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DETERMINATION OF TRANSIENT HEAT CONDUCTION THROUGH BUILDING ENVELOPES—A REVIEW

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ABSTRACT

A review of research on the determination of transient heat conduction through building envelopes is presented. Three methods—numerical techniques, harmonic analysis, and the z-transfer function method—are differentiated. The capabilities, limitations, and assumptions of these methods are given. Methods to determine z-transfer function coefficients are discussed. A limited comparison of experimental methods to derive z-transfer function coefficients is also presented.

Frequency response analysis using a binary multi-frequency sequence signal can give a thorough description of the dynamic thermal performance of a system. The z-transfer function coefficient can then be obtained using multi-linear regression techniques in the frequency domain.

INTRODUCTION

Transient heat conduction through walls can occur due solely to a change in outdoor weather conditions, or because of the space temperature control strategy, occupancy, and lighting use patterns, or due to combinations of both the outdoor and indoor conditions.

Transient effects of the indoor and outdoor conditions on heat conduction through walls are critical factors in predicting the space thermal loads of a building. Therefore, accurate calculation of transient heat conduction through building envelopes will save energy.

Calculation of transient heat conduction through walls is extremely important in designing HVAC systems, which involves the computation of a peak design load for a design day. Accordingly, the designer needs to know the transient heat flow through the inside surface of a wall while outside conditions for the design day vary.

On the other hand, knowledge of surface temperatures and heat flow through the inside surface of a wall is also essential information in thermal comfort studies.

Methods of analyzing the transient heat conduction through walls may be classified into the lumped-parameter technique, the analytical harmonic solution, and the z-transfer function method. The z-transfer function method has been commonly used for the past 20 years to predict transient heat conduction through walls. The major advantage of the z-transfer function method is that the dynamic thermal performance of a wall is characterized by z-transfer function coefficients (ZTFCs) and the desired output at any time is determined from current arbitrary input plus the input and output history values. In contrast to the lumped-parameter technique, the z-transfer function method models a wall thermal phenomenon as a distributed system and expresses the characteristics of the system as ZTFCs, which can be derived to a specified accuracy once and for all. Thus, there is no need to solve the whole system over again when facing another input condition. Therefore, the z-transfer function method is much more efficient in building thermal analysis. It is also more accurate because it does not require the heat transfer boundary conditions to be periodic as in analytical harmonic analysis.

Using the z-transfer function method, the desired ZTFCs can be determined from analytical calculation or estimated from experimental procedures. The analytical calculation of ZTFCs (Stephenson and Mitalas 1971) requires knowledge of the thermal properties of each layer of materials and is based on the assumptions that the walls are made of homogeneous materials and that heat conduction through walls is one-dimensional. Further investigation of the calculating method (Ceylan and Myers 1980) treats walls containing parallel thermal resistance as a two-dimensional problem. Recent research work has been increasingly attracted to the estimation of ZTFCs from laboratory measurements. Estimating the ZTFCs that characterize the multi-dimensional nature of conduction in wall systems becomes more useful as the issue of thermal bridges in wall systems has attracted more attention. The laboratory measurements are attempts to develop methods for determining the dynamic performance of wall systems with thermal bridges or nonconductive heat transfer, as well as novel wall systems.

This paper presents a review of techniques for determining transient heat conduction through walls and discusses in detail the estimation of transfer function coefficients from experimental data.

REVIEW OF PREVIOUS WORKS

The fundamental tool in the modeling of transient heat conduction through building components is the transmission matrix method (Pipes 1957). Namely, the heat flow conducted through a wall and the surface temperatures are related by a square transmission matrix,

$$\begin{bmatrix} T_o \\ Q_o \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} T_i \\ Q_i \end{bmatrix}, \qquad (1)$$

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where T and Q are the temperature and heat flux, and i and o refer to inner and outer surfaces, respectively. Elements in the transmission matrix are called transfer functions and are given either in hyperbolic function or in complex exponential function forms. The transfer functions depend on physical properties and thickness of a wall material. Given different boundary conditions, Equation 1 can be rearranged. For example, the surface heat flows can be expressed as a response to the excitation of surface temperatures. The transmission matrix of a multi-layer wall can be obtained by multiplication of the matrix of each layer of the wall. The thermal analysis of the whole wall can then be treated as for a single-layer wall.

