DEVELOPMENT AND USE OF EQUATION BASED SIMULATION TOOLS TO SUPPORT AUDIT OF COMMERCIAL BUILDINGS

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ABSTRACT

A series of new simulation tools are being developed to help the building energy auditor in establishing his diagnosis (including benchmarking and detailed analysis of actual energy consumption) and in evaluating the selected Energy Conservation Opportunities (ECOs).

The Building-HVAC System global models presented here include simplified models of building zones and of HVAC equipment. Only a limited number of easily identifiable parameters are required. The simplicity of the models and the use of an equation solver to run the simulation ensure good robustness and full transparency. The development, the implementation and the use of these models are discussed in the present paper. Focus is also given to the calibration of the tools to monthly utility bills.

INTRODUCTION

Environmental concerns and the recent increase of energy costs open the door to innovative techniques to reduce energy consumption. Buildings account for approximatly 40% of the European energy consumption. Non-residential buildings represent a significant part of this consumption and improvement of their energy performances is a major challenge of the 21th century. To this end, the European Commission approved the European Directive on Energy Performance of Buildings on 16 December 2002.

To promote improvements in the HVAC installations of existing buildings, the article 9 of the EPBD directive establishes mandatory audits and inspections of air-conditioning systems. The development of auditing tools and procedures and the training of future auditors are the main objectives of the HARMONAC project launched in 2007.

Four audit stages are generally distinguished (Adnot et al., 2007):

1. The "benchmarking" helps in deciding if it is necessary to launch a complete audit procedure; it's based mainly on energy bills, basic calculations and comparisons to reference values ("benchmarks").
2. The aim of the "pre-audit" is to identify the main defects and "energy conservation opportunities" (ECO's). Its results are supposed to orient the future "detailed" audit. The inspection consists in a visual verification of HVAC equipment, in an analysis of operating data records and in a systematic disaggregation of recorded energy consumptions.
3. The "detailed" audit consists in a detailed and comparative evaluation of the ECOs previously selected.
4. The "investment grade" audit concerns the detailed technical and economical engineering studies, justifying the costs of the retrofits. This fourth audit stage brings the system (building + HVAC) to a new life cycle: new design, call for tenders, submissions, evaluations, installations, commissioning, etc.

Answering these questions requires some diagnosis, which has to be established on the basis of the very scarce information currently available: technical data contained in as-built files actually available and, generally, very global recordings of energy consumptions (fuel and electricity).

A series of equation-based simulation tools are being developed in the frame of the HARMONAC project (2008) to help the auditor in establishing his diagnosis despite of the lack of data.

The main modeling issues and the methodology are discussed in the first part of the paper. Focus is given to the parameterization work and to the calibration process in the second part of the paper. Finally, some retrofit options are assessed and compared.

METHODOLOGY

This section shows how the proposed simplified building energy simulation tools can be used through the step-by-step audit procedure proposed here above.

Benchmarking

The first tool, called "BENCHMARK" (Bertagnolio, 2008a), is used to compute the "theoretical" (or "reference") consumptions of the actual building, supposed to be equipped with a “typical” HVAC system, including air quality, temperature and humidity control. The building is considered as a unique zone, described by very limited number of parameters. This first simulation tool should help the auditor in getting, a very first impression about the performances of the system and very first interpretation of the recorded consumptions.

The “typical” HVAC system is a classical air-and-water system including an equivalent (or consolidated) single-duct CAV AHU ensuring air quality (hygienic ventilation flow rate) and humidity control (indoor RH varying between 40 and 60%). Temperature setpoint is maintained by means of classical heating-cooling fan coil units located in the zone. Heat and cold production are ensured by, respectively, natural gas boilers and air cooled chillers. The nominal performances of each component are defined basing on rule of the thumb values or on European Standards (prEN 13053 and 13779). Occupancy and operating profiles used are typical profiles selected in accordance with the use of the building.

Audit and calibration

The second tool, called “SIMAUDIT”, supports the auditor during the inspection and the detailed audit stages. The building is still considered as a unique zone or cut in up to 5 zones. Each zone is coupled to a more realistic HVAC system, representing the actual one. This second tool allows making a more detailed analysis and a disaggregation of the actual building energy use.

