APPROXIMATE ANALYTICAL SOLUTION FOR THE HEAT TRANSFER IN PACKED BEDS FOR SOLAR THERMAL STORAGE IN BUILDING SIMULATORS

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ABSTRACT

Schumann solution for the heating (cooling) of one-dimensional packed beds by the passage of a hot (cool) fluid is extended by the incorporation of a small solid thermal conductivity by means of using perturbation methods based on the Laplace transform and a Picard iteration for the Green's functions for the heat transfer of both phases. The new solution shows smooth heat front propagation through the medium during either the heating or the cooling (depending on its initial temperature) and can be easily incorporated into solar thermal storage simulators currently using Schumann solution, but with increased thermal accuracy.

NOMENCLATURE

x	axial position along the packed bed
y	nondimensional axial position
L	length of the solid medium
t	time coordinate
t	nondimensional time coordinate
T_f , T_s	temperature of fluid and solid, resp.
c	nondimensional temperature of the solid
e	nondimensional temperature of the fluid
r_f , r_s	density of fluid and solid, resp.
c_s	specific heat of the solid
c_f	fluid specific heat at constant pressure
v_f	fluid flow velocity
h	fluid-solid heat transfer coefficient
h_b	fluid-solid boundary heat transfer coefficient
g	nondimensional boundary heat transfer ratio
l_s	solid thermal conductivity
b^2	nondimensional solid thermal conductance
p	"effective" porosity
T_a	ambient temperature
T_{f0}	initial temperature of the packed bed
n	fluid to solid heat capacity ratio

INTRODUCTION

Schumann (1929) obtained the analytical solution for the problem of heating (cooling) of one-dimensional porous media (packed bed) by the passage of a hot (cool) fluid. Such solution has been rediscovered several times during 20th century (Nusselt, 1911; Anzelius, 1926; Nusselt, 1930), using alternative mathematical formulations which are completely equivalent (Baclic & Heggs, 1985; Lach & Pieczka 1985). The inclusion of a finite axial heat conduction in the Schumann problem complicates its theoretical analysis because exact analytical approaches cannot be applied in such a case. Numerical have been widely applied (e.g. Sözen & Vafai, 1993; Kuwahara et al., 2001; Hayes et al., 2008). However, asymptotic analysis is scarce in the scientific literature. Kuznetsov (1994; 1995; 1997) uses perturbation methods based on Fourier series for rectangular packed beds using a two-equation model for the fluid temperature and the difference between temperatures of the fluid and the solid phases. Kuznetsov (1997) uses the inverse of the product of the fluid-to-particle heat transfer coefficient between the solid and fluid phases by the specific surface area as small parameter. Under this assumption, the temperature difference between both phases is found to be small compared to the difference between the inlet temperature of the fluid and the initial temperature of solid. Spiga & Morini (1999) also developed an asymptotic solution for the two-equation model but assuming an infinite velocity for the travelling wave in the gas phase. Such assumption is reasonable in typical air-rock bed installations but do not results in a proper characterization of the stratification in the packed bed. Such stratification is very important in charging/discharging conditions of the energy storage unit, requiring the consideration of a finite convection velocity in the gas.

The main contribution in this paper is the development of an asymptotic approximate solution valid for small thermal conductivity under the assumption of a semi-infinite medium. In our approximation the temperature difference between both phases is not assumed to be small and a finite travelling velocity for the gas phase is considered.

Packed or rock beds are the cheapest energy storage unit for solar heating and cooling systems since heating demands and solar irradiation are time-dependent functions do not matching between them. Thermal energy storage provides a reservoir of energy to adjust this mismatch and to meet the energy needs at all times. Figure 1 shows a simplified scheme of a air-rock bed, thermal storage unit in a solar air heating installation (Duffie & Beckman, 1991; Schmidt & Willmott, 1981). Note that for installations using hot water or other fluids a heat exchanger must be incorporated in the sketch shown in Fig. 1. In charging the rock bed heat storage unit, hot air from the solar air collectors, or hot fluid through heat exchangers, is passed through the bed tank from its top and cold air is collected from the other side, where it is again passed to the solar collector, or to the inlet of the heat exchanger for liquids. When the heat is required, the flow of the air in the packed bed storage unit is reversed (not shown in Fig. 1). This paper considers only the modelling of the charging (discharging) of the packed bed without taking into account the rest of the system.

