BUILDING SIMULATION OF THERMAL ENVIRONMENT USING RESPONSE FACTOR ANALYSED BY THREE-DIMENSIONAL CFD

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ABSTRACT

Indoor climate has a three-dimensional spatial distribution caused by three-dimensional airflow. To obtain the accurate knowledge of building performance, it is demanded to integrate the spatial distribution into building simulations. Thus, CFD analysis is necessary in design process. However, usually only a few case of CFD could be executable in real design process, because of the large computational load. The main subject of this study is the development of a method; how to extract heat transport phenomena in rooms from such limited CFD analyses, and how to integrate the data into a nodal analysis. Using the method, we can calculate indoor environment, including the spatial distribution of temperature with a very light computational load and with almost the same degree of precision as CFD simulation.

INTRODUCTION

The main subject of this paper is to structurally analyze the transport properties of air and heat in an air-conditioned room, and to develop effective methods to design and control indoor environments.

In this paper, we define "advection-diffusion thermal response factor", which indicates the time-serial response from inputted heat source within a room. The thermal response factor indicates the phenomenon of heat transport in air-conditioned room. We can carry out the transient calculation of heat transport by a convolution operation on the quantities of heat produced and the thermal response factors. In these contexts, if we carry out only a few times of CFD analysis in short period to calculate the thermal response factors, we carry out the transient three-dimensional heat transport simulations without CFD analysis in long period.

DEFINITION OF ADVECTION-DIFFUSION THERMAL RESPONSE FACTOR

Ishida and Kato (2008) defined a concentration response factor, which is the time-serial response from inputted unit concentration pulse in an air-conditioned room. It is based on the response factor method (Mitalas et al. 1967) that is used widely in
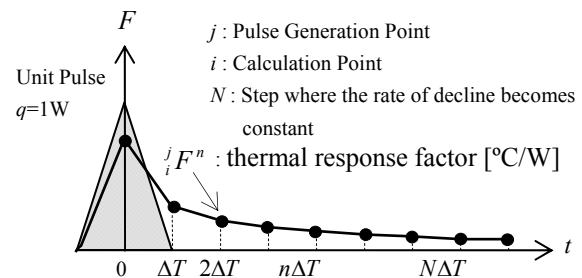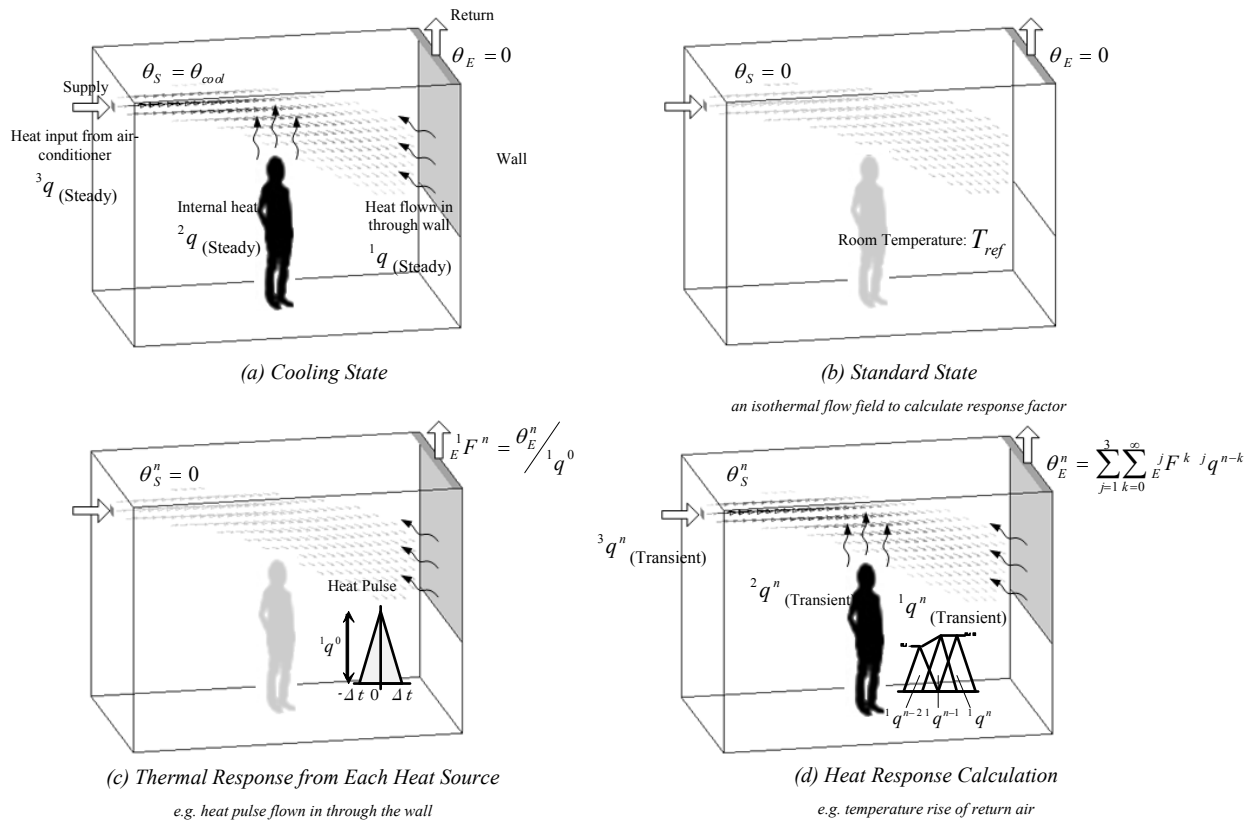

