FINITE ELEMENT MODELLING OF COUPLED HEAT AND MOISTURE TRANSFER IN TYPICAL EARTH-SHELTERED BUILDING ENVELOPE

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ABSTRACT

It is necessary to do research on the heat and moisture transfer in the earth-sheltered building for getting a better underground environment. Based on Philips and De Vries' and Zhang's theory (Philip et al., 1957; Zhang et al., 2006), this paper establishes a detailed 2-D transient mathematical model to understand the heat and moisture transfer exactly in the earth-sheltered building envelope and the surrounding soils. A general FEM (Finite Element Method) software is adopted to solve the highly nonlinear governing equations. The numerical simulation showed that the existence of waterproof layer makes the temperature field continuous and the moisture field discontinuous. The results may afford the basis of heat and moisture load calculation for the earth-sheltered building.

INTRODUCTION

With the development of economy and society, the underground space is explored widely in China to enlarge the urban space (Carmody et al., 1982; Tong, 2005; Qian et al., 2007). However, moisture is one of the most important factors limiting an earth-sheltered building's service life. Moisture damage has been identified as one of the main reasons for building envelope deterioration. More recently recognized are the potential serious health hazards of mould and other organisms which flourish in buildings and constructions with excessive moisture (Geving, 1997). Therefore, it is necessary to do research on the heat and moisture transfer in the earth-sheltered buildings for getting a better underground environment.

For decades, many researchers have devoted to the work of modelling of heat and moisture transfer in ground building envelope (Straube et al., 2001). As early as 1960s, Luikov (Luikov, 1966) has firstly proposed a mathematical model for simultaneous heat and mass (moisture) transfer in building porous materials. The conservation equations include the mass, the momentum and the energy conservation equations. The constitutive equations are described by Darcy's law, Fick's law and Fourier's law. However, the solutions are either numerical or complicated involving complex eigenvalues. Cary and Taylor (Cary et al., 1962; Cary et al., 1962) uses irreversible thermodynamics to describe the interactions of the forces and fluxes involved. A mechanistic approach based on physical models of the phenomenological processes that occur in the soil was established by Philips and De Vries (Philip et al., 1957; De Vries, 1958). Whitaker (Whitaker, 1973; Whitaker, 1977; Whitaker, 1980) averaged the transport equation on a representative elementary volume (REV) at the continuum level and obtained the governing equations in a higher level. This modelling method overcomes the modelling difficulty that porous media are heterogeneous. But the transport coefficients of the model can not be get easily without the complex experiments. These models have been adopted widely by many researchers (Deru, 2001; Lu, 2002; Lu, 2002; Janssen et al., 2004; Qin et al., 2006). According to the models of Philip, De Vries, Luikov and Whitaker, Liu (Liu et al., 1995; Liu et al., 1998) developed the multiphysics-phase change-diffuse model recently which based on the Navier-Stokes equation. This model has seven field variables. Besides, there are more models established by the researchers worldwide (Thomas, 1987; Pedersen, 1990; Matsumoto et al., 1997; Mendes et al., 2002; dos Santos et al., 2006; Tariku et al., 2006).

Compared to the heat and moisture research in the ground buildings, the literature referred to earth-sheltered building is limited. Ogura analysed the heat and moisture behaviour in underground space by quasilinearized method (Ogura et al., 1999). Zhang had presented a mathematical model which described with temperature and relative humidity as driving potential for coupled heat and moisture transfer in porous wall materials. This model could make the discontinuity on the moisture content profile continuous. However, both the models did not take the influence of the waterproof layer into consideration and the quasilinearized method could not reflect the heat and moisture variations actually.

Then the purpose of this paper is to clarify the heat and moisture behaviour in the earth-sheltered building envelope under the natural condition.

PHYSICAL AND MATHEMATICAL MODEL

To analyse the coupled heat and moisture transfer in the envelope of the typical earth-sheltered building, this paper takes an underground building in Nanjing (China) as an example, as depicted in Figure 1. The length (L), with (W), height (h) and the arc height (h_1) is 50m, 6m, 5.3m and 1.5m, separately. The distance between the soil surface and the top of the building is 1.5m and the thickness of the envelope is 0.5m (Suppose the thickness in the different area of the envelope is same).

