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A Systematic Approach for Derivation of Transfer Function Coefficients of Buildings from Experimental Data

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ABSTRACT

Modern control theory can contribute in different ways to minimize energy consumption while maintaining the thermal comfort in a building. For this purpose, it is essential to identify the dynamic behaviour of the building. The dynamic thermal behaviour of a building in response to external and internal changes is not well known. The pre-calculated coefficients recommended in the ASHRAE Handbook of Fundaments, 1985, are based on simplifying assumptions. Moreover, they are only for three types of construction (light, medium and heavy), which do not cover all possible design alternatives.

In this study, the transfer function coefficients for three buildings are determined by experimental identification. The weight levels of these buildings are 46, 130, 535 kg/m^2 of floor area. The derived coefficients complement those values given in the ASHRAE Handbook.

INTRODUCTION

Knowledge of the dynamic thermal response of a building is important in the design of thermal systems, in the renovation of existing buildings and in the operation of energy management systems. It has been claimed that an accurate building dynamic model and proper functioning of the HVAC systems could save up to 75% of the required load [1].

The transfer function method has been used extensively for building load calculations. The pre-calculated coefficients (response factors) given in the ASHRAE Handbook [2] are valid for typical light, medium, and heavy construction which are characterized by 146, 312 and 635 kg/m^2 of floor area, respectively, and do not cover lighter or heavier construction. Several approaches have been used for the derivation of dynamic models to predict the thermal behaviour of buildings. One approach is to develop a set of differential equations that describe the room air temperature as a function of ambient weather conditions, using knowledge of the physical characteristics of building. This set of differential equations could be either deterministic [3] or stochastic [4]. This technique requires numerous assumptions and approximations to specify the actual condition.

Another approach is to derive a dynamic model from field measurements using system identification techniques [5]. The main advantage of this approach is that it does not require simplifying assumptions and it can take into account the material deterioration and thermal bridges. Most recent experiments performed have shown that air-conditioning cooling load calculated by the ASHRAE method is different from the cooling load needed practically [6, 7]. This lack of agreement was also observed when *in-situ* measurements from three buildings were compared with the predicted results by TARP (Thermal Analysis Research Program) [8]. The results are shown in Figs. 1 - 3. The TARP program uses the conduction transfer function method to calculate the transient heat transfer through walls and has the capability to compute hourly room temperature and load profiles. This lack of agreement between measured and predicted energy consumption was also confirmed in ref. 6.

The use of system identification methods to determine parameters of a system is well

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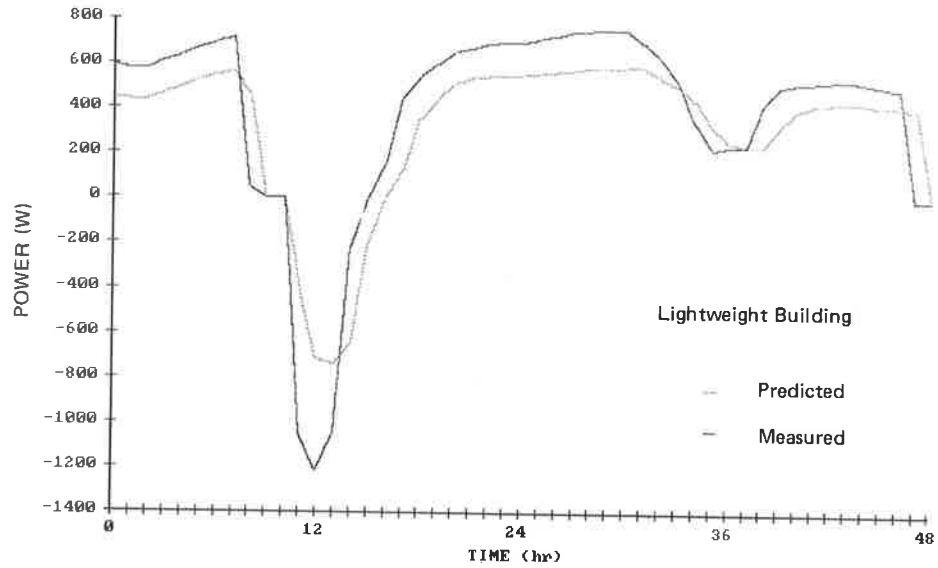

Fig. 1. Comparison of the power consumption of a super-lightweight building predicted by TARP simulation program, with the measured data for December 27 - 28, 1980.

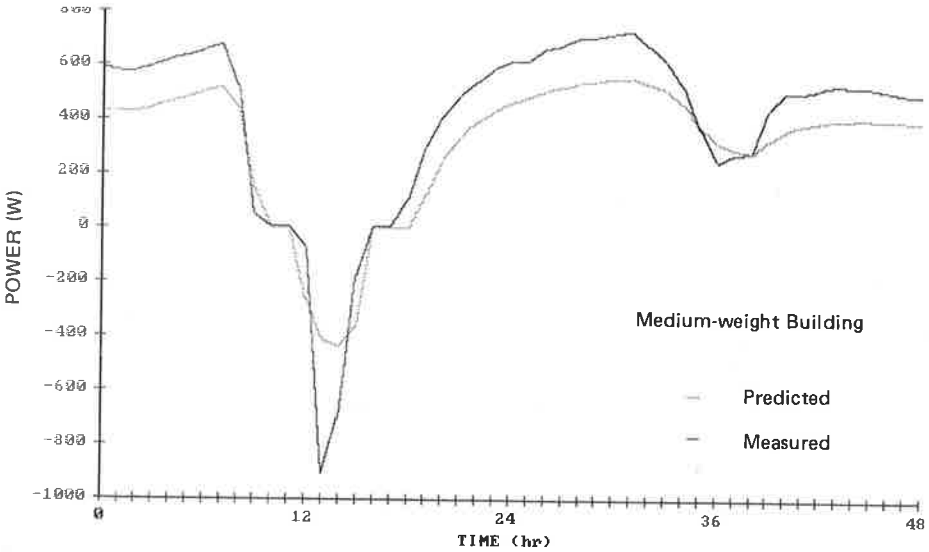

Fig. 2. Comparison of the power consumption of a medium-weight building predicted by TARP simulation program, with the measured data for December 27 - 28, 1980.