In the frame of a research project initiated by ASHRAE, Reddy (2006) identified four main types of calibration methods: based on manual, iterative and pragmatic intervention; based on graphical comparative displays; based on special tests and based on mathematical methods. The three first methods are heuristic methods mainly based on user’s knowledge. In the frame of this project, a complete, systematic and quite complex calibration methodology involving heuristic and numerical methods has also been developed and implemented in DOE-2 program.

To keep the tools as simple as possible, only heuristic calibration guidelines, similar to the one proposed by Yoon et al. (2003), are used here. These guidelines are applied to the simplified building energy simulation tool presented above. This calibration method is fully integrated into the audit procedure described above and is based on detailed the study of collected data and a graphical analysis of energy records.

Analysis and evaluation of selected ECOs

After having been calibrated to the recorded data, the baseline model can be used to identify the main energy consumers (lights, appliances, fans, pumps …) and quantify their contribution. After that, the baseline model can be modified to assess the performance of the selected ECOs.

MODELING

Reddy and Maor (2006) have identified five types of building energy simulation tools:

1. Spreadsheet programs
2. Simplified system simulation method
3. Fixed schematic hourly simulation programs
4. Modular variable time step simulation programs
5. Specialized simulation programs

Spreadsheet programs (1) and steady state simplified methods (2) have shown their limits in predicting the energy use of modern buildings. Specialized programs (5) are mostly dedicated to the simulation of particular phenomena (such as contaminants movement, air stratification…). The aim of this work is to develop simulation tools associating the simplicity of quasi-steady state fixed schematic hourly simulation (3) programs (generally based on the LSPE sequential approach: loads - secondary system - primary system - economics) and the flexibility of modular simulation programs (4) based on more realistic models and taking all the interactions (building/system/control) into account.

This section presents the main modeling issues of the two simulation tools mentioned above. Both tools include two parts: a simplified dynamic building model is coupled to a steady-state model of the primary and secondary HVAC equipments. They mainly differ in the amount of information used to tune the building and HVAC simulation models:

- BENCHMARK is supplied with just a minimum of geometrical and scale data. The HVAC system is fixed to a typical configuration.
- SIMAUDIT is supplied with much more detailed information on building and HVAC system in such a way to fit as well as possible with the actual energy consumptions.

Zone Model

A simplified dynamic zone model is used. It is based on the lumped capacitance method as proposed by Laret (1981) and further developed by, among others, Masy (2008). The chosen model is a third-order lumped model (Figure 1). Heavy external walls and heavy internal walls (as ceilings and floors, considered as adiabatic) are modeled by, respectively, a 2RC branch and a 1RC branch. The third capacitance simulates the indoor environment.

The values of the resistances and capacitances are computed and adjusted through analytical calculations based on the admittance method (Masy, 2008).

Analytical, empirical and comparative validations have been performed (Bertagnolio et al., 2008b) and have shown that the simplified model developed is able to predict indoor conditions and building heating and cooling demands with a sufficient accuracy.

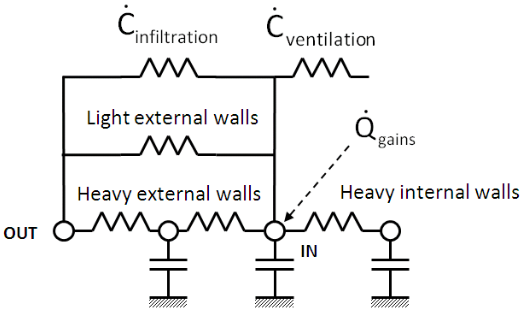

Figure 1 Building model RC network

Secondary HVAC System Model

Steady-state models are used to simulate secondary and primary HVAC components.