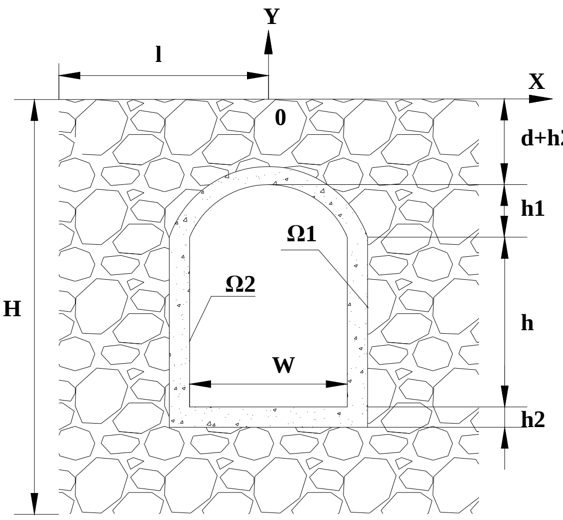

Figure 1 Cross-section of typical shallow buried underground engineering

Before the mathematical modelling, there are some assumptions.

- Homogeneous and isotropic porous media (envelope and the surrounding soil) with no distension and contraction is considered.
- Local thermal equilibrium is satisfied throughout the porous media.
- Capillary pressure on the interfaces of the different material layers is equal, i.e. the hydraulic continuous.
- The moisture absorption and desorption progress is isothermal without considering the influence of the capillary hysteresis.
- The length of the building is relative long compared to the with and height, the two dimensional is adopted.
- The drive potential for the liquid water in the porous material and the water vapor is density gradient and capillary pressure gradient.
- The waterproof layre can stop the mositure transfer completely.
- The thermal parameters are constants for given material.
- There is no heat or moisture source in the envelope and the surrounding soils.

As the Philips and De Vries' model (Philip et al., 1957) was adopted worldwide and the merit of Zhang's model (Zhang et al., 2006), the coupled heat and moisture transfer model in this paper is derived according to the two. Based on the assumption above, the governing equations describing the heat and moisture transfer in the envelope and the surrounding soils are as follows.

$$\rho_m c_m \frac{\partial T}{\partial \tau} = \nabla\left(k_{eff} \nabla T\right) \tag{1}$$

$$\xi \frac{\partial \varphi}{\partial \tau} = \nabla\left(D_\varphi \nabla \varphi + D_T \nabla T\right) \tag{2}$$

Where

$$k_{eff} = k_m + \frac{L(T) D_v \, p_{v,sat} \, \varphi}{R_v \, T^2}\left[\frac{L(T)}{R_v} - 1\right] \tag{3}$$

$$D_\varphi = \frac{D_v \, p_{v,sat}}{R_v \, T} + \frac{D_l \, \rho_l \, R_v \, T}{\varphi} \tag{4}$$

$$D_T = D_l \, \rho_l \, R_v \left(\ln \varphi\right) + \frac{D_v \, p_{v,sat} \, \varphi}{R_v \, T^2}\left[\frac{L(T)}{R_v} - 1\right] \tag{5}$$

D_l in Equation (4) is determined by Equation (6) (Valen, 1998),

$$D_l = \frac{D_v \, \varphi \, \rho_{v,sat}}{R_v \, T \, \rho_l} \tag{6}$$

The meaning of the symbols and notations used in the equations are given in the Nomenclature.

The upper surface of the physical domain is exposed to solar and long-wave radiations, convection heat and moisture transfer. This way, for $Y = 0$, the boundary condition becomes

$$\begin{aligned} -k_{eff} \nabla T\big|_{Y=0} &= h_{hs}\left(T_o - T\big|_{Y=0}\right) + \alpha I(\tau) \\ &-\varepsilon\sigma\left(T\big|_{Y=0}{}^4 - T_{sky}{}^4\right) + L(T) h_{ms}\left(\rho_o - \rho\big|_{Y=0}\right) \end{aligned} \tag{7}$$

and the mass balance is written as

$$-D_\varphi \nabla \varphi - D_T \nabla T = h_{ms}\left(\rho_o - \rho\big|_{Y=0}\right) \tag{8}$$

The heat and moisture balance on Ω_2 is described as

$$\begin{aligned} -k_{eff} \nabla T\big|_{Y=0} &= h_{hi}\left(T_i - T\big|_{\Omega_2}\right) \\ &+ L(T) h_{mi}\left(\rho_i - \rho\big|_{\Omega_2}\right) \end{aligned} \tag{9}$$

$$-D_\varphi \nabla \varphi - D_T \nabla T = h_{mi}\left(\rho_i - \rho\big|_{\Omega_2}\right) \tag{10}$$