established [9 - 11]. These are methods of obtaining a mathematical model for a system on the basis of analysis of input and output signals. This requires both selection of the form of the model (i.e., the equation) and estimation of values for the parameters in the model. The existence of many different solutions for a given system is common. Therefore, the selection of the model depends upon the purpose of the identification and the experience of the user. While system identification techniques may employ non-linear models, the techniques described in this paper are applicable only when linearity can be assumed since the z-transfer function is valid only for linear systems. Heat transfer problems are normally considered linear, but this may be inappropriate in some cases such as when heat transfer is primarily by radiation.

Frequency response analysis is a classical technique in control engineering and may be used for identification of systems. The application of frequency domain methods to the study of a class of multivariable systems is given in the literature [12]. Frequency analysis has been used in the past both in design and in parameter estimation of buildings [13, 14]. Regression techniques have also

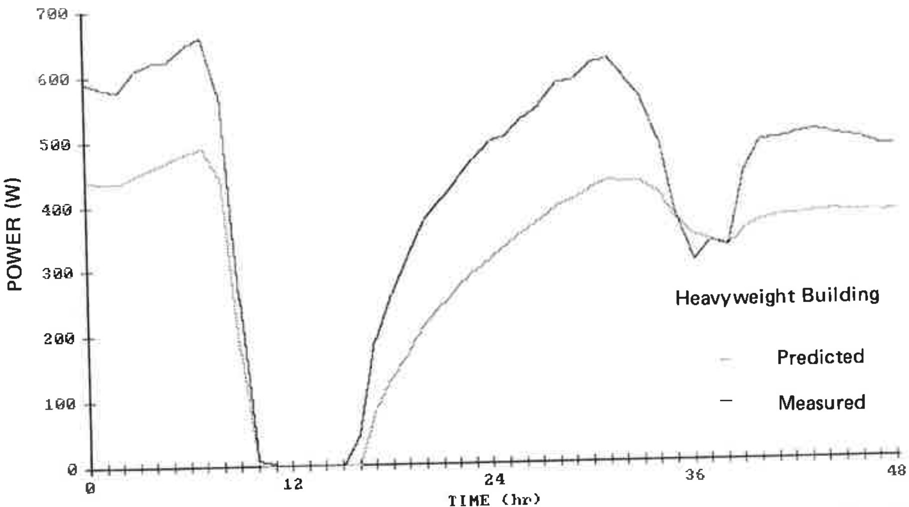

Fig. 3. Comparison of the power consumption of a heavyweight building predicted by TARP simulation program, with the measured data for December 27 - 28, 1980.

been employed to estimate, or identify quantitatively, the mathematical relationships representing the process [15]. Crawford and Woods [16] modeled a building as a system with several inputs and a single output, and then used a simple least-squares technique to determine the values of the system parameters. A sequential least-squares algorithm has also been used to study the parameters of an existing building [16, 17].

These techniques assume a static model and result in time invariant controls. The static model is not ideal for this application due to the parameter variation from season to season, due to aging materials, deterioration of components, wind direction, and the pattern of air distribution in the room [16, 17]. These variations could be included by the use of modern estimation and control algorithms for performing the tuning process in real time. Using adaptive control, the controller parameters are continously updated, adjusting to changes in internal load, external weather, and building envelope, etc. Adaptive control is the ideal choice for energy management systems applications [18 - 21]. First, the dynamic behavior of a building is slowly varying, allowing for the estimation of the parameters on line. Second, to maintain the comfort conditions in a building, a closed-loop control system is required. This type of control is ideal for closed-loop systems. The usefulness of using microprocessors for tuning controller has been recognized in building energy management systems [18 - 21].

The main objective of this paper is to develop a methodology for derivation of a dynamic model of thermal behavior of buildings from experimental data. This study also provides a technique for developing a dynamic thermal model of a building for application in building automation.

THE TRANSFER FUNCTION TECHNIQUES

The transfer function techniques for calculation of thermal loads and room air temperature are described in the ASHRAE Handbook [1] and DOE Manual [22]. This calculation is performed in a two-step process. The block diagram of these two steps is shown in Fig. 4. In the first step, the room air temperature is assumed to be constant at a reference value

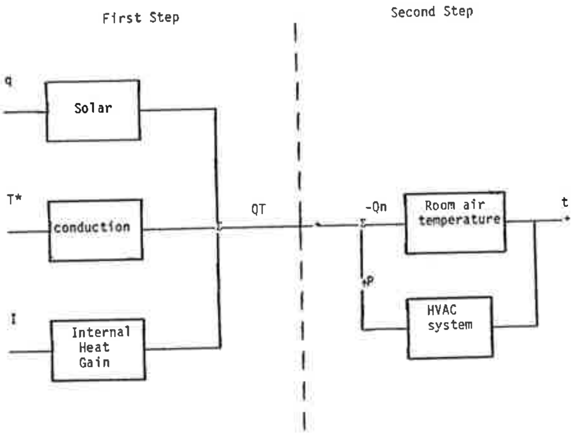

Fig. 4. Block diagram of the system.

and the instantaneous heat gains/losses are calculated based on the reference room air temperature. These instantaneous heat gains (losses) are solar radiation entering the room through windows, internal heat gains, and conduction through the building envelopes. The transfer function of each component is used to calculate the cooling (heating) load. The total load is the summation of the individual terms. This load is defined as the rate of heat which must be removed (added) from the room in order to maintain the air temperature constant at the reference value.

In the second step, as shown in Fig. 4, the total load, along with data about the HVAC system, and the information about the room air-temperature transfer function are applied to determine the actual heat-extraction rate (net cooling load) and the actual room air temperature (see the Appendix).