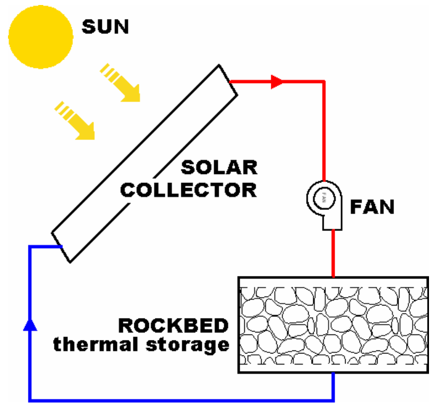

Figure 1 Rock bed as thermal storage system integrated in a solar collector installation.

The contents of the paper are as follows. Next section presents the mathematical formulation of the problem and its physical assumptions. Schumann solution, the leading order solution for both the fluid and the solid phases, is recalled in Section 2. Sections 3 and 4 present the first-order approximation for, respectively, solid and fluid temperatures. In Section 5 the main results are discussed. Finally, Section 7 is devoted to the general conclusions and further lines of research.

MATHEMATICAL MODEL

The heat transfer in a packed bed is modelled by the two-phase problem (Nield et al., 1999)

$$p\, r_f\, c_f \left(\frac{\partial T_f}{\partial t} + v_f \frac{\partial T_f}{\partial x} \right) = -h\left(T_f - T_s\right), \tag{1}$$

$$(1-p)\, r_s\, c_s \frac{\partial T_s}{\partial t} = h\left(T_f - T_s\right) + (1-p)\, l_s \frac{\partial^2 T_s}{\partial x^2}. \tag{2}$$

Let us assume, as initial condition, that both phases start in thermal equilibrium, i.e., with the same temperature equal to the ambient one,

$$T_f\left(0,x\right) = T_a, \quad x > 0, \tag{3}$$

$$T_s\left(0,x\right) = T_a, \quad x > 0. \tag{4}$$

Let us also assume that the fluid is injected in the solid at x=0, with constant flow velocity and temperature,

$$T_f\left(t,0\right) = T_{f0}, \quad t > 0. \tag{5}$$

and apply a Robin boundary condition at the inlet boundary of the solid matrix given by

$$l_s \frac{\partial T_s}{\partial x}(t,0) = h_b\left(T_s(t,0) - T_f(t,0)\right). \tag{6}$$

Finally, let us also assume that the length of the solid medium is large enough such that it can be considered as semi-infinite ($L \to \infty$). In such a case, the outlet boundary condition is given by

$$\lim_{x\to\infty} T_s(t,x) = \lim_{x\to\infty} T_g(t,x) = 0. \tag{7}$$

Equations (1)–(7) may be nondimensionalized by introducing the following variables

$$t = \frac{h\,t}{(1-p)\, r_s\, c_s}, \qquad y = \frac{n\,h\,x}{p\, r_g\, c_g\, v_f},$$

where

$$n = \frac{p\, r_f\, c_f}{(1-p)\, r_s\, c_s},$$

and defining $c(t,y)$ and $e(t,y)$ through the relationships

$$T_s = T_a + c\left(t,y\right)\left(T_{f0} - T_a\right),$$

$$T_f = T_a + e\left(t,y\right)\left(T_{f0} - T_a\right),$$

resulting in the dimensionless form

$$n\left(\frac{\partial e}{\partial t} + \frac{\partial e}{\partial y} \right) = c - e, \tag{8}$$