Figure 1 Definition of pulse release and corresponding response factor

air-conditioning load calculations. The calculation method (Ishida et al. 2008, Hiyama et al. 2008) using this concentration response factor was developed as a simulation to calculate the three-dimensional concentration transport in an air-conditioned room with a very light computational load. In the method, the concentration response factors are previously calculated by CFD analysis in short period. If we could regard that the advection-diffusion equation as linear in a steady state flow field, we can carry out the transient calculation of concentration transport by a convolution operation on the quantities of concentration produced and the concentration response factors. Thus, it permits omitting the CFD-dependent calculations in the process of simulation to calculate concentration transport. In this study, we develop the method to simulate the heat transport in an air-conditioned room.

The response factor used in this thermal response calculation is defined as advection-diffusion thermal response factor $_i^j F^k$, as is the concentration response factor (Fig. 1). The response factor indicates the temperature rise (fall) at arbitrary reference point i, k steps after input of a unit heat pulse at heat source j (● in Fig. 1). The unit for the thermal response factor is [ºC/W].

When there is only one heat source in a room, the temperature rise at arbitrary reference point is obtained by inner product calculation using the thermal response factors as shown in Equation (1).

$$\theta_i^n = \sum_{k=0}^{\infty} {_i^j F^k}\, {^j q^{n-k}} \tag{1}$$

(a) Cooling State

(b) Standard State

an isothermal flow field to calculate response factor

$_E^1 F^n = \theta_E^n / _1 q^0$

(c) Thermal Response from Each Heat Source

e.g. heat pulse flown in through the wall

$\theta_E^n = \sum_{j=1}^{3} \sum_{k=0}^{\infty} {}_E^j F^k \, {}^j q^{n-k}$

(d) Heat Response Calculation

e.g. temperature rise of return air

${}^1 q^n$: Heat flown in through wall in time step n θ_S^n : Temperaure rise of supply air in time step n ${}_E^j F^n$: Heat response factor for return air

${}^2 q^n$: Internal heat in time step n θ_E^n : Temperature rise of return air in time step n (Note 3) T_{ref} : Reference temperature

${}^3 q^n$: Heat for air-conditioning in time step n θ_{cool} : Temperature rise of supply air on *Cooling State*

Figure 2 Concept of heat response calculation method

${}_i^j F^k$: Response factor from heat source j to reference point i in time step k [°C/W]

${}^j q^k$: Heat flow occurring from heat source j in time step k.   [W]

θ_i^n : Temperature rise at reference point i in time step n. A rise in temperature is stated as a positive rise, whereas a drop in temperature is stated as a negative rise. [°C]

n : Time step.

i : Reference point number.

j : Heat source number (includes information about the point of its occurrence)