The energy balance for the room air temperature results in:

$$Q_s + Q_e + Q_R = P \qquad (1)$$

where Q_s is the cooling load due to solar gain through window, Q_e is cooling load due to heat gain through envelope, Q_R is the heat release by room thermal storage and P is the auxiliary energy required to keep the room air temperature within given limits. The corresponding transfer functions can be written as:

$$Q_s(z) = \frac{v_0 + v_1 z^{-1}}{1 + w_1 z^{-1}}\, q(z) \qquad (2)$$

$$Q_e(z) = \frac{a_0 + a_1 z^{-1}}{1 + w_1 z^{-1}}\, T^*(z) \qquad (3)$$

$$Q_R(z) = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2}}{1 + w_1 z^{-1}}\, T(z) \qquad (4)$$

where $q(z)$, $T^*(z)$ and $T(z)$ are z-transforms of the instantaneous heat gain, sol-air temperature, and room air temperature deviation. The symbols of a, v, g and w_1 are the z-transfer function coefficients (see the Appendix). Now, by substituting eqns. (2), (3), and (4) in eqn. (1) we have

$$\begin{aligned}&(v_0 + v_1 z^{-1})q(z) + (a_0 + a_1 z^{-1})T^*(z)\\ &\quad + (g_0 + g_1 z^{-1} + g_2 z^{-2})T(z)\\ &\quad = (1 + w_1 z^{-1})P(z)\end{aligned}$$

or in the time domain

$$\begin{aligned}P(t) = &\sum_{i=0}^{1} v_i q(t - i\Delta) + \sum_{j=0}^{1} a_j T^*(t - j\Delta)\\ &+ \sum_{k=0}^{2} g_k T(t - k\Delta) + w_1 P(t - \Delta) \qquad (5)\end{aligned}$$

where t is the time and Δ is the time interval.

These coefficients vary from season to season due to deterioration of building components, variation in wind speed and direction, and pattern of air distribution in the space. The coefficients can be derived from actual system performance data, using modern control techniques mentioned earlier.

EXPERIMENTAL FACILITY

The buildings selected for this study were three one-story insulated wood-frame huts [23]. The field measurements were performed during December 16 - 28, 1980, in the unoccupied huts. These huts are nearly identical in construction, except for their interior finishing. They are 79.6 m^2 and 2.47 m high inside. The details of the huts and the wall construction are given in Figs. 5 and 6. Each hut consists of a south and north room with a connecting door. The south and north rooms have a window of 2.6 and 1.0 m^2 net glass area, respectively. All the interior wall surfaces of unit 1 (light construction) consist of a single 12.7 mm layer of gypsum board; unit 2 (medium construction) consists of four layers of 12.7 mm gypsum board on walls and two layers on the ceiling; and unit 3 (heavy construction) contains a layer of 100 mm of solid cement bricks. All the units are pressurized with corridor air (20 °C) to eliminate the infiltration effect. The floors of the units are insulated with a thick layer of insulation (7 m^2 °C/W) to reduce the basement heat gains/losses. The basement is heated and kept at 21 °C and the corridor temperature is kept constant at approximately 20 °C. Each test unit is heated to 20 °C (low set-point) with an electric baseboard heater. The north zone is kept at 20 °C constant, while the control strategy for the south zone consists of temperature set-points in the range of 20 - 27 °C. To avoid overheating, the south rooms are equiped with an exhaust fan to cool the room with outdoor air whenever the room temperature exceeds 27 °C. This wide range of temperature fluctuation in the south zone provides a wide range

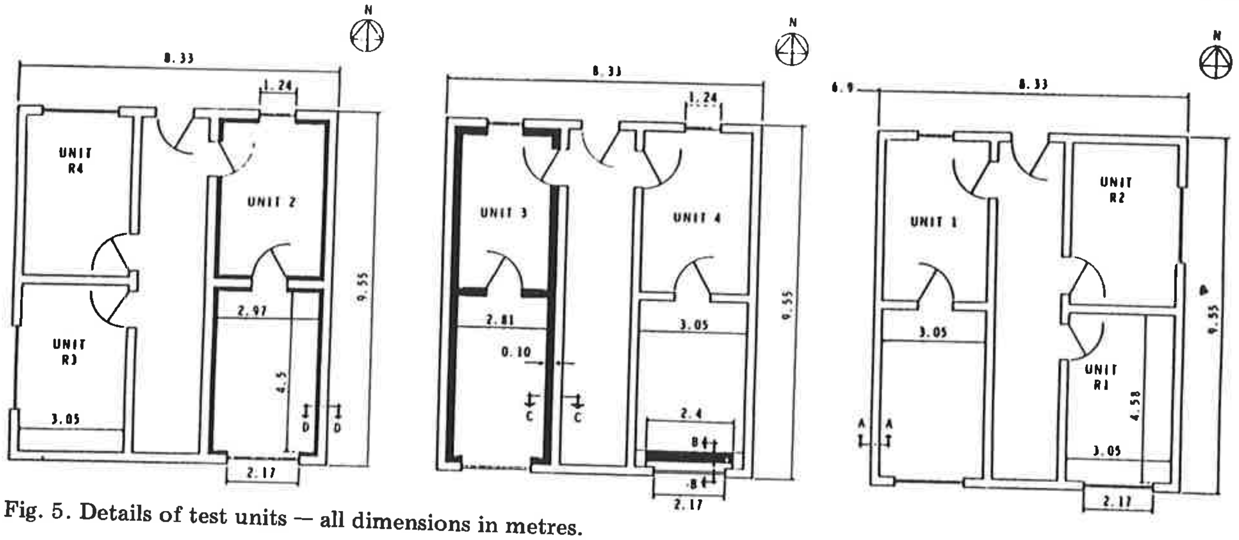

Fig. 5. Details of test units — all dimensions in metres.