In both BENCHMARK and SIMAUDIT, the secondary HVAC system model includes a model of a complete Air Handling Unit (AHU) and models of different heating and/or cooling Terminal Units (TU). The AHU considered includes (Figure 2):

- Air-to-air recovery system
- Economizer (or mixing system)
- Adiabatic or steam humidification system
- Dehumidifying or cooling coil
- Post heating coil
- Extraction and supply fans
- Filters

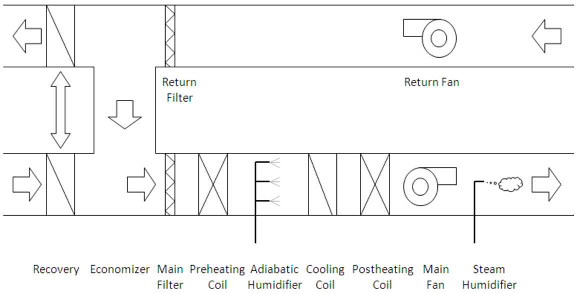

Figure 2 AHU components available in the tool

Emission systems include radiators, induction or fan coil units, heating floor and cooling ceiling and are directly controlled to maintain indoor temperature.

Of course, these components are never to be selected all together at the same time.

Primary HVAC system model

The heating and cooling plant system considered includes two water networks, two main pumps, one heat production system (e.g. a gas boiler) and one cold production system (e.g. an air cooled chiller). The performance curves of these components are typical curves extracted from manufacturer data (Underwood and Yik, 2004) or generated by means of reference models (Adam et al., 2006).

Control

Building and System entities are generally modeled separately, and called in a sequential way during a simulation process. This is not the case here; building and HVAC system models are directly coupled. This approach allows to take into account of HVAC components limited capacities and to model the interactions between the building and the system

Ideal proportional control laws are used to control all the HVAC system components (AHU coils, TUs…). The control variable X varies between 1 and 0 (eq. 1).

$$X_{control}=\mathrm{MIN}\left[1,\mathrm{MAX}\left[0,C_{control}*\left(t_{setpoint}-t\right)\right]\right] \qquad (1)$$

This control allows simple and robust simulation. Of course, the proportional control gain $C_{control}$ has to be chosen as a compromise between accuracy and reactivity of the control.

Zoning

The simulation tool presented here can simulate 1 to 5 zones. The four first zones have external walls (i.e. in contact with the outdoor) facing one or several orientations. The fifth zone interevene only in the peripheral-core configuration (Figure 3).

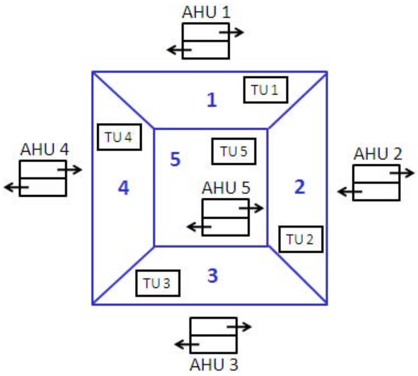

Figure 3 Five zones configuration

The conductive heat transfer between zones is taken into account by connecting the different indoor nodes with additional branches. Only purely resistive branches are used to simplify the calculation. Each zone is equipped with its equivalent AHU and TU. Of course, the different zones can be characterized by different internal gains, operating schedules…

The mass transfer between zones is not taken into account.

Inputs, parameters and outputs

The outputs, inputs and parameters are selected according to the specific needs of the user.

The main outputs of the tool are:

- Air quality and hygrothermal comfort achievements: CO2 contamination, temperature, humidity, PPD and PMV indexes
- Power distribution and energy consumptions (Fuel and Electricity)
- HVAC components specific demands

The main inputs (provided in tables) are:

- Weather data: hourly values of temperature, humidity, global and diffuse radiations
- Nominal occupancy loads (in W/m²), occupancy rates
- Comfort requirements: air renewal, temperature and humidity set points
- HVAC installation functioning rates
- Control strategies and set points: feedback on indoor temperature and relative humidity, feedforward on occupancy schedules and calendar.

The main parameters are:

- Dimensions, orientation and general characteristics of the building envelope;
- Main characteristics of the HVAC system (type, nominal performances, sizing data)

If they are not available, the parameters of the model, as HVAC system characteristics and nominal performances and capacities, can be automatically computed through a pre-sizing calculation or estimated by means of some rules of thumb.

Implementation in an equation solver

The presented model is implemented in an Engineering equation solver (Klein, 2008). This implementation ensures a full transparency for the user and makes easier the continuous improvement and development of the tools. It is also very easy to develop additional HVAC components models and to connect them to the existing ones. Moreover, the present equation solver is very well adapted to solve differential equations systems as used to model the thermal behaviour of the building zone.