CALCULATION METHOD

Taking an air-conditioned room in summer, as shown in Fig. 2, as an example, we shall explain the concept of the calculation method using the advection-diffusion thermal response factor. First, assume that the room is being cooled. The room has heat flowing in through the wall and internal heat. This condition is called a cooling state (Fig. 2 (a)). Next, assume the non-isothermal flow field in the cooling state as the representative flow field in summer while the room is cooled. Then, set up an isothermal flow field with the same velocity field as the representative flow field. This condition is called the standard state (Fig. 2 (b)). The room temperature is set at reference temperature T_{ref} . The advection-diffusion thermal response factors are obtained by generating unit heat pulse ${}^j q^k$ from a heat source, and solving Transport Equation (2) for temperature rise θ (Fig. 2 (c)). In this paper, any element that supplies heat to air is called a heat source. For example, even a wall which does not generate heat by itself is handled as a heat source if heat flowing through it generates a heat flow from the outside of the room to the inside. θ is based on the reference temperature. In this case, it is assumed that kinematic eddy viscosity ν_t is proportional to velocity and that θ does not influence the flow field. Therefore, in the flow field, θ is calculated as a passive contaminant.

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta u_j}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{P_{rt}}\frac{\partial \theta}{\partial x_j}\right) + \frac{q`}{\rho C_p} \qquad (2)$$

u_j : Velocity   [m/s]

ν_t : Kinematic eddy viscosity   [m²/s]

P_{rt} : Turbulent flow Prandtl number
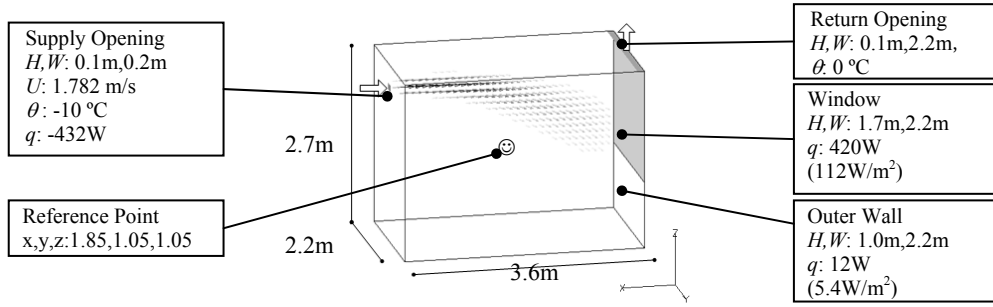
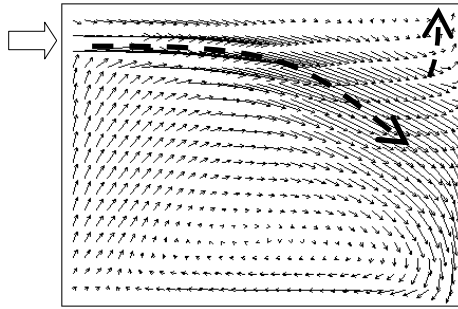$q`$: Generated heat per unit volume   [W/m³]

Figure 3 Room for case study

Figure 4 Result of CFD analysis
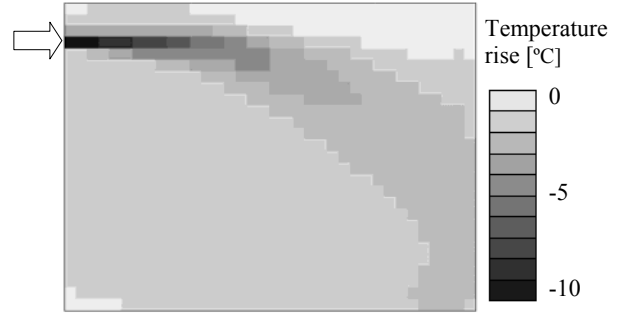*Velocity on X-Z section, Y=1.1m*

Figure 5 Result of CFD analysis
Temperature rise on X-Z section, Y=1.1m

Table 1 Results of a coupled simulation of air current and radiation. Heat generation is stated as positive heat, while heat absorption is stated as negative value

	Input Heat[W]	Convection [W]	Radiation [W]
Window	420.3	140.0	280.3
Outer wall	11.9	19.7	-7.8
Inner wall		35.6	-35.6
Side wall 1		68.3	-68.3
Side wall 2	0.0	70.0	-70.0
Roof		36.6	-36.6
Floor		62.1	-62.1
Supply	-432.2	-432.2	0.0

ρ : Air density   [kg/m^3]

C_p : Air specific heat   [J/(K kg)]

As can be seen from Equation (2), in a flow field in which the air velocity and diffusion factor distributions are fixed, the transport equation for the temperature rise is linear (Ishida et al. 2008). Namely, the temperature rise at any reference point can be calculated by superposing the cumulative temperature rises from the individual heat sources (Fig. 2 (d)).