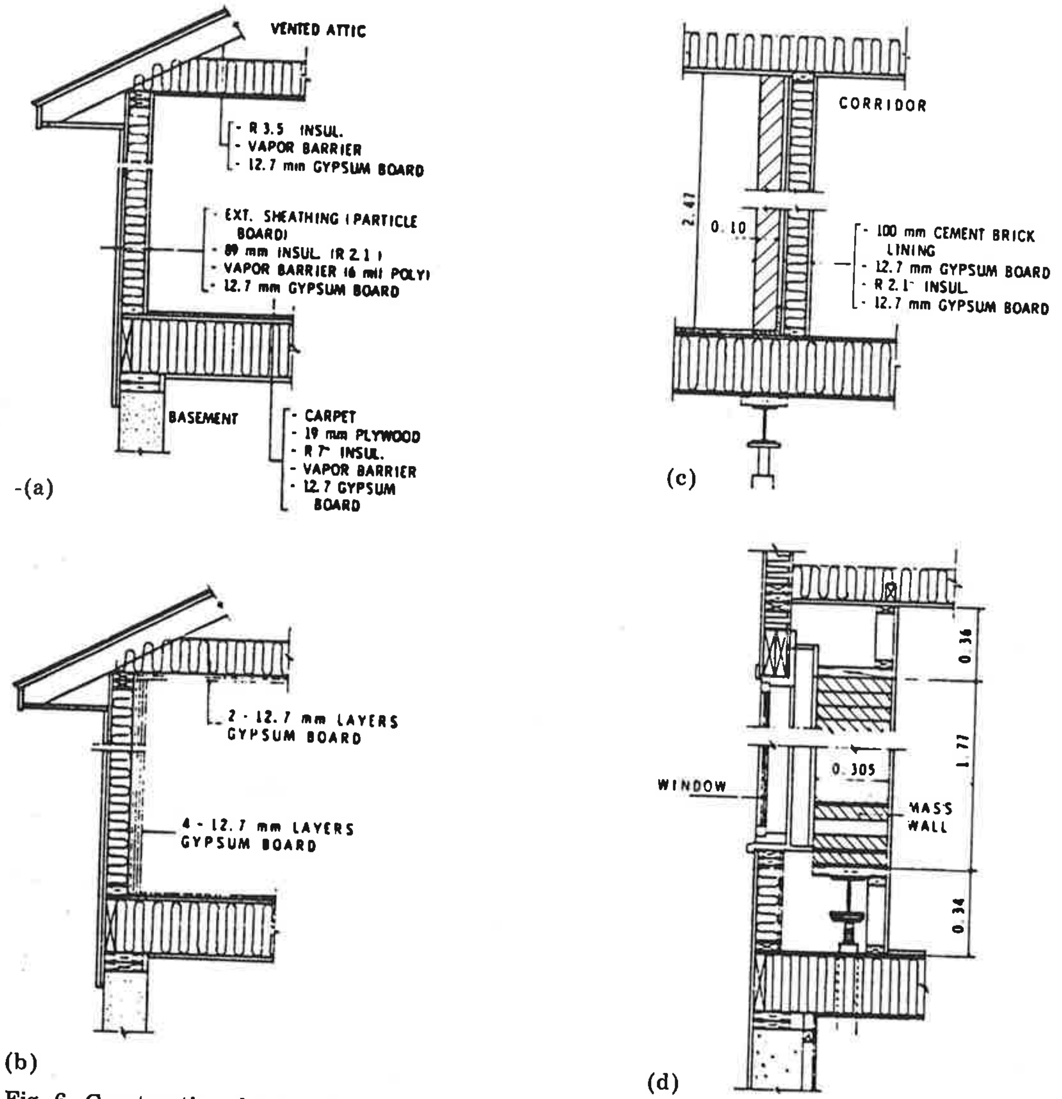

Fig. 6. Construction details of test facility. (Thermal resistance values in m^2 °C/W). (a) Wall construction. Unit 1. (b) Wall construction. Unit 2. (c) Wall construction. Unit 3. (d) "Mass-wall" construction. Unit 4 [23].

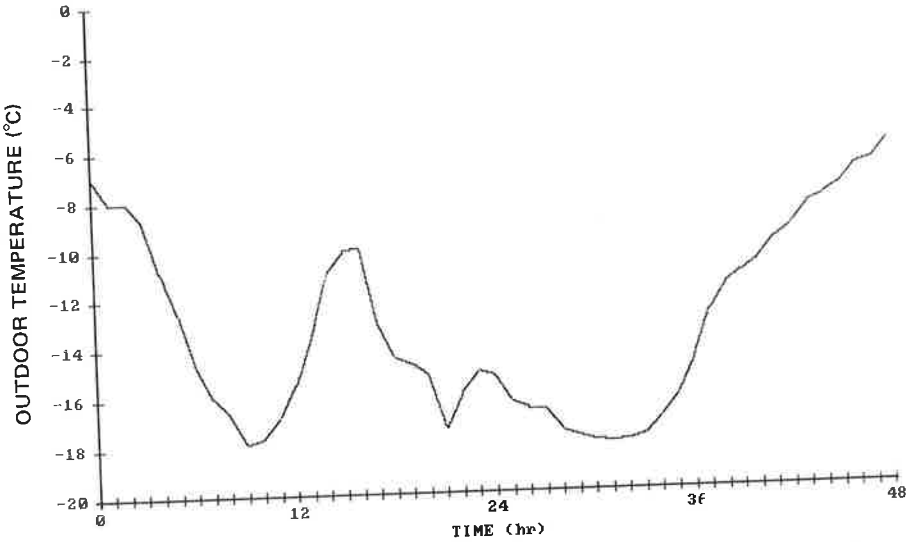

Fig. 7. Example of measured outdoor temperature for December 26 - 28, 1980.

