm the observation made on Figure 2: the use of IWLS and WLOC method instead of OLS method reduces the standard deviation of the regression parameters. Secondly, the uncertainties calculated using both IWLS and WLOC methods fit better the standard deviation (i.e. the "real uncertainties") than using OLS method. This means that using the equations in section 2.1 to compute uncertainties in variables, WLOC and IWLS methods are more reliable regarding the calculation of regression parameters uncertainties. However, there is still a difference between calculated uncertainties and standard deviation for WLOC and IWLS. Since this series of test meet the assumptions for WLOC and IWLS regression methods, the difference between real and calculated uncertainties implies that the method for computing variables uncertainty is still underestimating real uncertainties.

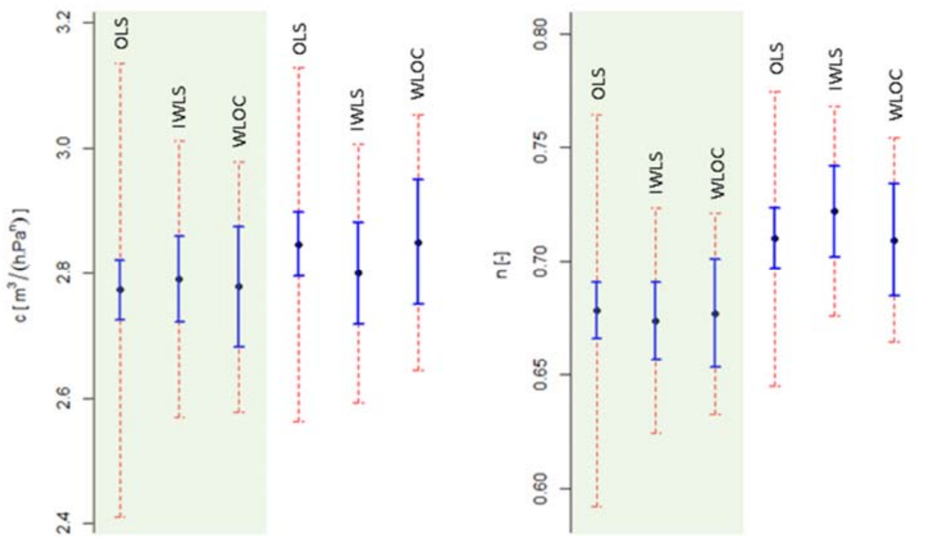

Figure 3: Average value (black dot), calculated uncertainty (full blue line) and standard deviation (dashed red line) of regression parameters in depressurization (green zones) and pressurization (white zones).

3.2 Change in sequence of pressure

Figure 4 shows the average of airflow rate values for the 30 tests at multiple pressure differences computed using OLS with four different pressure sequences. The three first sequences have all the same airflow averages but the fourth sequence (*5:5:50*, see section 2.3) shows a slightly higher average airflow rate at high-pressure.

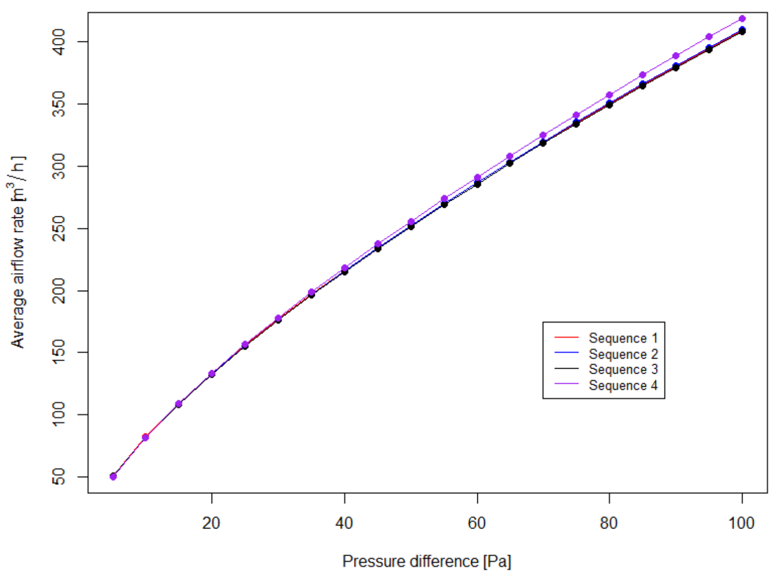

Figure 4: Average airflow rate for the 30 tests as a function of the pressure difference computed with OLS in four different pressure sequences