Of course, the use of an equation solver to solve complex equation systems implies longer computation time than other simulation softwares, but the continuous increase of computer performances tends to reduce this inconvenience. At present time, about 20 minutes are necessary to simulate a mono-zone building and its complete HVAC system (including AHU, terminal units and heat and cold production and distribution) hour by hour on one year with a classical computer equipped with a 2.00GHz processor.

The model and its implementation have been detailed in a previous paper (Bertagnolio and Lebrun, 2008a).

EXAMPLE

An example of application of the audit procedure and of use of both tools is presented hereafter. The building under study is a medium-size office building (around 26700 m² of air-conditioned floor area), built in Brussels at the end of the sixties.

Building description

The building is composed of three blocks, has an "H" shape (Figure 4) and is North-South oriented. The nine floors of the building include a lobby and eight levels of landscaped offices and meeting rooms. The five basement levels are dedicated to cars parking.

The frontages of the lobby are made of single-glazed windows. The rest of building envelope is made of about 1000 double-glazed modules, equipped with external solar protection (Figure 4). The global heat transfer coefficient of the envelope is about 23 kW/K.

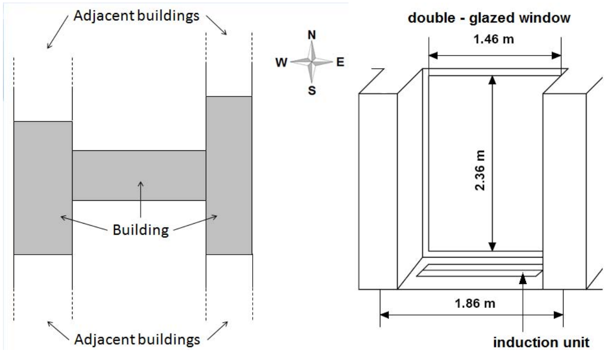

Figure 4: Case study building (left) and envelope module (right)

HVAC system

About 1000 four-pipes heating and cooling induction units are installed in the offices. The CAV Air Handling Units provide together a total ventilation flow rate of about 290000 m³/h with a fresh air ratio of about 66% (i.e. about 190000 m³/h), 75 hours per week, to the nine conditioned storeys (from level 0 to 8). Extracted air is rejected in the basement storeys to ensure ventilation of the parking. This ventilation flow rate corresponds to about 2.4 air renewals per hour. According to the weather conditions, the supplied air can be heated and adiabatically humidified, or cooled and dehumidified.

Heat production is ensured by four fuel-oil boilers, giving together a nominal heating capacity of about 4 MW. Chilled water production is ensured by four water-cooled chillers, coupled to two cooling towers, giving a total cooling capacity of about 2.1MW.

RECORDED DATA ANALYSIS

Monthly records of fuel consumptions are available from 1971 to 2005 and monthly records of electricity consumption are available from 2005 to 2007. The fuel and electricity consumptions for 2005 are shown in Figure 5. The average electricity consumption of this period is floating around 520 MWh/month ± 10 %. No seasonal variation is noted.

The very high electricity consumptions are probably (but not completely) due to important plug-loads and lighting equipements installed in the building.

As seasonal variations are insignificant, it is not possible to identify the impact of the chillers or base load consumption. More detailed records would be required to go further in this analysis: hourly records and/or separate records for HVAC and non-HVAC consumptions.

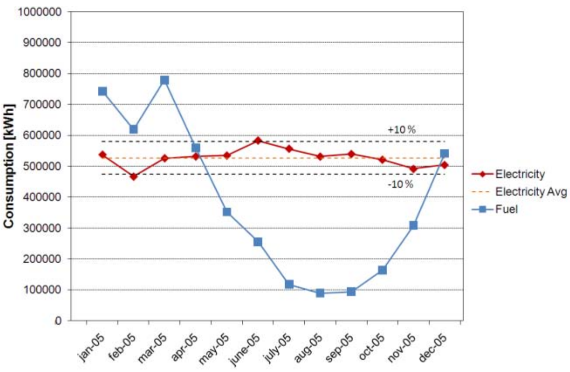

Figure 5: Monthly fuel and electricity consumptions (2005)

The fuel oil energy consumption shows much larger variations, around an average of 440 MWh/month.