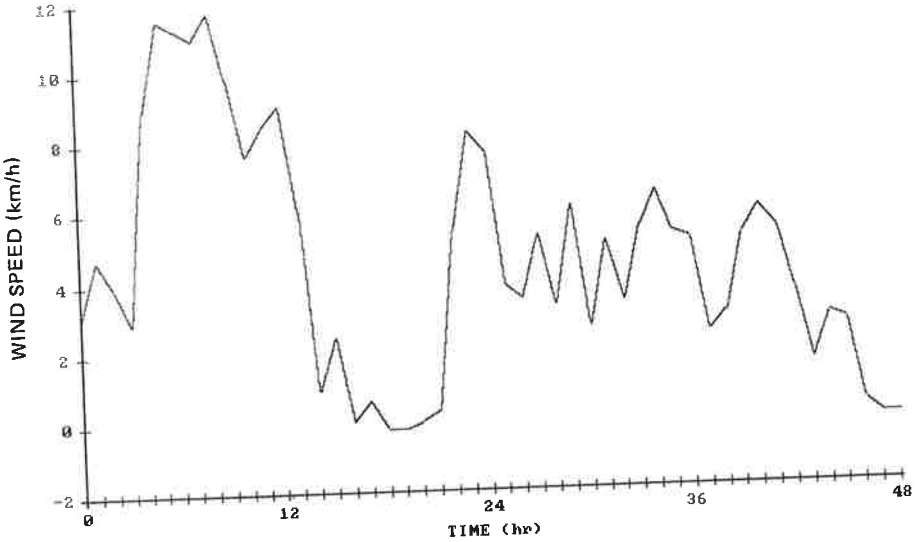

Fig. 8. Example of wind speed for December 26 - 28, 1980.

of dynamic test conditions for the internal thermal mass.

Figures 7 - 10 present examples of typical outside air temperature, solar radiation, and wind speed. This data was applied to develop a dynamic model and to derive the transfer functions coefficients.

ESTIMATING TRANSFER FUNCTION COEFFICIENTS FROM DATA

The analysis involves fitting an appropriate transfer function model to a set of the measured data. A standard least-squares technique is used to determine the coefficients in this model from the actual performance data [24]. The estimated transfer function coefficients of the cooling load due to solar gain through a window are shown in Table 1 along with the results of other investigators and ASHRAE. In general, the data follows the theoretical direction, which means the coefficient w_1 (decay coefficient) increases as the building weight increases. Figures 11 - 13 show the predicted hourly heating and cooling load using eqn. (13) along with the measured data, for three weight levels: light, medium and heavy.

The results indicate that this technique predicts the heating load within 5%, while TARP underestimates it by 23% (see Figs. 1 - 3).

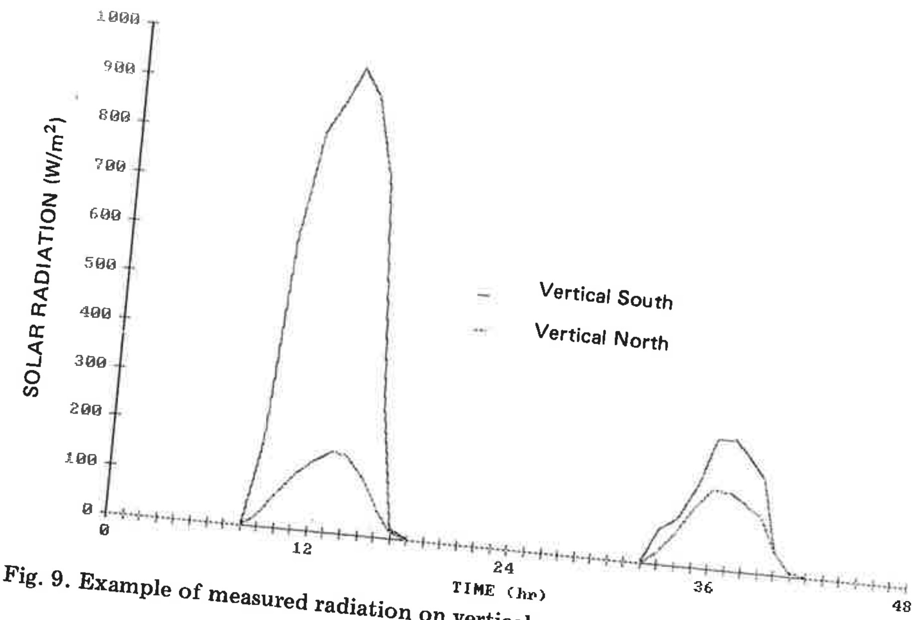

Fig. 9. Example of measured radiation on vertical south and north for December 26 - 28, 1980.

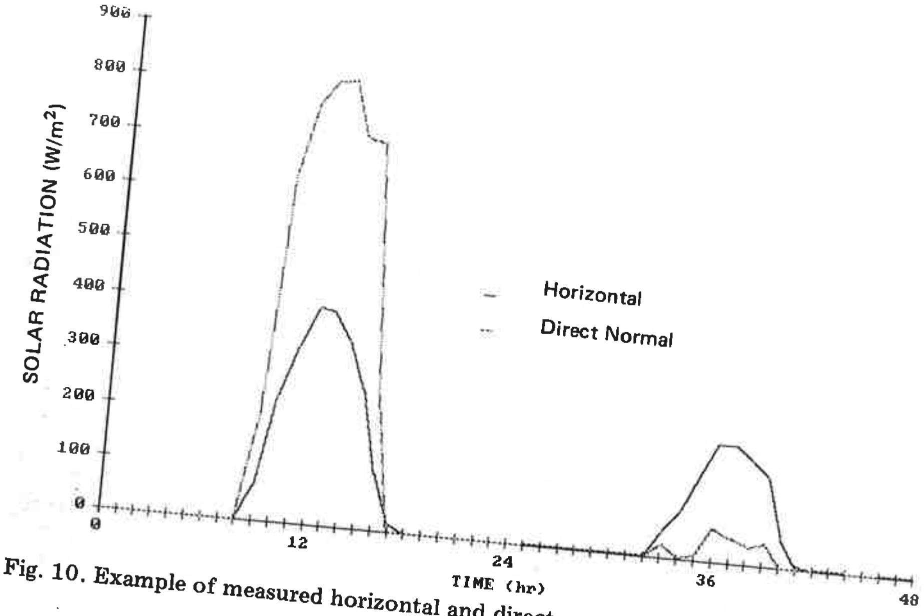

Fig. 10. Example of measured horizontal and direct normal radiation, for December 26 - 27, 1980.