Equation (3) shows the superposition of temperature rises from the three types of heat sources shown in Fig. 2. In Equation (3), the first term on the right shows the contribution of the heat flowing in through the wall to the temperature rise at the reference point. The second shows the contribution of the internal heat to the temperature rise at the reference point. And the third shows the contribution of the heat supplied for air conditioning; that is, the air supply temperature as the operating factor, to the temperature rise at the reference point.

$$\theta_i^n = \sum_{k=0}^{\infty} {}_i^1 F^k \, {}^1q^{n-k} + \sum_{k=0}^{\infty} {}_i^2 F^k \, {}^2q^{n-k} + \sum_{k=0}^{\infty} {}_i^3 F^k \, {}^3q^{n-k}$$

(3)

In Fig. 2, we assumed that there were only three heat sources for the sake of convenience. In actual calculations, as shown in Equation (4), the temperature rise at any reference point is obtained as the total sum of the results of superposition of all existing heat sources.

$$\theta_i^n = \sum_{k=0}^{\infty} {}_i^1 F^k \, {}^1q^{n-k} + \cdots + \sum_{k=0}^{\infty} {}_i^m F^k \, {}^mq^{n-k}$$

$$= \sum_{j=1}^{m} \sum_{k=0}^{\infty} {}_i^j F^k \, {}^jq^{n-k}$$

(4)

CASE STUDY

Analysis Object

Consider the living room model shown in Fig. 3. The room is provided with one supply opening and one return opening for air conditioning. As the thermal load, heat is supplied from the inner surfaces of a window and an outer wall. In this paper which forms the first step of the study, inner walls are assumed to be heat-insulating and the absorption of heat from the inner walls is excluded from consideration. Therefore, it is assumed that the outer walls only supply heat flowing through them and that they do not absorb any heat. It is assumed that the air conditioning is cooling the room in summer. For the purpose of calculation, the reference temperature is set to be 26ºC. The heat supplied for air-conditioning is set to be equal to one flown in through the window and the outer wall.

Table 2 Outline of numerical calculation

Air Supply	Flow rate : 128m³/h
	Velocity : -1.78 m/s (*x*-direction)
	Turbulent intensity : 0.1
Return	Gradient of normal direction equal 0 (velocity, *k*, *ε*, *θ*)
Wall Boundary	Generalized logarithmic law (constant E=9) (velocity, *k*, *ε*, *θ*)
	Emittance : 0.9
Turbulent flow Prandtl number	0.9
Division of Space	Division number : 36(*x*-direction) x 22(*y*) x 10(*z*)
	Space interval : Δ*x* = Δ*y* = Δ*z* = 0.1m
Division of Time	Time increment Δ*t'* : 0.1s
	Pulse interval Δ*t* : 300s
	Step where the rate of decline becomes constant *N* : 6

Thus, the temperature of return air becomes the reference temperature as long as there is only one exhaust opening.

The results of a coupled simulation of air current and radiation are shown in Figs. 4 and 5 and Table 1. From Figs. 4 and 5, it can be seen that since the air supplied was cool air, it descended from the supply opening under the influence of buoyancy. It can also be seen that the rise in room temperature was negative at every point in the room. In other words, the room temperature was generally lower than the reference temperature (26ºC). The temperature difference at the center of the room (reference point) was -1.6ºC (temperature: 24.4ºC).

Calculation of advection-diffusion heat response factor

In the room under consideration, there are three heat sources—the window, outer wall and air conditioner. However, even the heat-insulating walls (partition wall, ceiling, and floor) that do not emit heat by themselves receive radiation heat from the heat sources and emit the heat to the room interior by convection. Thus, heat is supplied to the room interior from all the heat-insulating walls. Here, all the heat-insulating walls are handled as one heat source for the sake of convenience. Therefore, there are four heat sources in total. In obtaining the response factors, therefore, it is necessary to calculate not only the flow field in the cooling state but also the transient response of temperature rise θ as a passive contaminant by CFD analysis in four different cases.