$$\frac{\partial c}{\partial t} - b^2 \frac{\partial^2 c}{\partial y^2} = e - c, \tag{9}$$

with initial conditions

$$e\left(0,y\right) = 0, \qquad c\left(0,y\right) = 0, \qquad y > 0, \tag{10}$$

and boundary conditions

$$e\left(t,0\right) = 1, \tag{11}$$

$$\frac{\partial c}{\partial y}(t,0) = g\left(c(t,0) - e(t,0)\right), \tag{12}$$

$$\lim_{y\to\infty} c\left(t,y\right) = \lim_{y\to\infty} e\left(t,y\right) = 0, \tag{13}$$

where, in Eqs. (8)–(13),

$$b^2 = l_s\, h\left(\frac{n}{p\, r_f\, c_f\, v_f} \right)^2, \text{ and } g = \frac{h_b\, p\, r_f\, c_f\, v_f}{h\, n\, l_s}.$$

In order to solve the quarterplane problem given by Eqs. (8)–(13), the Laplace transform method can be used. Let $\hat{e}\left(s,y\right)$ and $\hat{c}\left(s,y\right)$ be the Laplace transform on t of $e(t,y)$ and $c(t,y)$, respectively. The Laplace transform of Eqs. (8)–(13) yields

$$n\left(s\,\hat{e}\left(s,y\right) + \frac{\partial \hat{e}\left(s,y\right)}{\partial y} \right) = \hat{c}\left(s,y\right) - \hat{e}\left(s,y\right), \tag{14}$$

$$s\,\hat{c} - b^2 \frac{\partial^2 \hat{c}}{\partial y^2} = \hat{e}\left(s,y\right) - \hat{c}\left(s,y\right), \tag{15}$$

with boundary conditions

$$\hat{e}(s,0)=\frac{1}{s}, \tag{16}$$

$$\frac{\partial \hat{c}(s,0)}{\partial y}=g\left(\hat{c}(s,0)-\frac{1}{s}\right). \tag{17}$$

The exact solution of the problem given by Eqs. (8)–(13) cannot be obtained analytically for general b since the exact solution of the problem in Laplace space given by Eqs. (14)-(17) is not known. Therefore, for large b^2, Eqs. (8)–(13) require the use of numerical methods. However, since the exact solution for $b=0$ of Eqs. (8)–(12), is widely known, singular perturbation methods (Kevorkian & Cole 1996) can be applied in order to obtain an approximate asymptotic solution at least for $b^2 \ll 1$.

LEADING ORDER SOLUTION

Equations (8)–(13) for $b=0$ are usually referred to as either Nusselt or Schumann problem in the quarterplane, whose exact solution is widely known. Let c_0 and e_0 be such solution, and $\widehat{e_0}(s,y)$ and $\widehat{c_0}(s,y)$, respectively, their Laplace transforms in t. The solution of Eqs. (14)–(16) for $b=0$ yields

$$\widehat{e_0}(s,y)=\frac{e^{-ys}}{s}\exp\left(-\frac{s\,y}{n(s+1)}\right), \tag{18}$$

$$\widehat{c_0}(s,y)=\frac{e^{-ys}}{s(s+1)}\exp\left(-\frac{s\,y}{n(s+1)}\right). \tag{19}$$

The evaluation of the Bromwich integral for the inverse Laplace transform of Eqs. (18) and (19) yields (Klinkenberg 1954)

$$e_0(t,y)=q(t-y)e^{-y/n}\left\{e^{-t+y}I_0\left(2\sqrt{\frac{y(t-y)}{n}}\right)+\int_0^{t-y}e^{-u}I_0\left(2\sqrt{\frac{yu}{n}}\right)du\right\}, \tag{20}$$

$$c_0(t,y)=q(t-y)\mathrm{e}^{-y/n}\int_0^{t-y}\mathrm{e}^{-u}I_0\left(2\sqrt{\frac{yu}{n}}\right)du. \tag{21}$$

$$e_0(t,y)-c_0(t,y)=q(t-y)\mathrm{e}^{y-y/n-t}I_0\left(2\sqrt{\frac{(t-y)y}{n}}\right). \tag{22}$$