TABLE 1

Response factors

Solar transfer function coefficients	Weight/floor area (kg/m²)									
	46		130		146	341	535		635	—
	Present work	Ref. 6	Present work	Ref. 6	ASHRAE	ASHRAE	Present work	Ref. 6	ASHRAE	Ref. 7
v_0	0.64	0.635	0.52	0.400	0.224	0.197	0.28	0.335	0.187	0.32
v_1	−0.35	−0.240	−0.28	−0.140	−0.044	−0.067	−0.17	−0.205	−0.097	−0.25
w_1	−0.70	−0.605	−0.76	−0.740	−0.820	−0.870	−0.88	−0.870	−0.93	−0.93
f	1	1		1	1	1	1	1	1.31	1

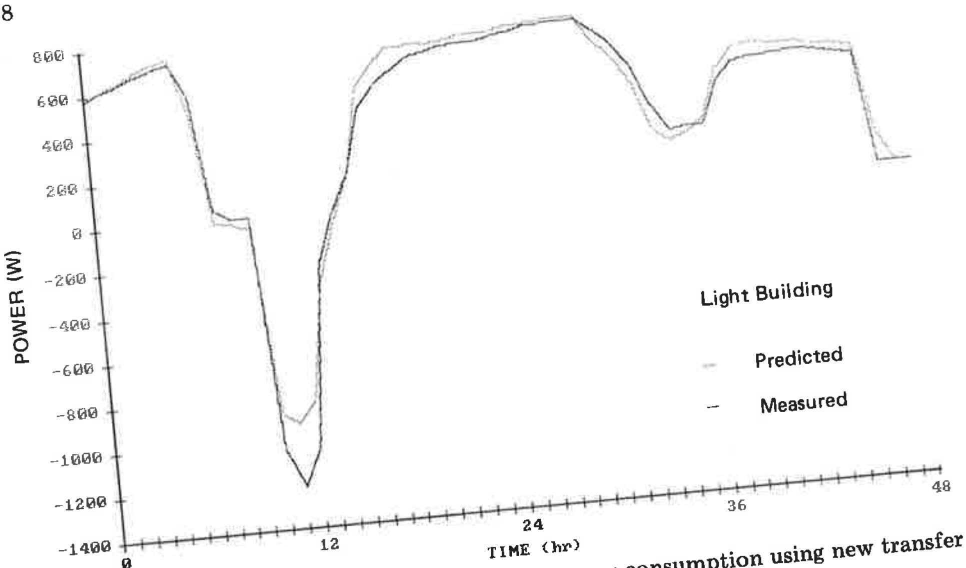

Fig. 11. Comparison of measured and simulation power consumption using new transfer function coefficients.

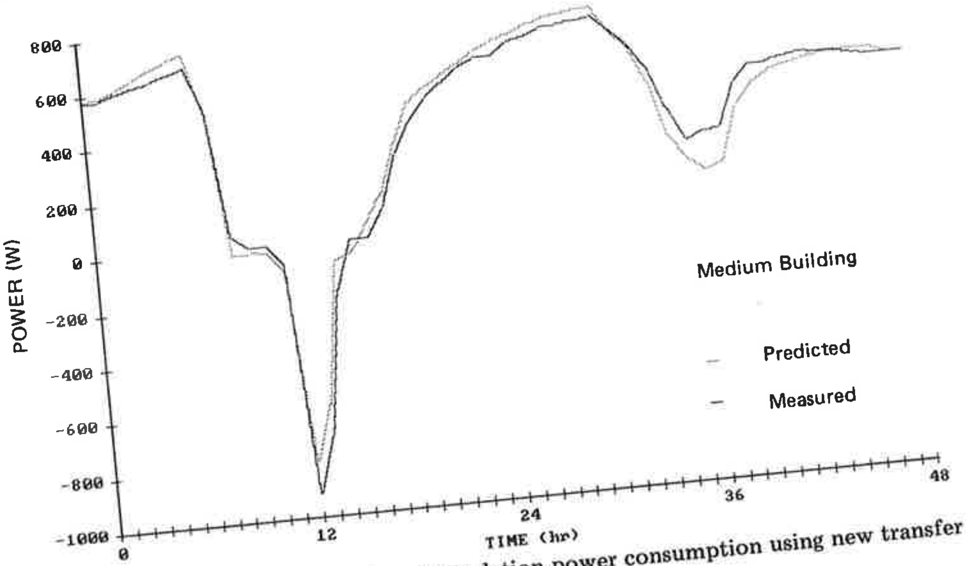

Fig. 12. Comparison of measured and simulation power consumption using new transfer function coefficients.