As the first step in the response factor calculation, the indoor thermal environment and velocity distribution in a cooling state that can be regarded as almost steady are calculated using a three-dimensional CFD analysis based on a standard k-ε type 2-equation model coupled with radiation (Omori et al. 2003). Next, a flow field in standard state is created with the velocity distribution fixed, the room temperature kept at the reference temperature (26ºC) and the heat supplied from heat sources set at zero. Under this condition, a heat flow from a certain heat source is generated by an isosceles triangular pulse to calculate the transport of the transient temperature rise θ as a passive contaminant. In so doing, the time series of responses at the reference point is output to collect response factors. In this case study, the reference point is located at the center of the room (*x*: 1.85, *y*: 1.05, *z*: 1.05). The calculation conditions are shown in Table 2 and Fig. 3.

The calculated response factors are shown in Figs. 6 through 9. The triangles shown by thick lines in Figs. 6 through 9 indicate the time series for heat generation. The value at the vertex of each of the isosceles triangles is assumed to be 1W regardless of the vertical axis of the graph. For the purpose of comparison, response factors obtained when the heat supplied to the room interior was perfectly mixed instantly are also shown (Ishida et al. 2008). The calculated response factors in Figs. 7, 8 and 9 show a time series resembling that of the response factors for which perfect mixing was assumed. This implies that since the room has a small volume and employs an air-conditioning system which permits the air to circulate through the room easily. Thus, the heat supplied for air conditioning, the heat radiated from the outer wall, and the heat transferred from the heat insulating-walls by convection mix well in the room. On the other hand, the response factors for the heat from the window that is close to the exhaust opening (Fig. 6) reach a maximum when *n* = 1, whereas the other response factors maximize at *n* = 2. The reason for this is that the heat flow from the window is moved directly to the return opening right above the outer wall by the upward air current in its neighborhood and exhausted from the room, while the heat flows from the other heat sources are mixed together to some degree before they are exhausted from the room. Assuming that the heat flows from all heat sources are perfectly mixed, the temperature of the entire room becomes the reference temperature, since in the case study the heat supplied for the air-conditioning is equal to the heat flown in. However, as Fig. 6 shows, the heat flow from the window is exhausted from the room before it is perfectly mixed with the heat flows from the other heat sources. As a result, the room temperature is lower than the reference temperature. This can be well understood from the response factors shown in Figs. 6 through 9.
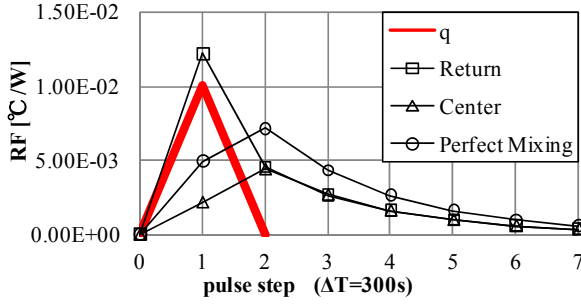
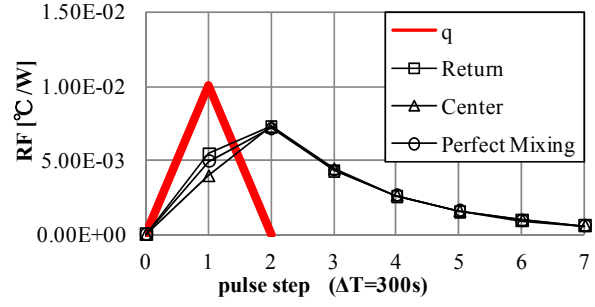Figure 6 Response Factor

Heat generation point : Window

Figure 7 Response Factor
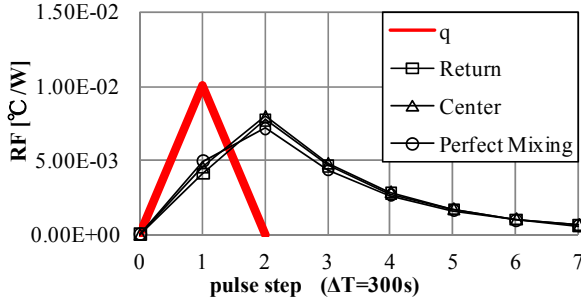
Heat generation point : Heat insulating wall