The interface Ω_1 represents the waterproof layer, so the heat is continuous while the moisture could not transfer from it. The boundary is

$$\left(-D_\varphi \nabla \varphi - D_T \nabla T\right)_{soil}\Big|_{\Omega_1} = 0 \tag{11}$$

$$\left(-D_\varphi \nabla \varphi - D_T \nabla T\right)_{envelope}\Big|_{\Omega_1} = 0 \tag{12}$$

$$T_{soil}\big|_{\Omega_1} = T_{envelope}\big|_{\Omega_1} \tag{13}$$

Through the model of natural soil temperature field, the depth formula of the constant temperature layer was deduced by Liu et al. (Liu et al., 2007). So in the depth of $H = 20$ m, it is supposed that the water table is there and the temperature is constant. As a result, the heat and moisture condition in this layer in Nanjing (China) is written as (Liu et al., 2007)

$$T|_{Y=20} = (17.5 + 273.15)\ \text{K} \tag{14}$$

$$\varphi|_{Y=20} = 90\% \tag{15}$$

If it is sufficient far away from the envelope, the heat and moisture transfer can no longer influence the soil temperature profile, where is defined as the adiabatic and impermeable. When it refers to the determination of l, it is another problem. It comes true only $l \to \infty$ in theory. However, it is unrealistic in the numerical simulation. $0.5l = 15-20$ m in many simulations (Adjali et al., 1998; dos Santos et al., 2003) is adopted here.
Symmetry reduces the modelling domain to half of the model. It gets a symmetry boundary on $X = 0$.

SIMULATION

The governing equations were solved using COMSOL Multiphysics (COMSOL AB, 2008). COMSOL Multiphysics is a powerful interactive environment for modelling and solving all kinds of scientific and engineering problems based on partial differential equations (PDEs). With this software you can easily extend conventional models for one type of physics into multiphysics models that solve coupled physics phenomena—and do so simultaneously. The software runs the finite element analysis together with adaptive meshing and error control using a variety of numerical solvers.

Suppose the material of the envelope consists of concrete, and the soils surrounding the underground building is made of lime rock. The thermophysical parameters of them are listed in Table 1.

Table 1
Thermophysical parameters of the building materials and the soils (Ma et al., 1986)

MATERIAL	LIME ROCK	CONCRETE
ρ [kg/m^3]	1700	2200
c [J/(kg K)]	930	840
K [W/(m K)]	0.93	1.28
D_v [m^2/s]	6.9×10^{-6}	1.4×10^{-5}

The CSWD, Chinese Standard Weather Data, (China Meteorological Bureau Climate Information Center Climate Data Office et al., 2005) for the city of Nanjing was used, with a constant convection heat transfer coefficient of 15 and 8.14 W/(m^2 K) for h_{hs} and h_{hi} (Yuan, 2005), an absorptivity of 0.5 (dos Santos et al., 2003) and emissivity of 0.9 (Liu et al., 1983). Figure 2-Figure 5 present the annual variation of temperature, relative humidity, horizontal global irradiation and equivalent sky temperature in Nanjing.