The cooling load response factors due to the heat gain through the envelope (construction) and the room air-temperature response factors were also estimated. The estimated values could not be directly compared with the values given in the ASHRAE Handbook, since $T^*(t)$ (which is defined as the temperature drop across the surfaces of a wall) is not the same for each wall, and also in eqn. (5) the values of a include wall overall heat transfer coefficients (UAs) and that again is not the same for each wall. It should be also noted that a_0 in ASHRAE does not include UA. In this study, the temperature difference is defined as,

$$\begin{aligned} \Delta T_i^* &= T_0 - T_R \quad \text{for outside surfaces} \\ &= T_i - T_R \quad \text{for inside surfaces} \end{aligned} \tag{6}$$

where T_0 is the sol-air temperature, T_i is the adjacent space room air temperature and T_R is the room air temperature. $T^*(t)$ is a weighted temperature difference defined as:

$$T^*(t) = \frac{\sum_{i=1}^{n} A_i \Delta T_i^*}{\sum_{i=1}^{n} A_i} \tag{7}$$

where A_i is the area of each surface. A more accurate result can be obtained if the overall

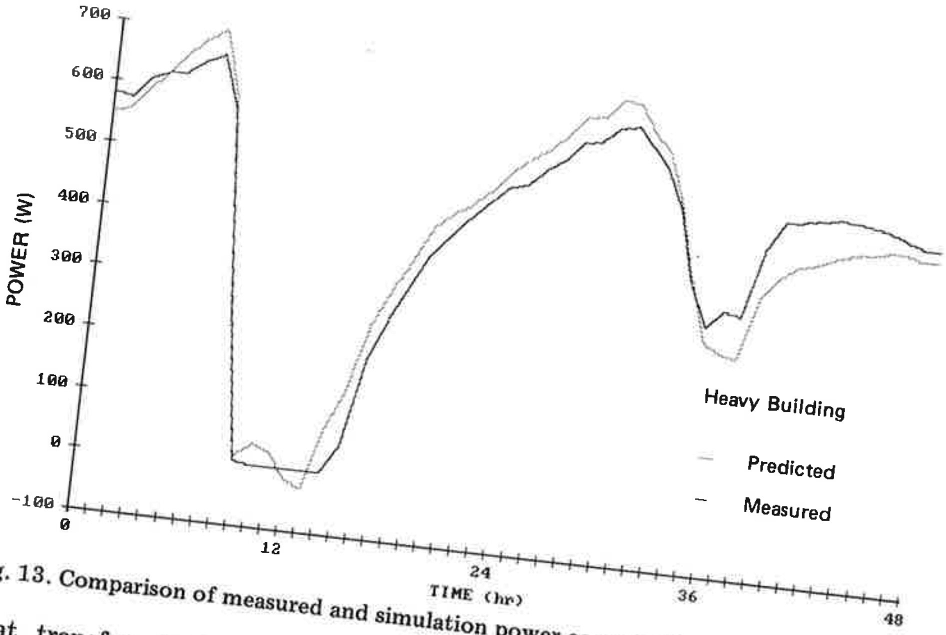

Fig. 13. Comparison of measured and simulation power consumption using new transfer function coefficients.

heat transfer coefficient of each surface is known, then the weighted temperature difference can be defined by:

$$T^*(t) = \frac{\sum_{i=1}^{n} (UA)_i \Delta T_i^*}{\sum_{i=1}^{n} (UA)_i} \qquad (8)$$

For similar reasons, the estimated air-temperature response factors cannot directly be compared with those given in the ASHRAE Handbook due to inclusion of inside film coefficient in calculation of conduction response factors.

SUMMARY AND CONCLUSION

A methodology for determination of the building transfer function coefficients using system identification techniques has been demonstrated. This method involves the following steps: select the form of the model; measure inputs and output variables; fit the model parameters to the measured data.

The obtained results have been demonstrated and compared with the values given in the ASHRAE Handbook and the results of other workers. The method has been shown to be quite applicable and reliable for modelling the dynamic performance of three particular buildings. As mentioned earlier, the values of transfer function coefficients depend on the input and output. Therefore, the transfer function coefficients obtained from this study (solar) can be used in the design of other buildings, and are complementary to those in the ASHRAE Handbook.

The experiment on the three test units indicates that system identification could be a powerful tool in determining dynamic thermal response. By developing an appropriate test procedure, one could apply it to determination of the other transfer function coefficients of eqns. (3) and (4). Since this type of testing has long been used in process industries, it could be well suited to determining the dynamic characteristics of the other elements, such as HVAC system components.

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REFERENCES

1 K. J. Canfield, Controlling building energy, *Military Eng.*, (509) (1986) 445 - 447.
2 *ASHRAE Handbook of Fundamentals*, American Society of Heating, Refrigeration and Air Conditioning Engineers, 1985.
3 F. Haghighat and M. Chandrashekar, System-theoretic models for building load calculation, *ASME J. Solar Energy Eng.*, *109* (1987) 79 - 88.
4 F. Haghighat, M. Chandrashekar and T. E. Unny,

Thermal behaviour of building under random conditions, *Appl. Math. Modelling, 11* (5) (1987) 121 - 133.

5 F. Haghighat and D. M. Sander, Experimental procedures for determination of dynamic thermal response using identification techniques, *Thermal Insulation, 11* (1987) 120 - 143.

6 S. A. Barakat, Experimental determination of the Z-transfer function coefficients for houses, *ASHRAE Trans., 93* (Part 1) (1987) 146 - 160.

7 J. P. Shan, The development of researches on air conditioning load calculations method of building in China, *Proc. Clima 2000, Copenhagen, 1985*, VVS Kongres-VVS Messe, Charlottenlund, Denmark, 1985.

8 G. N. Walton, *Thermal Analysis Research Program Reference Manual, NBSIR 83-2655*, U.S. National Bureau of Standards, 1983.

9 K. J. Astrom and P. Eykhoff, System identification — A survey, *Automatic, 7* (2) (1971) 123 - 162.

10 G. A. Bekey, System identification — An introduction and a survey, *Simulation, 14* (4) (1970) 151 - 166.

11 N. K. Sinha and B. Kuszta, *Modelling and Identification of Dynamic Systems*, Van Nostrend Reinhold Co., 1983.

12 J. O. Gray and P. M. Taylor, Computer aided design of multivariable nonlinear control systems using frequency domain techniques, *Automatica, 15* (1979) 281 - 297.

13 A. T. Kirckpatrick and C. B. Winn, Spectral analysis of the effective temperature in passive solar buildings, *J. Solar Energy Eng., 106* (1984).