Integration by parts in Eq. (20), recalling that $I_0=I_1$, and $I(0)=1$, results in

$$e_0(t,y)=\mathrm{e}^{-y/n}\left\{1+\int_0^{t-y}\mathrm{e}^{-u}I_1\left(2\sqrt{\frac{yu}{n}}\right)\sqrt{\frac{y}{un}}\,du\right\}. \tag{23}$$

Plots of the numerical evaluation of Eqs. (21) and (23) can be find, e.g., in (Lach & Pieczka 1985).

FIRST-ORDER APPROXIMATION

Let us start determining the Green's function $G_b(t,y;t',y')$ for the solid temperature, i.e., the heat equation (15) with the initial condition (10) and boundary conditions (12)–(13), given by the solution of

$$-b^2\frac{\partial^2 G_b}{\partial y^2}+\frac{\partial G_b}{\partial t}=d(t-t')d(y-y'),\quad t,y>0, \tag{24}$$

$$G_b(0,y;t',y')=0,\qquad y>0, \tag{25}$$

$$\frac{\partial G_b}{\partial y}(t,0;t',y')-g\,G_b(t,0;t',y')=-g,\quad t>0, \tag{26}$$

where Eq. (11) has been used and $d(\cdot)$ denotes the Dirac delta function. The expression for Green's function of the quarterplane problem of the heat equation is given by (Polyanin 2002)

$$G_b(t,y;t',y')=q(t-t')\frac{\exp\left(\frac{-(y-y')^2}{4b^2(t-t')}\right)+\exp\left(\frac{-(y+y')^2}{4b^2(t-t')}\right)}{2b\sqrt{p(t-t')}}-g\,q(t-t')\exp\left(g\left(b^2g(t-t')+y+y'\right)\right)\times\operatorname{erfc}\left(\frac{2b^2g(t-t')+y+y'}{2b\sqrt{p(t-t')}}\right), \tag{27}$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function.

Using the Green's function (27), the formal solution of equation (15) with Eqs. (10)–(13) is given by (Polyanin 2002)

$$c(t,y)=\int_0^t\left(\int_0^\infty G_b(t,y;t',y')\left(e(t',y')-c(t',y')\right)dy'\right)dt'+g\,b^2\int_0^t G_b(t,y;t',0)\,dt'. \tag{28}$$

The first-order correction for the solid temperature may be obtained by substituting Eqs. (22) and (27) into Eq. (28), after evaluation of the integral in the boundary condition term in Eq. (28), yielding

$$c_1(t,y)=\operatorname{erfc}\left(\frac{y}{2b\sqrt{t}}\right)-e^{g\,y+b^2g^2t}\operatorname{erfc}\left(\frac{2b^2g\,t+y}{2b\sqrt{t}}\right)+\frac{1}{2b\sqrt{p}}\int_0^t\frac{K_{b,0}(y,t,t')}{\sqrt{t-t'}}dt'-g\int_0^t\exp\left(g^2b^2(t-t')\right)L_{b,0}(y,t,t')\,dt', \tag{29}$$

where

$$K_{b,0}(y,t,t')=\int_0^{t'}\left(\exp\left(\frac{-(y-y')^2}{4b^2(t-t')}\right)+\exp\left(\frac{-(y+y')^2}{4b^2(t-t')}\right)\right)\times\exp\left(y'-\frac{y'}{n}-t'\right)I_0\left(2\sqrt{\frac{(t'-y')y'}{n}}\right)dy',$$

$$L_{b,0}(y,t,t')=\int_0^{t'}\exp\left(g\,(y+y')\right)\operatorname{erfc}\left(\frac{2b^2 g\,(t-t')+y+y'}{2b\sqrt{p\,(t-t')}}\right)\times$$
$$\exp\left(y'-\frac{y'}{n}-t'\right)I_0\left(2\sqrt{\frac{(t'-y')\,y'}{n}}\right)dy'.$$