Figure 5 shows the impact of different pressure sequences on the standard deviation of airflow rate. Since the difference between WLOC and IWLS standard deviations is much lower than the difference with OLS (Figure 2), Figure 5 compares the four sequences using WLOC and OLS only. The Sequence 4 (*5:5:50*) has higher standard deviation at low and high pressure than other. This is coherent with the recommendation of ISO 9972:2015 standard (ISO-9972, 2015) that suggests to perform the linear regression with a highest-pressure station higher than 50 Pa. Sequence 1 (*5:5:100*) has higher standard deviation at low- and high-pressure than Sequence 2 (*10:5:100*) and Sequence 3 (*10:10:100*). This indicates that the use of low-pressure station when performing the regression increases the variability of airflow rate at low-pressure. It seems logical since the uncertainty in explanatory variable is higher at low-pressure stations (Figure 1). Performing the regression using 19 points (*Sequence 2 – 10:5:100*) lead to lower standard deviation than using 10 points (*Sequence 3 – 10:10:100*). However, the results are close and the use of 10 points reduces the time of testing and therefore reduces the uncertainty related to zero-flow pressure approximation. In our point of view, the advantage of taking more points to perform the linear regression is not straightforward.

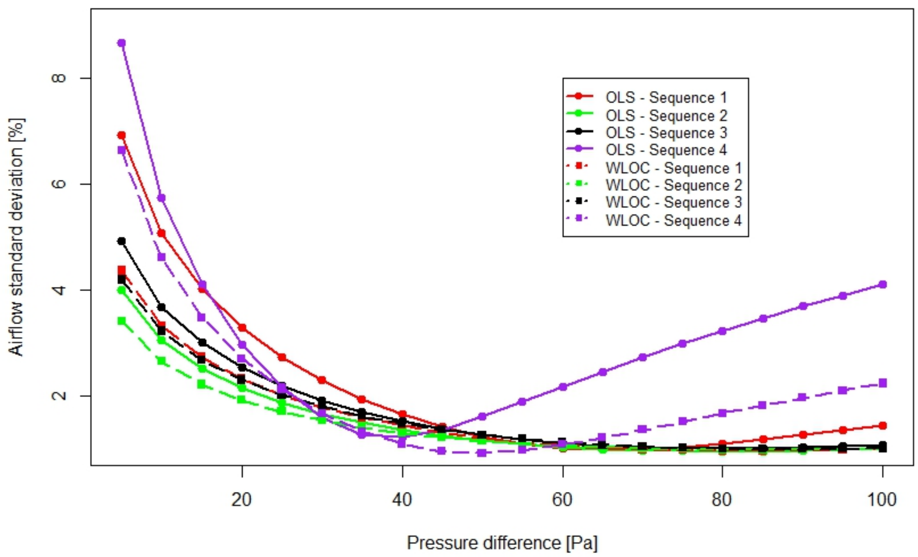

Figure 5: Airflow rate standard deviation (in %) for the 30 tests as a function of the pressure difference for four different pressure sequences computed using OLS (full lines) and WLOC (dashed lines).

Another interesting observation is that WLOC method reduces the airflow standard deviation, but it also reduces the impact of pressure sequences. Indeed, the range of possible standard deviations for different pressure sequences is lower when using WLOC than OLS. One could expect such a result since in WLOC, the stations with higher uncertainties (and therefore leading to an increase of standard deviation) have a lower weight in the determination of regression parameters. This highlights another advantage of WLOC method over OLS method.

4 CONCLUSIONS AND FURTHER WORK

This paper presents the results of a series of 30 tests in reproducibility conditions performed within a period of 10 days during October 2017. This series shows similar relative standard deviation than previous repeated measurements found in literature. Regarding the regression method applied on the 30 tests, IWLS and WLOC methods show similar averages and lower standard deviation than OLS for the airflow rate. In addition, the calculated uncertainties using IWLS and WLOC methods are closer to the real uncertainty (i.e. the standard deviation) than OLS. Regarding the sequence of pressure, results show that the recommendations of European standards were consistent, principally the need for a highest-pressure station higher than 50 Pa. Results also showed that the lowest-pressure station should be taken as high as possible (within the standard requirements) to reduce the airflow rate standard deviation.