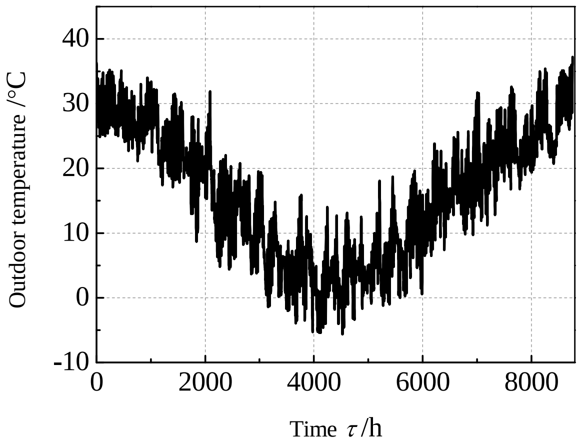

Figure 2 Annual variation of the dry bulb temperature Nanjing (China) form CSWD

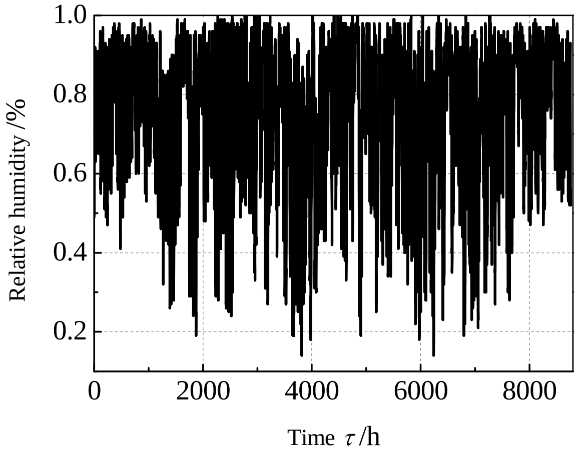

Figure 3 Annual variation of relative humidity Nanjing (China) form CSWD

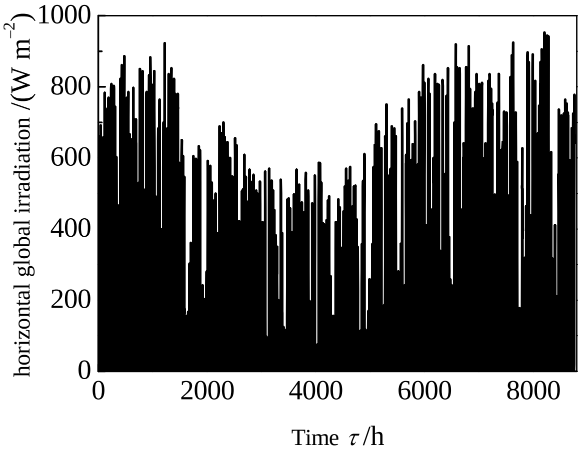

Figure 4 Annual variation of horizontal global irradiance Nanjing (China) form CSWD

The approximate expression of mass convection coefficient is determined by (Galbraith, 1992)

$$h_m = 9.28\times10^{-4}\times h_h \tag{16}$$

The moisture absorption and desorption process lines for concrete and lime rock are provided with the following equations (Ma et al., 1986; Pei et al., 1999),

$$w = \frac{0.000392\varphi}{1 - 1.8504\varphi + 0.9874\varphi^2} \tag{17}$$

$$\begin{aligned} w = {} & 0.014\varphi^4 - 0.021\varphi^3 + 0.014\varphi^2 \\ & + 0.0014\varphi + 0.00042 \end{aligned} \tag{18}$$

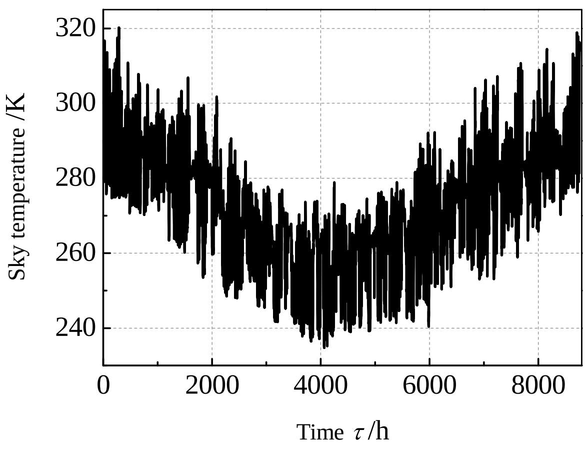

Figure 5 Annual variation of the equivalence sky temperature Nanjing (China) form CSWD

RESULTS AND DISCUSSIONS

In this numerical simulation, the finite element method is triangle and 945 mesh points, 1760 elements were used. Take 15:00 on July 20th, which is the time of outdoor maximal temperature (37.2°C), as the initial time for the simulation. For initial condition for the soil, a temperature of 17.5°C and a relative humidity of 50% were utilized. While for the envelope, temperature of 17.5°C and a relative humidity of 90%. Suppose the underground space is air-conditioned by the HVAC system, and the temperature and relative humity maintains 26°C and 60%, separately. The simulation period is 2 year (17520 hours) and time step is 1 hour (Wang, 2007).

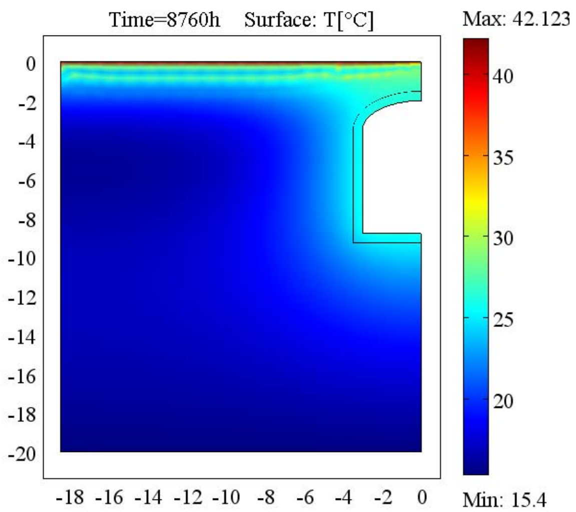

Figure 6 Temperature profile of the earth-sheltered building envelope and the surrounding soils at the time of 2 years