Let us differentiate Eq. (8) on t and substitute Eq. (9) resulting in

$$n\frac{\partial}{\partial y}\left(\frac{\partial e}{\partial t}+e\right)+\frac{\partial^2 e}{\partial t^2}+(n+1)\frac{\partial e}{\partial t}=b^2\frac{\partial^2 c}{\partial y^2}. \qquad (30)$$

This equation is used instead of Eq. (9) for the first-order approximation to e, because it lacks explicit dependence on b.

The Laplace transform of Eqs. (30) and (11) yields

$$\frac{\partial \hat{e}(s,y)}{\partial y}+s\left[1+\frac{1}{n(s+1)}\right]\hat{e}(s,y)=\frac{b^2}{n(s+1)}\frac{\partial^2\hat{c}(s,y)}{\partial y^2}, \qquad (31)$$

and $\hat{e}(s,0)=1/s$.

The first-order ordinary differential equation (31) can be easily solved by using an integrating factor yielding

$$\hat{e}(s,y)=\frac{b^2}{n(s+1)}\int_0^y\frac{\partial^2\hat{c}(s,x)}{\partial x^2}\exp\left[(x-y)s\left(1+\frac{1}{n(s+1)}\right)\right]dx+$$
$$\frac{1}{s}\exp\left[-ys\left(1+\frac{1}{n(s+1)}\right)\right], \qquad (32)$$

which, by means of using Eqs. (18) and (19), can be rewritten as

$$\hat{e}(s,y)=\widehat{e_0}(s,y)+\frac{sb^2}{n}\int_0^y\widehat{c_0}(s,y-x)\frac{\partial^2\hat{c}(s,x)}{\partial x^2}dx. \qquad (33)$$

A first-order approximation of $\hat{e}(s,y)$ on b^2 can be obtained by replacing the function $\hat{c}$ in Eq. (33) by $\widehat{c_0}$, taking into account Eq. (18), (19), and the second partial derivative of Eq. (19), i.e.,

$$\frac{\partial^2\widehat{c_0}(s,y)}{\partial y^2}=\frac{s}{s+1}\left(1+\frac{1}{n(s+1)}\right)^2\exp\left[-\frac{sy}{n}\left(n+\frac{1}{s+1}\right)\right], \qquad (34)$$

resulting in

$$\widehat{e_1}(s,y)=\widehat{e_0}(s,y)+b^2\,\widehat{\Delta e_0}(s,y), \qquad (35)$$

where

$$\widehat{\Delta e_0}(s,y)=\frac{s^2}{n(s+1)^2}\left(1+\frac{1}{n(s+1)}\right)^2\times$$
$$\int_0^y\widehat{c_0}(s,x)\exp\left(-\frac{s(y-x)}{n}\left(n+\frac{1}{s+1}\right)\right)dx$$
$$=\frac{s\,y}{n(s+1)^2}\left(1+\frac{2}{n(s+1)}+\frac{1}{n^2(s+1)^2}\right)\times$$
$$\exp\left(-\frac{s\,y}{n}\left(n+\frac{1}{s+1}\right)\right). \qquad (36)$$

By applying the inversion formula of the Laplace transform to Eq. (35), then

$$e_1(t,y)=e_0(t,y)+b^2\,\Delta e_0(t,y), \qquad (37)$$

where

$$\Delta e_0(t,y)=\frac{1}{2p\,\mathrm{i}}\int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty}\exp(st)\,\widehat{\Delta e_0}(s,y)\,ds$$
$$=q\,(t-y)\frac{1}{2p\,\mathrm{i}}\int_C\frac{s\,y}{n(s+1)^2}\exp\left(s(t-y)-\frac{s\,y}{n(s+1)}\right)\times$$
$$\left\{1+\frac{2}{n(s+1)}+\frac{1}{n^2(s+1)^2}\right\}ds, \qquad (38)$$

and C is the simple closed contour shown in Figure 2. The integration in the C_R arc of C is null if and only if $t-y>0$ as shown in the appendix.