The conclusions drawn from this series of tests lead to multiple possible further work. Firstly, these conclusions relate to the apartment tested. An important further work is the comparison of different regression methods on other series of tests in reproducibility condition to confirm and generalized the results found here. Secondly, further researches could also investigate other regression methods as grouping methods or instrumental variables (Gillard, 2010). Thirdly, even if the uncertainties computed with WLOC and IWLS methods seem better than with OLS, there are still differences between computed and real uncertainties. The quantification of uncertainties should be a priority in researches dealing with fan pressurization uncertainties.

5 ACKNOWLEDGEMENTS

Authors want to thank the company *Jacques Delens* for its mobilization in this work and the availability of their construction sites. This work is part of the "AirPath50" project funded by INNOVIRIS.

6 REFERENCES

Bracke, W., Laverge, J., Bossche, N. V. D., & Janssens, A. (2016). Durability and Measurement Uncertainty of Airtightness in Extremely Airtight Dwellings. *International Journal of Ventilation, 14*(4), 383-394.

Carrié, F. R., & Leprince, V. (2016). Uncertainties in building pressurisation tests due to steady wind. *Energy and buildings, 116*, 656-665.

Delmotte, C. (2013). *Airtightness of buildings-Calculation of combined standard uncertainty.* Paper presented at the 34th AIVC Conference "Energy conservation technologies for mitigation and adaptation in the built environment: the role of ventilation strategies and smart materials", Athens, Greece.

Delmotte, C. (2017). *Airtightness of Buildings - Considerations regarding the Zero-Flow Pressure and the Weighted Line of Organic Correlation.* Paper presented at the 38th AIVC Conference "Ventilating healthy low-energy buildings", Nottingham, UK.

Delmotte, C., & Laverge, J. (2011). *Interlaboratory tests for the determination of repeatability and reproducibility of buildings airtightness measurements.* Paper presented at the 32nd AIVC conference and 1st TightVent Conference: "Towards Optimal Airtightness Performance", Brussels, Belgium.

Gillard, J. W. (2010). An overview of linear structural models. *Revstat, 8*(1), 57-80.

Helsel, D. R., & Hirsch, R. M. (2002). Statistical Methods in Water Resources *Techniques of Water Resources Investigations* (Vol. Book 4, Chapter A3, pp. 522): U.S. Geological Survey.

ISO-9972. (2015). NBN EN ISO 9972:2015 - Performance thermique des bâtiments - Détermination de la perméabilité à l'air des bâtiments - Méthode de pressurisation par ventilateur (ISO 9972:2015) CEN, Brussels, Belgium.

JCGM. (2008). Evaluation of measurement data—guide for the expression of uncertainty in measurement. (pp. 134): Joint Committee for Guides in Metrology.

Kim, A. K., & Shaw, C. Y. (1986). Seasonal variation in airtightness of two detached houses *Measured Air Leakage of Buildings: A Symposium* (pp. 16-32). Philadelphia, USA: ASTM International.

Novak, J. (2015). *Repeatability and reproductibility of blower door tests - four years' experience of round-robin tests in Czech republic*. Paper presented at the 9th International Buildair Symposium, Germany.

Okuyama, H., & Onishi, Y. (2012). Reconsideration of parameter estimation and reliability evaluation methods for building airtightness measurement using fan pressurization. *Building and Environment, 47*, 373-384.

Persily, A. (1982). *Repeatability and accuracy of pressurization testing.* Paper presented at the ASHRAE/DOE Conference Thermal Performance of the Exterior Envelopes of Buildings II, Las Vegas, USA.

Saqr, A., & Khan, S. (2014). New weighted geometric mean method to estimate the slope of measurement error model. *Journal of Applied Statistical Science, 22*(3/4).

York, D., Evensen, N. M., Martınez, M. L., & Delgado, J. D. B. (2004). Unified equations for the slope, intercept, and standard errors of the best straight line. *American Journal of Physics, 72*(3), 367-375. doi:10.1119/1.1632486