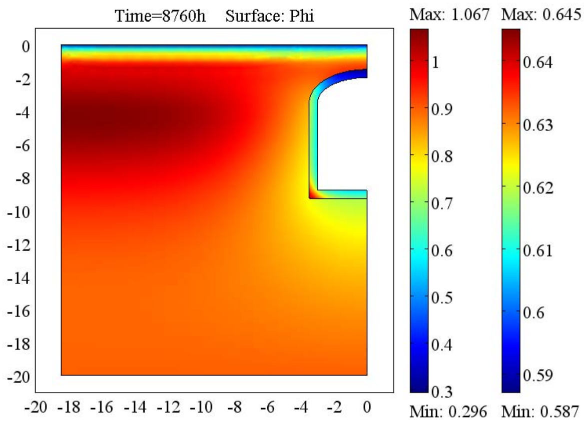

Figure 7 Relative humidity profile of the earth-sheltered building envelope and the surrounding soils at the time of 2 years

Figure 6 and Figure 7 present the temperature and relative humidity spatial distributions at 14:00 on July 20th. It is possible to notice in Figure 6 that the existance of the earth-sheltered building had influenced the soil's temperature distribution a lot. For the indoor air have a constant temperature of 26°C, the heat is tranfered from the envelope to the soil in summer. While the waterproof layer had not affect the heat transfer progess on the interface of the envelope and the rock, it did effect the moisture transfer (see in Figure 7). Note that in Figure 7, the left colorbar is for the rock and the right one for the envelope. After a whole year, the moisture had absorped by the upper lay of soil in this simulation. Due to the waterproof layer, the moisture in the soils didn't transfer to the envelope, and the moisture in the envelope had been transferred into the indoor environment afer construction.

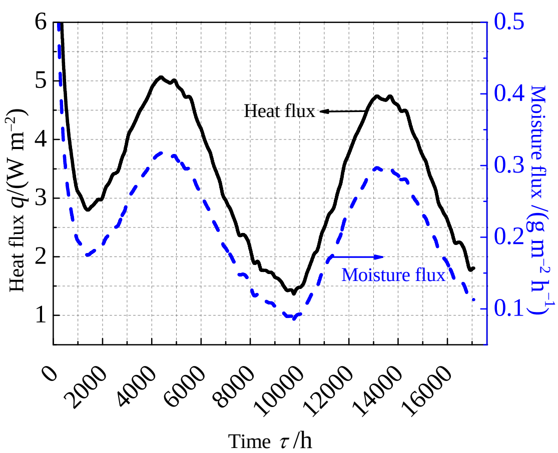

Figure 8 Variation of the heat and moisture flux on the wall of the earth-sheltered building

Figure 8 shows the variation of the heat and moisture flux on the wall of the earth-sheltered building. The moisture flux varies similarly with the heat flux. The maximum moisture flux in this simulation is 3.03g/(m^2 h) and it fluctuates between 0.1-0.3 g/(m^2 h), Which meets the experimental results of 1.7g/(m^2 h) and 0.12 g/(m^2 h) in China (Editorial Group, 1983). So the models in this paper is validated based on the assumption that the waterproof layre can stop the mositure transfer completely.

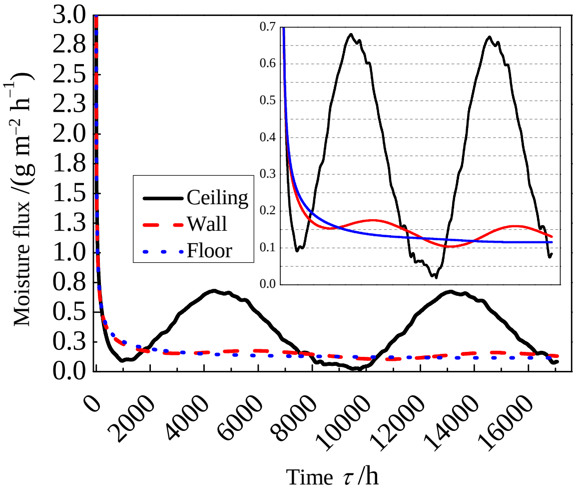

Figure 9 Variation of the moisture flux in the ceiling, floor and wall of the earth-sheltered building