Figure 6 shows the distribution of fuel oil power defined in montly average (from 1997 to 2005), as function of the external dry bulb temperature.

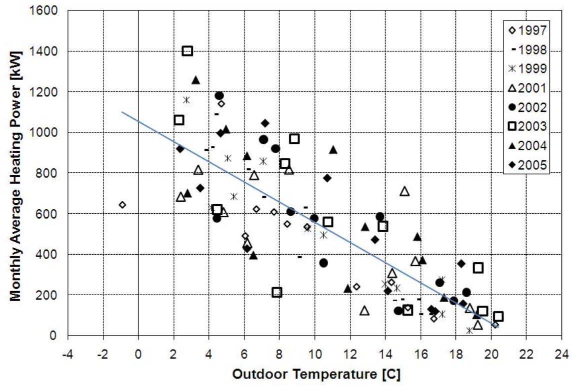

Figure 6: Building thermal signature

This thermal signature allows identifying a useful linear regression. The slope of this law (about 50 kW/K) should be of the same order of magnitude as the average heat transfer coefficient of the building (including the heat losses through the envelope and the ventilation needs, eq. 2).

$$K = 23\left[\frac{kW}{K}\right] + 64\left[\frac{kW}{K}\right] * \frac{75}{168} \approx 52\left[\frac{kW}{K}\right] \qquad (2)$$

Both values are in fair agreement and confirm that the fuel consumption is mainly due to transmission and ventilation losses.

Theoretically, the value indentified thanks to the thermal signature should have been a little higher, because of the latent heat power consumed to humidify the air, which is not taken into account in the heat transfer coefficient K. The remaining error found in this building "signature" is very probably due to the effect of the inside temperature control (the temperature inside the building is "sliding" down slowly with the outside temperature) and to errors in the fuel consumption measurements.

The poor correlation coefficient (R^2=66%) are explained by the fact that the fuel consumption is not only influenced by the outdoor temperature but also by other parameters:

- The solar gains;
- The heterogeneous use of HVAC equipment (due to variable comfort requirements, variable occupancy rate, variable internal loads…).

However, considering the fact that the building is equipped with very efficient solar protections, the first influence can be neglected and the discrepancies in the recorded data should be due to the variable occupancy, variable internal loads associated and variable way of using the HVAC system.

Bertagnolio and Hannay (2008c) give a more detailed analysis of the available as-built and recorded data.

USE OF SIMULATION TOOLS

It has been decided to compare the computed fuel and electricity consumptions to averaged data because of the following reasons:

- The very heterogeneous use of the HVAC equipment and the important dispersion of the recorded data (Figure 6) make the calibration to one-year data very difficult and arbitrary;
- There are some uncertainties in the fuel consumption recording schedule;
- As it is often the case, actual weather data are not available for the considered period.

The use of energy records averaged on up to 35 years should make the consumption profiles smoother, hide the heterogeneities in the building operation an tend to minimize the errors due to the use of a typical weather data set. Indeed, actual, recent and complete weather data are not available in the present case and typical hourly weather data are used all along this analysis. So, the aim of the calibration performed here is not to represent the behavior of the building for a given year but in average.

Benchmarking

The "typical" system considered at the benchmarking stage has to be able to satisfy temperature, humidity and air quality requirements.

So, a hygienic ventilation flow rate is fixed to 45 m³/h of fresh air per hour and per occupant. The global AHU is equipped with an adiabatic humidification system and a standard cross-flow air-to-air heat recovery system. Lighting, plug loads and equipment performances are defined according to prEN standards and rule of thumb values.

As expected, the fuel and the electricity consumptions are largely underestimated by the software (Table 1). This suggests the existence of important energy savings potentials.