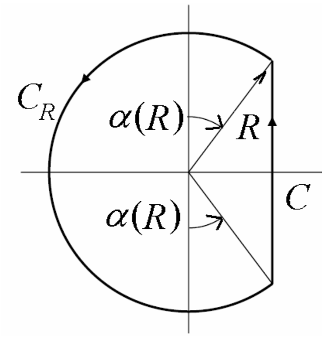

Figure 2 Simple closed contour C.

Recalling that the generating function of the modified Bessel functions I_k is given by (Lebedev 1972)

$$\exp\left(\frac{z}{2}\left(s+\frac{1}{s}\right)\right)=\sum_{k=-\infty}^{\infty}s^k I_k(z),$$

then the exponential function in the integrand of Eq. (38) may be written as

$$\exp\left((s+1-1)(t-y)-\frac{(s+1-1)\,y}{n(s+1)}\right)$$
$$=\exp\left(y-t-\frac{y}{n}\right)\exp\left((s+1)(t-y)+\frac{y}{n(s+1)}\right)$$
$$=\exp\left(y-t-\frac{y}{n}\right)\sum_{k=-\infty}^{\infty}s^k I_k(z), \qquad (39)$$

where

$$z=2\sqrt{\frac{(t-y)\,y}{n}},\quad s=\frac{s+1}{g},\quad g=\sqrt{\frac{y}{n(t-y)}}. \qquad (40)$$

The substitution of Eq. (39) in Eq. (38) results in

$$\Delta e_0(t,y)=q\,(t-y)\frac{e^{y-t-y/n}}{2p\,\mathrm{i}}\int_C\frac{1}{n(g\,s)^2}\sum_{k=-\infty}^{\infty}s^k I_k(z)\times$$

$$\left\{(g\,s-1)\,y\left(1+\frac{2}{n\,g\,s}+\frac{1}{(n\,g\,s)^{2}}\right)\right\}g\,ds$$

$$=\frac{y}{n}q\,(t-y)\frac{e^{y-t-y/n}}{2p\text{ i}}\int_{C}\left(\sum_{k=-\infty}^{\infty}s^{k}I_{k}(z)\right)\times$$

$$\left\{\frac{1}{s}+\frac{(2-n)}{g\,n}\frac{1}{s^{2}}-\frac{(2n-1)}{(g\,n)^{2}}\frac{1}{s^{3}}-\frac{1}{g^{3}n^{2}}\frac{1}{s^{4}}\right\}ds\,, \qquad (41)$$

which can be expanded to

$$\Delta e_{0}(t,y)=\frac{y}{n}q\,(t-y)e^{y-t-y/n}\left\{\frac{1}{2p\text{ i}}\int_{C}\sum_{k=-\infty}^{\infty}s^{k-1}I_{k}(z)ds\;+\right.$$

$$\frac{(2-n)}{g\,n}\frac{1}{2p\text{ i}}\int_{C}\sum_{k=-\infty}^{\infty}s^{k-2}I_{k}(z)\,ds\;-$$

$$\frac{(2n-1)}{(g\,n)^{2}}\frac{1}{2p\text{ i}}\int_{C}\sum_{k=-\infty}^{\infty}s^{k-3}I_{k}(z)\,ds\;-$$

$$\left.\frac{1}{g^{3}n^{2}}\frac{1}{2p\text{ i}}\int_{C}\sum_{k=-\infty}^{\infty}s^{k-4}I_{k}(z)\,ds\right\}. \qquad (42)$$