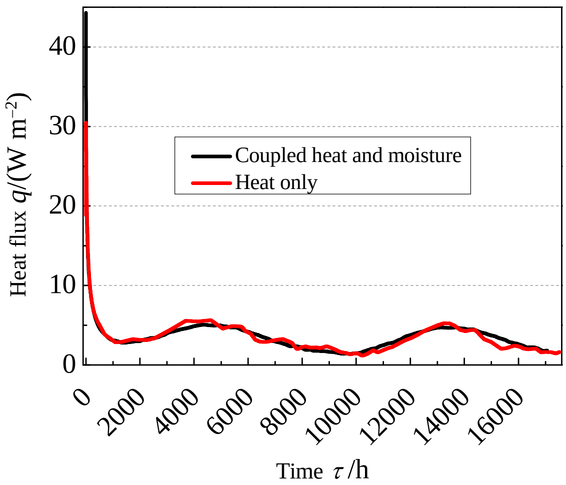

Figure 10 Comparison of the heat flux by the model in this paper and the heat transfer only model

The difference of moisture flux in the ceiling, floor and wall of the earth-sheltered building is depicted in Figure 9. The moisture flux on the ceiling fluctuated more than the one on the floor or wall. It may due to the influence of the vaiation of outdoor meteorologic paratmeters. Figure 10 compares the heat flux on the surface of the underground engineering envelope based on the coupled heat and moisture transfer model and heat transfer model, separately. The mean relative error during the simulation is 11.4%. Shen did an an investigation of transient, two-dimensional coupled heat and moisture flow in soils (Shen, 1986), results from the coupled model were compared to results obtained when the governing equations were decoupled. For the sand simulation, a 9% greater wall heat loss during the winter and almost a 50% increase in summer heat gain occurred due to coupling. Due to the lack of thermophysical parameters of the building materials, only the lime rock and concrete were studied in this paper. However, both the results indicates that the couple heat and moisture transfer model should be considered in some cases.

CONCLUSION

Based on the theory of Philips and De Vries' and Zhang's model, this paper built a mathematical model to analyze the coupled heat and moisture transfer in the envelope of earth-sheltered building and the surrounding soils. The COMSOL Mutiphysics was utilized to solve the governing equations in the porous materials. The software could couple the temperature field with the moisture field and solve the nonlinear problem effectively. It may be conclude from the results that,

1. The waterproof layer could prevent the moisture tranfering between the envelope and the surrounding soils, while the temperature field is still coutinuous. The mathematical model could describe the coupled heat and moisture transfer in the different porous material simultaneously and independently.
2. The moisture flux in the wall, ceiling and floor of the earth-sheltered building is different. Of the three, moisture flux in the ceiling is the biggest. Becasuse the heat transfer in the ceiling is affected by the ourdoor meteorologic paratmeters more. Accordingly, the moisture transfer is infuenced by the temprature.
3. The couple heat and moisture transfer model should be considered in some cases when do investigation of heat transfer in the porous materials.

The research may reflect the heat and moisture transfer in the underground engineering envelope and the surrounding soils objectively and the results could afford the basis of heat and moisture load calculation for underground engineering. As the results were obtained by the numerical simulation, it is necessary to do verification and analysis by both the experiments in the lab and field test further.

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NOMENCLATURE

c_m	-	Specific heat of material [J/(kg K)]
D_l	-	Permeability coefficient [kg/(m s Pa)]
D_v	-	Diffusion coefficient of water vapor [m^2/s]
h_h	-	Convection coefficient [W/m^2 K]
h_m	-	Mass convection coefficient [m/s]
k_{eff}	-	Effective thermal conductivity [W/(m K)]
k_m	-	Thermal conductivity of material [W/(m K)]
$L(T)$	-	Latent heat of vaporization [J/kg]
$p_{v,sat}$	-	Saturation pressure of water vapour [Pa]
R_v	-	Gas constant of water vapour [J/(kg K)]
T	-	Temperature [K]
w	-	Moisture content [g/kg]
α	-	Solar absorptivity [-]
ε	-	Radiation emissivity
φ	-	Relative humidity [-]
ρ_l	-	Water density [kg/m^3]
ρ_m	-	Material density [kg/m^3]
$\rho_{v,sat}$	-	Density of saturated water vapour [kg/m^3]]
σ	-	Stefan-Bolzmann's constant [W/(m^2 K^4)]
τ	-	Time [s]
ξ	-	slope of the moisture equilibrium curve, $\xi = \partial w / \partial \varphi$