Carslaw and Jaeger (1959) also showed that surface heat flows and temperatures are related by linear equations when the temperatures at each surface of a homogeneous slab vary sinusoidally. The linear equations can be expressed in the matrix form, which is the same as the transmission matrix above.

In accordance with the analogy between heat transfer and electricity, analyses of the transient thermal behavior of walls were explored by using lumped-parameter techniques. These techniques model a physical thermal system as a number of discrete elements, or "lumps," and express the system by using resistance-capacitance (RC) as parameters in the transmission transfer functions. Then numerical techniques may be applied to obtain the solution (Stephenson and Mitalas 1961; Sonderegger 1977a; Davies 1983; Hammarsten et al. 1988).

Lumped-parameter techniques can be used to solve both linear and nonlinear system problems, but it is often necessary to take a large number of "lumps" at frequent intervals to avoid numerically induced oscillations in the solution, and the complete internal temperature distribution must be computed at each time step. Also, it is necessary to solve the complete problem over again when the same structure is subjected to another input time history. Therefore, the numerical techniques are costly and time-consuming, although computers have greatly reduced their disadvantages.

Analytical harmonic analysis, using the Fourier heat conduction law as a starting point, can solve the distributed model of the heat conduction differential equation. If a wall is considered as a thermal system and, ideally, that the change of outside air temperature or solar radiation incident on the outside surface of the wall is repeated in a 24-hour cycle, the harmonic solution can be obtained assuming the heat transfer parameters are constant with time and the radiant heat transfer is linearized (Mackey and Wright 1944, 1946; Van Gorcum 1951; Muncey and Spencer 1966; Maeda 1969; Gupla 1971; Sonderegger 1977b). The drawbacks of analytical harmonic analysis are obvious—the outside excitations include sudden changes in temperature that are not sinusoidal functions, and the sol-air temperature variation does not repeat every day.

The response factor concept revolutionized transient heat transfer analysis. The earliest response factor method (Nessi and Nissole 1925) was to determine a multi-layered slab's heat flux response to a unit step temperature change. The response for a given wall selection was determined once by solving the lumped-parameter RC analog so as to obtain a set of so-called "response factors." Thereafter, any flux was determined by applying the response factors to the actual temperature profile as approximated by rectangular pulses.

Overcoming the weakness in the lumped-parameter techniques and analytical harmonic analysis, Stephenson and Mitalas (1967) developed the thermal response factors (TRFs) method. This method approximates temperature as overlapping triangular pulses and treats the multi-layered slab by exact analysis rather than by lumped-parameter models. TRFs are defined as the time series output resulting from an input triangular pulse and are calculated by solving for roots of the characteristic equation of the system transfer function and summing the residues at each of these pulses. Then the response of the wall to any input can be obtained by approximating the input as a series of triangular pulses and applying the superposition principle. By properly selecting the time interval in the time series expression and using a given number of response factor sets, the TRFs method for calculating heat conduction transients showed adequate accuracy.

Hittle (1981) provided a comprehensive mathematical development for the solution of the thermal response factor method to determine the transient heat conduction through walls.

Studies have been made to solve the transient heat conduction problems by expanding the thermal response factor method. Mitalas (1968) presented a procedure for calculating transient heat flow through walls under periodic or nonperiodic boundary conditions and variable heat transfer coefficients. Kusuda (1969) expanded the thermal response factor method to a multi-layer structure with various curvatures of finite thickness and to semi-infinite systems.

Thermal response factors characterize the thermal performance of walls. The evaluation of response factors involves a numerical search for the roots of the characteristic equation of the Laplace transform solution to the heat conduction equation. Once the eigenvalues are known, residue calculus is used to find the inverse transform that yields response factors. For a homogeneous slab, response factors can be calculated by exact analysis given the thermal properties, dimensions of the slab, and the time interval (Mitalas and Stephenson 1967). Mitalas and Arseneault (1967) developed a computer program that considerably improved the root-finding procedure for the calculation of TRFs.

Hittle and Bishop (1983) developed improved root-finding techniques that calculate the response factors more efficiently. By discovering that the roots of transfer function B are separated by roots of transfer function A (see Equation 1), the method for finding the roots of the characteristic equation eliminated the need for an extremely fine step size while numerically searching for roots and ensured that roots will not be missed.