The use of Cauchy's residue theorem (Antimirov et al., 1998) yields

$$\Delta e_{0}(t,y)=\frac{y}{n}q(t-y)e^{y-t-y/n}\left\{I_{0}(z)+\frac{2-n}{g\,n}I_{1}(z)-\right.$$

$$\left.\frac{2n-1}{g^{2}n^{2}}I_{2}(z)-\frac{1}{g^{3}n^{2}}I_{3}(z)\right\}. \qquad (43)$$

PRESENTATION OF RESULTS

The main result of this paper are the nondimensional solutions (29), (37) and (43), which depend on the parameter n, *b*, and *g*, and may be evaluated by means of any numerical quadrature rule; here, adaptive Gauss-Kronrod quadrature (Press et al., 1992) has been used. In solar thermal storage simulators, fast and accurate numerical results are required. Equation (29) has an oscillatory integrand which results in slow convergence of the numerical method. Therefore, the authors recommend the use of Eq. (21) for $c(t,y)$ and Eqs. (37) and (43) for $e(t,y)$.

Figures 3 and 4 show the solutions $e_{1}(t,y)$ for $b=0.1$ and $n=0.5$, respectively, for several values of the other parameters at times $t=1,2,\ldots,10$, corresponding to curves plotted sequentially from the left to the right in each plot. Both figures show that the heat propagates through the medium from the boundary condition at $y=0$ as a temperature wavefront approximately constant velocity, equal to unity in nondimensional units. Figures 3 and 4 show snapshots of the fluid heat wave propagating from the left to the right presenting a sharp front at position $y=t$ due to the neglected gas heat diffusivity, mathematically corresponding to the Heaviside function in Eqs. (20) and (43). Figures correspond to the heating of the solid by the gas, but note that the cooling process may also be easily analyzed.

Figure 4 shows that the main effect of the value of *b*, the diffusivity of the solid phase, in the fluid heat wave. For $b=0.1$, the solution is practically the same as for a null value, given by Eq. (20). The gas inside the packed bed becomes hotter as time increases, resulting, for large time, in a constant value equal to the boundary value in the whole length of the medium. However, for larger β, this behaviour changes, due to the rise in temperature of the solid matrix far away from the front of the gas due to the non-null solid heat conduction. Hence, the asymptotic value of the gas temperature as time increases does not reach the inlet value. This difference is more important as β increases.

Figure 4 also shows that the heat flux (spatial derivative of the temperature) is not continuous at the heating front position ($y=t$). The size of this discontinuity decreases as time evolves, until reaching a null value.

CONCLUSION

Perturbation methods based on Green's function and Laplace transform have been applied in order to obtain analytical approximations for the solution of the heat transport in a packed bed for solar energy storage under the hypothesis of small solid diffusity and negligible fluid one. The resulting solution is a correction of the Schumann one, currently used in the majority of building simulators. The new solution for the fluid temperature is explicit and can be evaluated in an accurate and fast manner. However, the expression for the solid temperature is difficult to be evaluated numerically since they present a slowly converging integrand. Since, in practical simulators the most important expression is that of the fluid, this limitation is of minor value.

The new solution for the solid temperature depends on the boundary heat transfer coefficient between the fluid and the solid, so can be used in order to incorporate its effects in current simulators. In such a case, the authors recommend that the integrand in Eq. (29) be interpolated in the domain of the two-dimensional integral and a fast numerical method for the interpolant be used. The boundary heat transfer coefficient is an additional parameter which must be introduced in the building simulator.

Our results can be used for the development of new design rules for the boundary conditions at inlet and outlet in heat storage tanks. It is very important to stress that the overall efficiency of a solar installation using a rock bed can be enhanced by using the greatest degree of temperature stratification. Such design criteria, depending strongly on the boundary heat transfer coefficient, are outside the scope of the present paper resulting in an interested line for further research.