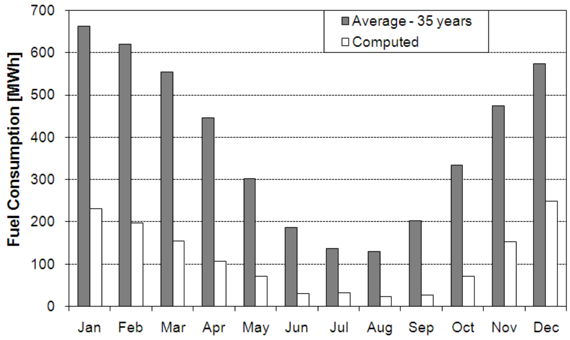

Figure 7: Recorded and computed monthly fuel consumptions (benchmarking)

Table 1 Annual recorded and computed fuel and electricity consumptions (benchmarking)

CONSUMPTIONS	FUEL	ELEC.
Computed [MWh]	1346	3232
Recorded [MWh]	4625	6187
Difference [%]	71	48

Model calibration and analysis

To simplify the parametrization and the calibration works, the mono-zone model, previously used in "BENCHMARK" is also used in the second part of the study. The results obtained with this simplifed global model should surely be improved by using the multi-zone capability of the tool. This fact will be discussed in further papers.

As aforementioned, the calibration method used here is totally heuristic and based on practical guidelines. The computed and recorded datawill be compared in terms of Mean Bias Error (MBE) and coefficient of variation of Root Mean Squared Error (CV(RMSE))

During the first step, the tuning of the parameters is based on as-built data and on the analysis of recorded data. In constrast with the benchmarking made before, this consists in implementing:

- The actual primary and secondary HVAC components;
- The actual setpoints and occupancy and operating schedules;
- The design air renewal rate.

Standard values coming from prEN standards and rule of thumb values are used for the undetermined HVAC components characteristics (as pump, fans and plant performances) and for the plug and lighting loads. At the end of this first step, it appears that the electricity consumption is largely underestimated and has to be re-evaluated. Even if the fuel consumption seems to be well estimated, the calibration of the whole system (building and HVAC system) has to be continued. Indeed, both electricity and fuel consumptions are intrinsincally linked and cannot be studied separatly.

During the second step, plug and lighting loads are adjusted basing on observations made during site visits and interviews. As expected, the electricity consumption increases and the fuel consumption decreases because of the increasing of density of lighting and plug loads (Table 2). At this step of the calibration work, the shapes of the consumption profiles are similar to the recorded ones but a "scale" difference remains.

Table 2 Calibration - simulation runs

RUN #	ERROR	FUEL	ELEC.
0 (benchmark)	MBE CV(RMSE)	-70.9 % 22.2 %	-47.8 % 13.8 %
1 (base case)	MBE CV(RMSE)	-4.4 % 4.2 %	-25.6 % 7.4 %
2	MBE CV(RMSE)	-11.9 % 4.5 %	- 9.4 % 2.8 %
3 (baseline)	MBE CV(RMSE)	-6.6 % 3.4 %	-3.2 % 1.6 %

The third step consisted in changing the performance of primary HVAC heating and cooling equipment (for which data ar not available) in more realistic values, depending on the type and age of the equipment. At the end of this third step, the results are satisfying and both MBE and CV(RMSE) are near the range defined by ASHRAE guideline 14-2002, respectively, +/- 5 % and +/- 15 %.

A comparison between computed and recorded electricity consumptions is shown in Figure 8. The consumption profile is quite well reproduced by the calibrated model. However, this calibration criteria is certainly not satisfying because of the constancy of the electricity consumption over the year and the small contribution of weather dependent loads (chiller) compared to the base load (lighting, appliances…) Indeed, the main part of the base load consumption is directly defined by simulation parameters (lighting density, occupation factors…).

Figure 9 shows a comparison between the thermal signature based on 5 years of recorded data and the one generated by the calibrated model (dotted line). Once more, the results are satisfying and the thermal behavior of the building and its system are well represented. The slope of both recorded and computed signatures, respectively 50 and 55 kW/K, are in good agreement with the building global heat transfer coefficient mentioned here above (52 kW/K).

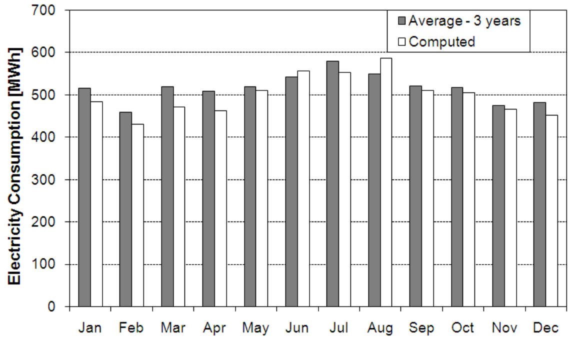

Figure 8 Recorded and computed monthly electricity consumptions (calibrated model)

The computed thermal signature (dotted line) is characterized by a better correlation coefficient (R^2=98%). This indicates that, with strictly well defined occupancy, corresponding internal gains and simplified HVAC control, the fuel consumption is mainly correlated to the outdoor temperature. So, as supposed above, the discrepancies observed with the recorded data (R^2=66%) should be mainly due to the variations in occupancy rate, internal gains and control strategy.