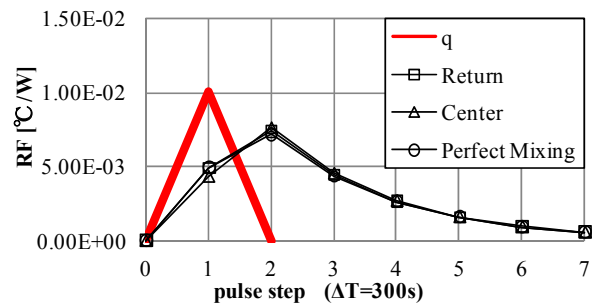Figure 8 Response Factor

Heat generation point : Outer wall

Figure 6 Response Factor

Heat generation point : Air conditioner

Calculation of transient heat transport using response factors

In this section, the heat response factors collected in the three-dimensional CFD analysis are substituted in Equation (4) and the heat quantity $^{j}q^{n}$ of each of the heat sources is input to perform a transient calculation of temperature rise θ to simulate the air conditioning. The case study is calculation the time-serial change in the controlled variable when a target value is set. The controlled variable is the temperature at the reference point at the center of the room. The equation for calculating temperature rise θ_{sen}^{n} at the reference point is shown in Equation (5). Although no upper limit is set on the total sum in Equation (4), the thermal response due to advection/mixing is assumed to be 0 when the ventilation time is about five times that of the nominal ventilation time τ_{n} or longer (Kato et al. 1998). Thus, in actual calculations only a product sum for ($\tau_{n}\times6$) time is calculated. In this calculation, since τ_{n}=10 min and response factor time slice Δt=5 min, the cumulative temperature rise using a 13-term response factor string, 65min, is calculated.

$$\theta_{sen}^{n} = \sum_{k=0}^{13} {}_{sen}^{sup}F^{k} \ {}^{sup}q^{n-k} + \sum_{k=0}^{13} {}_{sen}^{win}F^{k} \ {}^{win}q^{n-k}$$

$$+ \sum_{k=0}^{13} {}_{sen}^{outw}F^{k} \ {}^{outw}q^{n-k} + \sum_{k=0}^{13} {}_{sen}^{inw}F^{k} \ {}^{inw}q^{n-k} \quad (5)$$

$^{sup}q^{k}$: Heat flow generated from air conditioner in time step k [W]

$^{win}q^{k}$: Heat flow generated from window in time step k [W]

$^{outw}q^{k}$: Heat flow generated from outer wall in time step k [W]

$^{inw}q^{k}$: Heat flow generated from heat insulating wall in time step k [W]

$_{sen}^{sup}F^{k}$: Response factor from air conditioner to reference point [ºC/W]
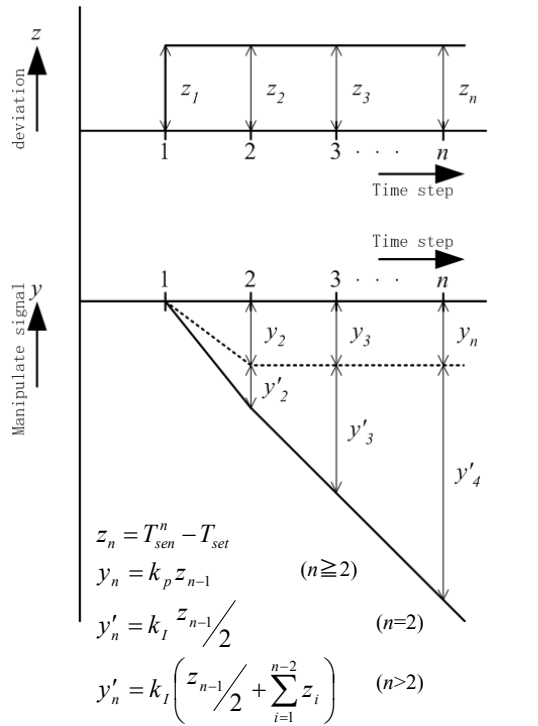
$_{sen}^{win}F^{k}$: Response factor from window to reference point [ºC/W]

$_{sen}^{outw}F^{k}$: Response factor from outer wall to reference point [ºC/W]

$_{sen}^{inw}F^{k}$: Response factor from heat insulating wall to reference point [ºC/W]

The heat flow $^{sup}q^{k}$ in Equation (5) is calculated from the amount of change ΔT_{sup}^{n} (Equation 6) in the supply air temperature determined by the PI control: Equation (7) (described later). Since the case study is intended to analyze the change in heat quantity of supplied air according to the change in target value, the heat quantities supplied from the other heat sources— $^{win}q^{k}$, $^{outw}q^{k}$, $^{inw}q^{k}$ —are kept constant at the values shown in Table 1 that were obtained by the coupled analysis of air currents and radiation.