Ouyang and Haghighat (1991) presented the state-space method to calculate TRFs for multi-layer walls. The approach consists of the following steps: first, expand transfer functions as rational fractions; then establish the state-space equation matrix based on the state-space theory; finally, determine the response factors from a series of multiplications of matrices. Using this method, the TRFs of a multi-layer wall were obtained without need for root finding.

To accurately determine the heat flux for thick walls, a large number of response factors are required. Peavy (1978) developed a method to reduce the number of terms; by relating the heat flux and a set of modified response factors in the conduction transfer functions, the computation time was reduced considerably.

Sherman et al. (1982) developed a simplified model based on surface temperatures and heat flux measurements. The model uses a set of simplified thermal parameters (STPs) to characterize the thermal performance of walls from an arbitrary temperature history. The STPs include the steady-state conductance (U-value), the time constant, and surface storage factors. The derivation of the STPs from the measured temperatures and heat flux can be carried out through a time-domain fitting procedure or by using the frequency domain analysis method. In addition, Sherman revealed the relation of STPs to the thermal response factors. As a result, the thermal response factors can be derived from the STPs. In principle, the developed model is based on the same concept as the thermal response factor method, but the former uses fewer STP terms. To obtain measurements, an envelope thermal test unit (ETTU) was devised as an experimental apparatus. Tests were conducted using a three-section "white-noise" spectrum as the driving function.

Peterson and Mouen (1973) attempted to determine TRFs experimentally. They tried to fit TRFs to measured data directly, but it was unsuccessful. They concluded that the direct procedure for determining TRFs is impractical because of the extreme sensitivity of such procedures to experimental error and the likelihood of error in a transient heat transfer experiment. Consequently, they applied system identification techniques in a two-step procedure: first, fit the thermophysical properties of the chosen model by applying system identification techniques, rather than adjusting the thermal response factors directly; then analytically determine TRFs from the thermophysical properties obtained. As a result, all of the requisite properties of response factors, such as the common ratio property and the overall summation property, were retained.

An extension of the thermal response factor method is the z-transfer function method (Stephenson and Mitalas 1971). The basis of the transfer function method is that the current output can be determined from the current input and the values of both the input and output at previous times:

$$O_{(t)} = \sum_{l=0}^{n} a_l I_{(t-l\Delta)} - \sum_{l=1}^{m} b_l O_{(t-l\Delta)} \qquad (2)$$

where

$O_{(t)}$ = output at time t

$I_{(t-l\Delta)}$ = input at time t and the previous times

$O_{(t-l\Delta)}$ = output at the previous times

Δ = time interval

a_l, b_l = z-transfer function coefficients.

The z-transfer function coefficients, like the TRFs, characterize the thermal performance of walls. The z-transfer function method, however, requires fewer coefficients than the response factor method, and is, therefore, more efficient.

Transfer functions have typically been used to model walls and roofs for which the predominant heat transfer mechanism is conduction. A discussion of the development and properties of the z-transfer function method (Mitalas 1978) indicated that the most important characteristic of the z-transfer function method in comparison with other methods is that the input and output are a sequence of values equally spaced in time. Thus, weather records of outside temperature and solar radiation, given on an hourly basis, can be used as an input with little preprocessing.

Stephenson and Ouyang (1985) analyzed the accuracy of z-transfer functions for walls using a sinusoidal function as excitation in the frequency domain. They indicated that the precision of heat flux calculated using the z-transfer function depends on how closely the frequency response of the transfer function approximates the true frequency response of the wall over the range of the driving function's frequencies. The effect of the time interval used in the transfer function method was discussed. They also indicated that if the number of eigenvalues used in the calculation is greater than four, the extra terms have little effect on the accuracy.

Ceylan and Myers (1980) developed the response-coefficient method to find the long-term solution of heat conduction transients. The response-coefficient method starts from the same fundamental idea as the z-transfer function method—the desired output at any time is expressed in terms of given current and previous inputs plus previously determined outputs. The time-dependent input forcing functions are each approximated by continuous, piecewise-linear functions, each having the same uniform time interval. The response coefficients are calculated by solving the generalized eigenvalue problem on a model that is obtained by discretizing the structure spatially in time. The method is applicable to one- and two-dimensional heat conduction problems. If walls containing parallel thermal resistance are treated as two-dimensional problems, the effect of wall studs on heat conduction can be handled by the response-coefficient method.