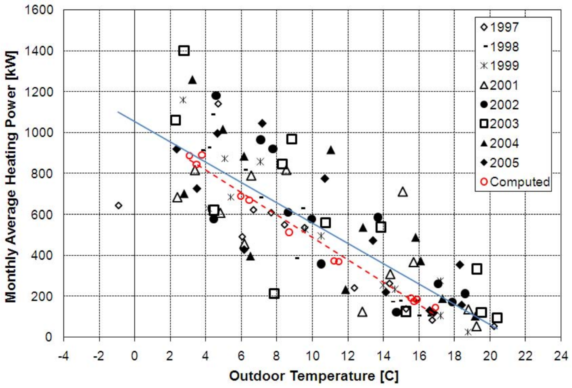

Figure 9 Recorded (full) and computed (dotted) thermal signatures

Even if the calibration has not been performed for a given year because of the reasons previously discussed, the simplified model used here seems to be able to represent the "average" behavior of the building and its system.

After calibration, the simulation tool is used to disaggregate the electricity consumption and to identify the main energy consumers.

As it appears in Figure 10, an important part of the monthly electricity consumption is due to lighting and plug loads (appliances). An important part is also due to ventilation fans, pumps and basement utilities (extraction fans to basement parking zone and parking lighting). Only 10% of the annual electricity consumption are due to the chillers.

Evaluation of retrofit options

About ten Energy Conservation Opportunities (ECOs) have been identified in a previous paper (Bertagnolio et al., 2008c). Only a few ones are evaluated here.

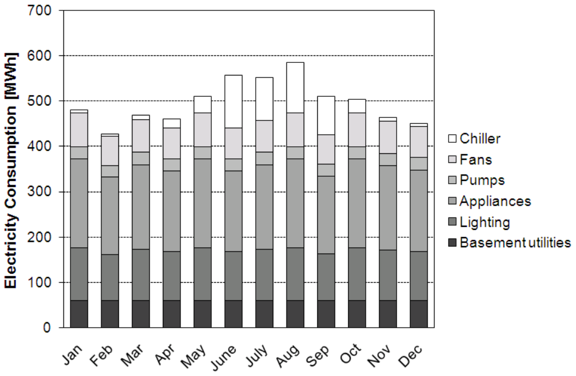

Figure 10 Monthly disaggregation of the computed electricity consumption

The simplest retrofit opportunity is certainly the implementation of a better fresh air management. It is proposed to reduce the air renovation time period from 75h to 55h per week, in order to fit better with the occupancy period.

The second retrofit option is to to increase air recirculation, whenever the induction units require more primary air than what is needed for IAQ. Currently, the fresh air flow rate per occupant is about 170 m³/h. It is proposed to reduce it to about 72 m³/h (corresponding to "high indoor air quality" as defined by prEN standard 13779).

The third retrofit option envisaged here is to install a classical air-to-air recovery system on the fresh air intake.

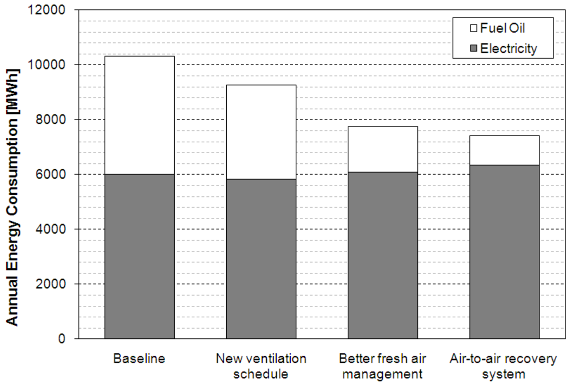

Figure 11 Retrofit options comparison

As shown in Figure 11 a better management of the ventilation schedule and the air recirculation allows a reduction of about 60% of the fuel consumption and causes only a slight increase of the electricity consumption (less than 2%, due to additional pressure drops in the economizer). The installation of an air-to-air recovery system leads to a smaller reduction of fuel consumption (15%) but causes a bigger increase of electricity consumption (6%).