Equation (6) gives the amount of change in temperature of the air supplied as air conditioning. It is determined by the time series for temperature at

$z_n = T_{sen}^n - T_{set}$

$y_n = k_p z_{n-1}$ $\qquad (n \geqq 2)$

$y_n' = k_I \dfrac{z_{n-1}}{2}$ $\qquad (n=2)$

$y_n' = k_I \left(\dfrac{z_{n-1}}{2} + \displaystyle\sum_{i=1}^{n-2} z_i \right)$ $\qquad (n>2)$

z_n : deviation of the controlled variable from the target value in time step n [ºC]

$T_{sen}^n = T_{ref} + \theta_{sen}^n$: controlled variable in time step n [ºC]

T_{set} : the target value in time step n [ºC]

y_n : manipulate signal of proportional control [ºC]

y_n' : manipulate signal of proportional control [ºC]

k_P : proportional gain [-]:magnification for calculating a manipulation signal (amount of correction) which is proportional to a given deviation

k_I : integral gain [-]:magnification for calculating a manipulation signal which is proportional to the time-integrated value of a given deviation

Figure 10 Outline of PI control The case when the constant deviation was appeared

the reference point at the center of the room using PI control.

$$\Delta T_{sup}^n = T_{sup}^n - \left(T_{ref} - 10 \right) = -\left(y_n + y_n' \right) \text{(see Fig. 10)}$$

$$= -k_P \left(T_{sen}^{n-1} - T_{set} \right) - \frac{1}{2} k_I \left(T_{sen}^{n-2} - T_{set} \right)$$

$$- k_I \sum_{i=1}^{n-2} \left(T_{sen}^i - T_{set} \right) \quad (n>2) \tag{6}$$

$${}^{sup}q^k = C_P \, G \left(T_{sup}^k - T_{ref} \right) = C_P \, G \left(\Delta T_{sup}^k - 10 \right) \tag{7}$$

ΔT_{sup}^n : Temperature rise of air supplied for air conditioning in step n from the temperature of air supplied for air conditioning in a *cooling state* [ºC]

T_{sup}^n : Temperature of air supplied for air conditioning in step n [ºC]

G : Flow rate of mass inflow from supply opening [kg/s]

Table 3 Calculation Case

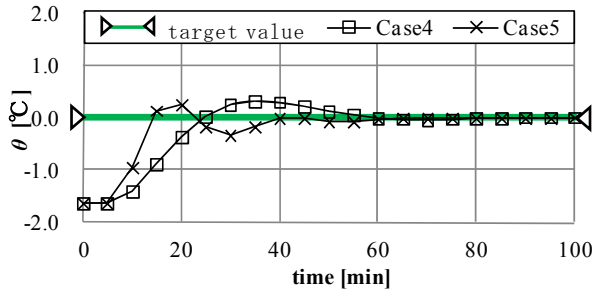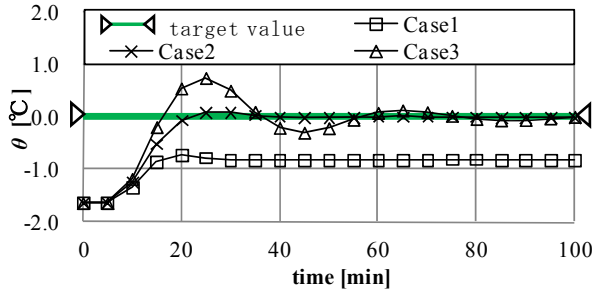|	Case1	Case2	Case3	Case4	Case5
Proportional gain k_P	1.0	1.0	1.0	0.5	2.0
Integral gain k_I	0.0	0.5	1.0	0.5	0.5

Figure 11 temperature rise of controlled variable

It can be seen from Equation (6) that based on the difference between the past temperature at the reference point and the preset temperature, the rise in supplied air temperature, ΔT_{sup}^n, is controlled and the temperature of the supplied air, T_{sup}^n, is corrected so that the temperature at the reference point in step n becomes the preset temperature.