ASHRAE has adopted the z-transfer function method to calculate the transient heat conduction through building envelope components. Z-transfer function coefficients for a multitude of different construction types are listed in *ASHRAE Fundamentals* (ASHRAE 1989). These were calculated using the combined outdoor air heat transfer coefficient, indoor air heat transfer coefficient, and material properties of wall constructions tabled in the *Handbook*. Application of these coefficients is limited to cases with sol-air temperature values similarly calculated with the combined outdoor heat transfer coefficients. ASHRAE also cited the computer program (Mitalas and Arseneault 1972) for the calculation of ZTFCs.

The analytical calculation methods for determining ZTFCs mentioned above are in terms of physically defined thermal properties of wall materials. They are based on the assumptions that walls are made of layers of homogeneous materials and heat flow is one-dimensional. Unfortunately, thermal properties of existing wall materials are often unknown. Further, walls contain framing members that act as thermal bridges. The existence of thermal bridges in walls significantly affects the heat transfer through the walls. A study to determine the thermal resistance of a wall experimentally (Fang and Grot 1985) showed the effect of wall framing members on heat conduction through the wall. Thermal resistance values determined from data obtained by a calorimeter were approximately 30% lower than values obtained with a heat flux transducer, and this was attributable to the effect of wall thermal bridges.

To estimate ZTFCs experimentally, research has been carried out in laboratories. A curve-fitting approach (Seem and Hancock 1985) was developed for characterizing the dynamic performance of a thermal storage wall based on measured data. The ZTFCs were obtained directly from measured data using linear least-squares regression. For different boundary conditions, the fitted ZTFCs successfully predicted the heat transfer response.

An experimental method (Stephenson et al. 1988) was developed to estimate the thermal dynamic performance of walls using a calibrated hot box. The test facility consists of three basic components: a metering chamber, a climate chamber, and a specimen support frame. The metering chamber was maintained to simulate a steady indoor condition and to serve as a calorimeter. The climate chamber was used to simulate outdoor conditions by generating a ramp excitation function. A full-scale wall that contains thermal bridges and anomalies was mounted in the support frame and sandwiched between the two chambers. The procedure to derive the ZTFCs was as follows. The transient heat flux through the inside surface of the wall was determined from the measurements in the metering chamber by taking an energy balance. Then the poles and residues for a ramp analytical solution were determined by analyzing the measured heat transfer response. The thermal transmittance of the test wall was also determined from the test measurements. Next, the thermal response factors were obtained from the thermal transmittance, the poles, and the residues of a triangular pulse excitation function. Finally, the ZTFCs were derived from the relation with the response factors. The predicted transient heat flux from the empirical ZTFCs agrees fairly well with the heat flux calculated from the analytical ZTFCs.

Burch et al. (1988) used curve fitting (Seem and Hancock 1985) to determine ZTFCs for a masonry wall specimen tested in a calibrated hot box. Four tests were conducted with different excitation functions. The measured specimen heat flux for each of the excitation functions was determined from an energy balance of the metering chamber. The ZTFC sets for the masonry wall were obtained, and the set of coefficients from the diurnal cycle dynamic test was used to predict the specimen heat flux for other tests. A comparison of the measured and predicted heat fluxes, using the fitted ZTFCs for the diurnal dynamic cycle tests, showed a good fit for three other tests. The results also indicated that the fitted ZTFCs were more successful in predicting the heat flux than an analytical model using tabulated thermal properties.

Burch et al. (1990) also applied the experimental procedure developed by Stephenson et al. (1988) to determine ZTFCs for a wall specimen. The dynamic test method was conducted on a multi-layered homogeneous masonry wall specimen without thermal bridges. The predicted diurnal performance of the wall from the obtained transfer function coefficients was successful.

Haghighat and Sander (1987) used system identification techniques to determine the dynamic response of walls. A wall was considered as a system with an output (the heat flux on the inside surface of the wall) and an input (the outside weather conditions) related by a "black box"—transfer function. A binary multi-frequency sequence (BMFS) was used as the input driving function. The coefficients of the z-transfer function, which was used to predict the heat flow through the inside surface of the single-layer wall sample due to the temperature change on the surface of the other side, were obtained using both frequency response analysis and least-squares regression in the time domain.