14 K. Subbarao, Building parameters and their estimation from performance monitoring, *ASHRAE Trans., 91* (Pt. 2) (1985) 1886 - 1892.

15 R. R. Crawford and J. E. Woods, A method for deriving a dynamic system model from actual building performance, *ASHRAE Trans., 91* (Pt. 2) (1985) 1859 - 1874.

16 D. V. Pryor, P. J. Burns and C. B. Wins, Parameter estimation in passive solar structures, *Proc. 5th National Passive Solar Conference, University of Massachusetts, Amherst, October, 1980*, AS/ISES.

17 C. B. Winn and D. V. Pryor, A sequential filter used for parameter estimation in a passive solar system, *Proc. 3rd Annual Conference on Systems Simulation, Economic Analysis/Solar Heating and Cooling Operational Results, Reno-Nevada, April/May, 1981*, ASME Solar Engineering Division.

18 R. M. C. De Keyser, F. A. Dumortier and A. R. Van Cauwenberghe, Adaptive control for domestic floor heating, J. of CIB, *Bat. Int. Build. Res. Pract., 14* (4) (1986) 239 - 244.

19 J. Florez and C. Barney, Adaptive control of central systems: Part 1: optimum start time control, *Appl. Math. Modelling, 11* (1987) 89 - 95.

20 A. L. Dexter, W. Danninger and W. J. Graham, *Design of Self-tuning Controllers for Wet Heating Systems, Report 1531-84*, Oxford University Engineering Laboratory, 1984.

21 D. R. Farris and T. E. McDonald, Adaptive optimal control — An algorithm for direct digital control, *ASHRAE Trans., 86* (Pt. 1) (1980) 880 - 893.

22 *DOE-2 Engineers Manual*, Energy and Environment Division, Building Energy Simulation Group, Lawrence Berkely Laboratory, University of California, and Group Q-11, Solar Energy Group, Energy Division, Los Alamos National Laboratory, November, 1982.

23 S. A. Barakat, *Passive Solar Heating Studies at the Division of Building Research, Building Research Note 188*, Ottawa, Canada, 1982, ISSN 0701-5232.

24 Åke Björck, Iterative refinement of linear least square solution II, *BIT, 8* (1968) 8 - 30.

APPENDIX A

In general, the transfer function, which relates the contribution of instantaneous heat gain to the cooling load of a component is given in eqn. (A1). To simplify the calculation, a three-term coefficient is used to describe the systems.

$$\frac{Q(z)}{q(z)} = \frac{v_0 + v_1 z^{-1}}{1 + w_1 z^{-1}} \tag{A1}$$

where $Q(z)$ and $q(z)$ are the z-transforms of the room cooling load and the instantaneous heat gain (e.g., solar radiation through window, conduction through the envelopes) and, v_0, v_1 and w_1 are the z-transfer function coefficients. These coefficients quantitatively determine how much of the energy that enters the room is stored and how rapidly that stored energy is released during later hours. Coefficients v_0 and v_1 depend on the type of heat gain (e.g., solar energy through window as compared to conduction through building envelope), while the coefficient w_1 shows the influence of building thermal storage (how much incoming energy is stored and how fast it is released) and depends only on the construction type. The equivalent of eqn. (A1) in the time domain then becomes

$$Q(t) = v_0 q(t) + v_1 q(t-1) - w_1 Q(t-1) \tag{A2}$$

In the second step, as shown in Fig. 4, the total cooling load, along with data about the HVAC system, and the information about the room air-temperature transfer function are applied to determine the actual heat extraction rate (net cooling load) and the actual room air temperature. In general, the actual heat-extraction rate differs from the total cooling load because of fluctuations in the room air temperature. The room air-temperature transfer function which relates the net cooling load to the deviation of the room air temperature from the reference value is defined by:

$$\frac{Q_N(z)}{T(z)} = \frac{g_0 + g_1 z^{-1} + g_2 z^{-2}}{1 + w_1 z^{-1}} \tag{A3}$$

where g_0, g_1, and g_2 are the transfer function coefficients and $Q_N(z)$ is the z-transform of the net cooling load, and $T(z)$ is the z-transform of the temperature deviation.

$$\begin{aligned} Q_N(z) &= Q_t(z) - E(z) \\ T(z) &= T_A(z) - T_R \end{aligned} \tag{A4}$$

where $T_A(z)$ is the z-transform of the air temperature, T_R is the reference temperature, $E(z)$ is the z-transform of the HVAC load heat extraction or addition, and $Q_t(z)$ is the net total cooling load. The equivalent of eqn. (A4) in the time domain is:

$$\begin{aligned} Q_N(t) = {} & g_0 T(t) + g_1 T(t-1) \\ & + g_2 T(t-2) - w_1 Q_N(t-1) \end{aligned} \tag{A5}$$

and solving for the room air temperature:

$$\begin{aligned} T(t) = {} & \frac{1}{g_0} [Q_N(t) + w_1 Q_N(t-1) \\ & - g_1 T(t-1) - g_2 T(t-2)] \end{aligned} \tag{A6}$$

Under steady-state conditions eqns. (A2) and (A5) have some interesting properties.

$$f = \frac{v_0 + v_1}{1 + w_1}$$

and

$$K_T = \frac{g_0 + g_1 + g_2}{1 + w_1} \tag{A7}$$

where f represents the fraction of the heat gain that appears as a cooling load; and K_T is the conductance of the room which includes all heat flow paths such as infiltration as well as conduction through the envelope.

In the ASHRAE Handbook, values of the air-temperature transfer function coefficients (g) are given as normalized factors. This normalization procedure is performed to eliminate the effect of the room conductance and the effect of room size. It is done as follows:

$$\begin{aligned} g_0^* &= (g_0 - K_T)/A_f \\ g_1^* &= (g_1 - w_1 K_T)/A_f \\ g_2^* &= g_2/A_f \end{aligned} \tag{A8}$$

where A_f is the floor area.