In the calculation, it is assumed that the cooling state is the initial state ($n = 0$, $t = 0$ min, θ_{sen}^n =-1.6ºC) and that in and after $n = 1$ ($t = 5$ min) the target value, T_{set}, is set at 26ºC and the control of θ_{sen}^n to make deviation z_n (see Fig. 10) 0 is started. The control of air conditioning used in the case study is such that deviation z_n in step n is reflected in the temperature of air supplied for air conditioning in step $n + 1$, it is in and after step 2 that the temperature of the air supplied as air conditioning is actually changed. The calculation method described in this report uses an isosceles triangular wave in calculating a response factor. Therefore, when the supply air temperature in step n is manipulated, the manipulation works proportionally from step n-1. In view of this characteristic, in the present case study, the supply air temperature in step n is decided based on the deviation in step n-1.

The calculation was performed in five cases with different integral gains and proportional gains to analyze the relationship between the design variables and the time series of a controlled variable. An outline of each of the cases is shown in Table 3. It should be noted that Case 1 omits integral control: only proportional control is implemented in this case. In the Cases 1, 2 and 3, only the integral gain is treated as a design variable and the proportional gain is assumed to be 1 in all three cases. In the Cases 4 and 5, the proportional gain is changed from Case 2.

Figs. 11 shows the time series of temperature rises in each case. The initial value of the controlled variable in the initial state of $n = 0$ ($t = 0$ min) is -1.6ºC (temperature rise in cooling state), and when $n = 1$ ($t = 5$ min), the target value is set to 0ºC and the control is started. In Case 1 in which only proportional control was implemented (Figs. 11), the controlled variable did not rise to the target value and an offset that kept the deviation from the target value constant (about -0.8ºC) appeared, indicating that the quantity of heat necessary to maintain the target value was not supplied properly. The phenomenon is often appeared when only proportional control is implemented. By contrast, in Cases 2, 3, 4 and 5 in which PI control is implemented (Figs. 11), the deviation in the controlled variable from the target value almost disappears after a certain period due to the effect of integral control. However, it can be seen that the difference in each gain significantly influences the process of convergence. In Case 3 in which the integral gain is relatively large, the amount of manipulation becomes excessive due to the large manipulation signal, readily causing an overshoot and cycling to occur. On the other hand, in Case 2 in which the integral gain is relatively small, the overshoot and cycling amplitudes are small as the manipulation signal is kept small. In Case 4 in which the proportional gain is smaller than Case 2, the amount of manipulation becomes excessive due to the large manipulation signal. The signal is accumulated by integral control since the small proportional gain cased manipulation delay. In Case 5 in which the proportional gain is larger than Case 2, the amount of manipulation becomes excessive due to the large manipulation signal caused by the large proportional gain.

In the calculation using the thermal response calculation method proposed in this paper, calculating the response factor using one node of the server (CPU: POWER5+ 1.5GHz) took some five hours, but a transient calculation of 100 min using the response factor required only milliseconds per case with a PC (CPU: CoreTM2 Quad 2.4 GHz). When the only three-dimensional CFD analysis was used, the transient calculation in same case took about 23 hours using one node of a server (CPU: POWER5+ 1.5GHz). If the analysis needing huge amounts of operations such as optimization calculation were performed by three-dimensional CFD analysis alone, it would take a year or so. However if the same analysis were performed by the thermal response calculation, it would take only a few seconds.

CONCLUSION

In this paper, we proposed a thermal response calculation method based on response factor using CFD analysis and presented an example of its application to simulate the control of air conditioning. Ordinarily, reproducing the phenomena shown in the calculation examples requires substantial manpower and costly computing resources for experimentation or CFD analysis. With our calculation method, it is possible to reproduce the same calculation model in a very short time with a PC. By proposing the thermal response calculation method, we showed the possibility of implementing, with a PC, those simulations which were formerly difficult to carry out without CFD, such as optimization problems and transient thermal load calculations taking into consideration the indoor temperature distribution.

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