The frequency response analysis method includes a two-step procedure. First, perform a Fourier transform on the measured data and determine the system transfer function that shows dynamic performance of the wall sample. Then employ multi-linear regression techniques fitting the ZTFCs to the frequency response obtained.

The time domain regression was also applied directly to the measured data. The predicted heat flow from the fitted ZTFCs was successful in both frequency and time domains.

Extending the method developed by Haghighat and Sander, the experimental procedure was also conducted on a multi-layer wall sample (Haghighat et al. 1991a). The predicted heat flow through inside surface of the wall sample agreed with that obtained from the analytically calculated ZTFCs.

To investigate the thermal performance of walls due to variations in both indoor and outdoor conditions, Haghighat et al. (1991b) used the frequency response analysis method mentioned above to determine the other transfer functions in Pipes' transmission matrix. The dynamic frequency response agrees fairly well with that obtained from analytical analysis.

Summary of Previous Results

Transient heat conduction through walls is calculated by three methods: lumped-parameter techniques, analytical harmonic analysis, and the z-transfer function method. The z-transfer function method analyzes a wall as a distributed system without constraints for boundary conditions. Once the ZTFCs of a wall are determined, the heat conduction through the wall caused by any excitations can be easily calculated.

Methods reviewed above for determination of ZTFCs are summarized in Table 1. The main conclusions are:

1. ZTFCs of either a homogenous material wall or a composite wall can be obtained from the analytical calculation methods that presuppose thermal properties of wall materials. If it is assumed that the walls are made of layers of homogenous materials and that heat flow is one-dimensional, the analytical calculation computer program (Mitalas and Arseneault 1972) can be used to derive the ZTFCs of the walls.

TABLE 1
Summary of Methods for Determining ZTFCs of Walls

Source	Thermal Properties	Construction Nature	Excitation Function	Derivation Procedure	Test Facility	Measurement Method
Hittle & Bishop(21)	Presupposed	HM&1-D	Triangular Pulses	Improved root finding		
Ouyang & Haghighat(22)	Presupposed	HM&1-D	Sinusoidal function	State space matrix		
Sherman et. al.(24)	Unknown	1-D	White-noise spectrum	Fit STPs and derive TRFs	ETTU	T/C on surfaces
Pederson & Mouen(25)	Unknown	1-D	Ramp function	Fit K, α, ... derive TRFs	Test chamber with baffle	T/C on surfaces
Stephenson & Mitalas(26)	Presupposed	HM&1-D	Variable excitation	Root finding		
Ceylan & Myers(29)	Given	1-D	continuous piecewise-linear func.	finite difference computation		
Seem & Hancock(33)	Unknown	Thermal storage wall	Sol-air temperature	Curve fitting	Passive solar calorimeter	Thermopile for air temperature
Stephenson et. al.(34)	Unknown	Composite wall	Fast ramp signal	Root finding	Calibrated hot box	T/C on baffle
Burch et. al.(36)	Unknown	HM&1-D	Diurnal cycle	Curve fitting	Calibrated hot box	T/C on baffle
Haghighat & Sander(37)	Unknown	Single layer	BMFS	Frequency response	Symmetric apparatus	T/C on surfaces
Haghighat et. al.(38)	Unknown	Composite wall	BMFS	Frequency response	Symmetric apparatus	T/C on surfaces

HM&1-D: Homogeneous Materials and 1-Dimension
1-D: 1-Dimension
T/C: Thermal Couples

For walls containing parallel thermal resistance, the response coefficient method (Ceylan and Myers 1980) can be applied for handling the two-dimensional problems.