Further research is also in progress on the incorporation of the finite length of the packed bed into the asymptotic solution. In such a case, a Fourier series expression is used for the Green's functions instead Laplace transform, but the asymptotic method remains valid. Furthermore, nonlinear boundary conditions of radiative heat transfer will also be incorporated into our solution by means of using perturbation methods.

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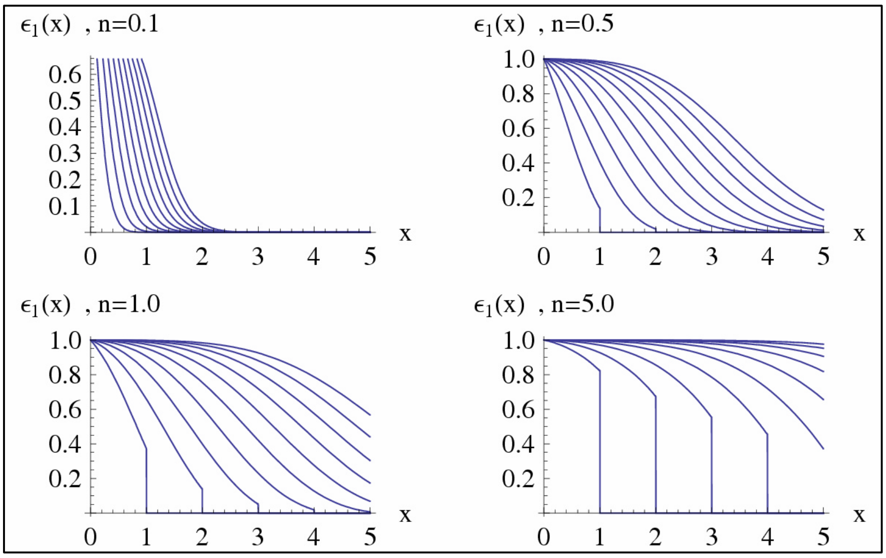

Figure 3 Evolution in time of the function $e_1(t, y)$ *for b=*0.1 *and several values of parameter* n *at times* τ=*1,2,3,…,10, corresponding to curves from left to the right.*

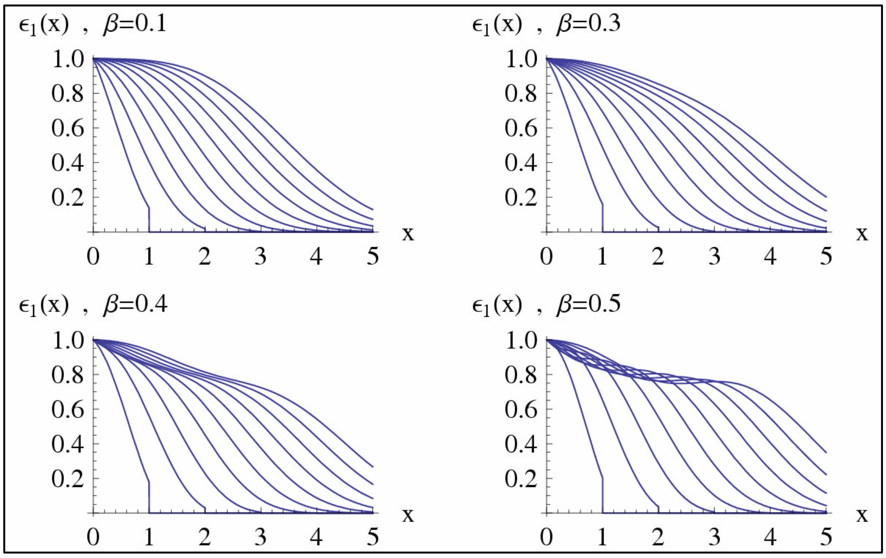

Figure 4 Evolution in time of the function $e_1(t, y)$ *for* n=*0.5 and several values of parameter b at times* τ=*1,2,3,…,10, corresponding to curves from left to the right.*