Of course, other retrofit options, such as replacement of lighting and appliances by less-consuming devices or reduction of the primary air flow rate should also be studied.

FUTURE WORK

Calibration is clearly an underdetermined problem: the presence of too many parameters and too limited input data (such as global consumptions records) should result in a non-unique solution. Reddy and Maor (2006) have shown that a good calibration cannot be based on only one set of values. So, it would be very convenient to make sensitivity analysis allowing the identification of a few realistic sets of parameters.

Another improvement would consist in using the multi-zone capability of the simulation tools to make a more detailed and more realistic representation of the building and its heating/cooling demands profiles.

CONCLUSION

Even if the heuristic calibration procedure applied here requires more time and skills than a more automatic calibration method, the application of this step-by-step calibration procedure to very simplified simulation tools allows the user (even beginner) to feel the influence of each parameter on the global energy consumption of the whole system and to perform quick and accurate enough calibration.

Two simplified building energy simulation tools have been presented here. Their use has been illustrated in the frame of a complete audit procedure. The two tools allow quick and useful evaluation and analysis of the behavior of the building. A heuristic calibration method has been used to calibrate the tool to averaged fuel and electricity consumptions. The use of averaged data has allowed us to avoid the effect of the variable operation of the building and its system.

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REFERENCES

ASHRAE (2002) ASHRAE Guideline: Measurement of energy demand and savings. ASHRAE Guideline 14-2002.

Adam, C., André, P., Georges, B., Lebrun, J., Lemort, V., Teodorese, I.V. (2006) From model validation to production of reference simulations: How to increase reliability and applicability of building and HVAC system simulation models. Proceedings of the 7th International Conference on System Simulation in Buildings, Liège, Belgium.

Adnot, J. et al. 2007. Field benchmarking and Market development for Audit methods in Air Conditioning (AUDITAC) final report. Intelligent Energy Europe.

Bertagnolio S., Lebrun J. (2008a) Simulation of a building and its HVAC system with an equation solver. Application to benchmarking. Building Simulation : An International Journal. Vol. 1, pp 234-250.

Bertagnolio S, Masy G, Lebrun J, André P (2008b). Building and HVAC System simulation with the help of an engineering equation solver. Proceedings of the Simbuild 2008 Conference, Berkeley, USA.

Bertagnolio, S., Lebrun, J., Hannay, J., Aparecida Silva, C. (2008c) Simulation – assisted audit of an air conditioned building.Proceedings of the 8th International Conference for Enhanced Building Operations, Berlin, Germany.

European Standard prEN13053 (2003) Ventilation for buildings - Air handling units - Ratings and performance for units, components and sections.

European Standard prEN13779 (2007) Ventilation for non-residential buildings – Performance requirements for ventilation and room-conditioning systems.

HarmonAC (2008) Harmonac project : Harmonizing Air Conditioning Inspection and Audit Procedures in the Tertiary Building Sector. Intelligent Energy Europe programme.

Klein SA (2008) EES: Engineering Equation Solver, User manual. F-chart software. Madison: University of Wisconsin, Madison, USA.

Laret, L. (1981) Contribution au développement de modèles mathématiques du comportement thermique transitoire de structures d'habitation. PhD thesis, University of Liège. (in French)

Masy, G. (2008) Definition and Validation of a Simplified Multizone Dynamic Building Model Connected to Heating System and HVAC Unit. PhD thesis, University of Liège.

Reddy, T.A., Maor, I. (2006) Procedures for reconciling computer-calculated results with measured energy data. ASHRAE Research Project 1051-RP. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

Underwood, C.P., Yik, W.H. (2004) Modelling methods for Energy in Buildings. Blackwell Publishing, Oxford, UK.

Yoon, J., Lee, E.J., Claridge, D.E. (2003) Calibration procedure for energy performance simulation of a commercial building. Journal of Solar Energy Engineering. Vol. 125, pp 251-257.