2. ZTFCs can be derived from TRFs. One of the calculation methods to determine TRFs is the root-finding and/or improved root-finding method. The other is the state space matrix method. These two calculation methods require advance knowledge of the thermal properties of walls and are based on the assumption of one-dimensional heat conduction. To estimate TRFs from experimental measurements, system identification techniques may be applied to determine TRFs from the thermal properties that were optimized from the measured data (Peterson and Mouen 1973). The simplified thermal parameters method (Sherman et al. 1982) can also be used to determine the STPs from experimental data TRFs can then be derived from the obtained STPs.
3. Experimental methods to estimate ZTFCs require no advance knowledge of the thermal properties of wall materials. Thermal bridges resulting in multi-dimensional heat conduction in wall systems can be handled. The curve-fitting method (Seem and Hancock 1985) determines ZTFCs directly from measured data using linear least-squares regression. The root-finding procedure (Stephenson et al. 1988) can be carried out to determine TRFs from experimental data; ZTFCs can then be determined. Frequency response analysis (Haghighat and Sander 1987; Haghighat et al. 1991a, b) can give a thorough description of the dynamic thermal performance of wall systems. Multi-linear regression techniques may then be used to fit the ZTFCs to the frequency response obtained.

Limited Comparison of Methods to Derive ZTFCs Experimentally

Methods of estimating ZTFCs experimentally can be compared as to the selection of test excitation functions, different data analysis procedures, and measurement techniques.

A test signal is used to simulate certain boundary conditions, and it may relate to the corresponding data analysis procedure. In the methods reviewed above, input signals are used to simulate sudden changes, variations in a short or long period, or a diurnal cycle. A ramp (or a fast ramp) excitation function can be used to simulate the sudden change of outside temperature and is the basic condition in the root-finding procedure (Stephenson et al. 1988) to determine TRFs. But a ramp excitation function cannot be used to simulate long or short period variations. The diurnal cycle may be used to simulate daily temperature profiles and, combined with the regression analysis procedure, produce successful results (Seem and Hancock 1985). But the sol-air diurnal cycle does not repeat every day, so a large number of tests are required if a solution is required for a long time period. To simulate variations in long or short periods and simulate sudden changes, BMFS signals (Haghighat and Sander 1987; Haghighat et al. 1991a, b) can be used. The BMFS signals simulate multi-variations and are easy to generate. The main advantage of using BMFS as excitation functions is that a multi-frequency response can be determined from one test. This requires less precise control and takes less time than test procedures that must be repeated for each frequency. BMFS signals also can be used to yield a best-fit-of-frequency response for a large number of frequencies by regression analysis.

The data analysis procedures described above contain the root-finding method (Stephenson et al. 1988), the linear regression techniques in a time domain (Seem and Hancock 1985), and the frequency response analysis (Haghighat and Sander 1987; Haghighat et al. 1991a, b). The root-finding method may be used to obtain the response to a sudden change of temperature, but the analytical procedure to determine ZTFCs is indirect and costly. The linear regression techniques in the time domain can be used to obtain the overall system performance, but it is less accurate since it shows the performance statistically. The frequency response analysis can be used to reveal the response to sudden changes at specific frequencies, and, if the excitation function contains multi-frequencies, a thorough description of the system response profile can be obtained. Further, the frequency response analysis is more accurate than the direct regression techniques in the time domain.

Measurement techniques refer to the application of instruments to measure energy quantities and temperatures. To measure the heat flow rate through the surface of a wall, the commonly used instrument is the heat flow meter (HFM). Available HFMs are not fast enough to give accurate readings at higher frequencies and do not easily satisfy the test requirements. The symmetrical test apparatus (Haghighat and Sander 1987; Haghighat et al. 1991a, b) can be set up to meet the desired test requirements, but imprecise control may result in a noisy power reading. The calibrated hot box (Stephenson et al. 1988) can be devised to simulate both indoor and outdoor conditions, and it gives closer control under the desired test requirements. Also, the measured heat flow rate can be calculated by taking an energy balance in the metering chamber.

To satisfy test requirements and achieve accurate readings at higher frequencies, the test apparatus mentioned above should be revised, or new measurement technique should be devised.

CONCLUSION

Research on the determination of transient heat conduction through walls has been reviewed. The z-transfer function method provides an efficient and accurate solution of transient heat conduction through building components. The ZTFCs can be analytically calculated or experimentally estimated. As shown in the limited comparison of methods above, more research is needed to revise and develop experimental methods for determining ZTFCs of walls as well as complex buildings. This includes using test signals that simulate multi-variations and require less precise control, employing frequency response techniques to obtain a thorough description and yield more accurate analysis, and using a calibrated hot box or a new test apparatus to achieve the desired experimental conditions.

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