

Convection Between Zones with Non-Linear Temperature Distributions

Large temperature
differences of $\sim 12^\circ\text{C}$
→ isothermal zone
approximation OK?

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ABSTRACT

Interzonal natural convection is an important process in the redistribution of thermal energy in passive solar enclosures. In this paper, interzonal natural convection in a two zone full scale building with non-linear zone temperature distributions is analyzed. Measurements of interzonal convection were taken in a doorway joining two rooms of the National Bureau of Standards Passive Solar Test Facility. A Bernoulli interzonal air flow model based on isothermal zone temperatures is modified to account for the non-linear zone temperature distributions. The measured zone temperature distributions are used in the modified model to predict interzonal mass flow rate, heat transfer rate, and velocity profiles in the doorway. The experimental values and values predicted by both the non-linear and isothermal model are compared and found in good agreement.

LIST OF SYMBOLS

A	Area of doorway (m^2)
C_m	Mass flow rate discharge coefficient (-)
C_p	Specific heat ($\text{J/kg}^\circ\text{K}$)
C_q	Heat transfer rate discharge coefficient (-)
g	Gravitational constant (m/s^2)
H	Doorway height (m)
h_k	Height at base of section k (m)
\dot{m}	Total doorway mass flow rate (kg/s)
\dot{m}_{ij}	Positive mass flow rate from zone i to zone j (kg/s)
\dot{m}_{ijk}	Positive mass flow rate from zone i to zone j in section k (kg/s)
N	Number of enclosure sections
P_{oi}	Pressure in zone i at $y=0$ (N/m^2)

P_{oj}	Pressure in zone j at $y=0$ (N/m^2)
Q_{net}	Net interzonal heat transfer rate (W)
T_i	Temperature of zone i at mid-height of doorway ($^\circ\text{C}$)
T_j	Temperature of zone j at mid-height of doorway ($^\circ\text{C}$)
T_{ik}	Temperature of zone i in section k ($^\circ\text{C}$)
z_k	Height at top of section k (m)
$V_{ij}(y)$	Doorway velocity distribution for flow from zone i to zone j (m/s)
W	Width of doorway (m)
Y_{nh}	Distance of the neutral height above the floor (m)

CHANGES

β	Coefficient of thermal expansion $\frac{1}{\rho} \frac{d\rho}{dT}$ ($1/^\circ\text{K}$)
$\bar{\rho}$	Average enclosure density (kg/m^3)
ρ_i	Density of zone i (kg/m^3)

INTRODUCTION

Solar performance and the thermal comfort of passive solar buildings can be improved by including interzonal convective heat transfer into the building design process. Interzonal convective air flow which is driven by the difference in zone pressures has been modeled through the application of the Bernoulli equation by assuming inviscid flow and isothermal zone temperatures [1-3]. Previous two zone full scale natural convection experiments [3,5] have shown that the values predicted by the isothermal model, with the neutral height located at the mid-height of the door, correlates well with experimental data. In reference 5, the location of the neutral height was obtained by applying the equation of continuity in

the doorway. This allows the model to include multizone enclosures, infiltration, and forced air systems with supply and return in different zones.

The isothermal model has been modified to include non-linear zone temperature distributions. The reason for developing the more complex non-linear model is to validate the interzonal flow model used in the building simulation code described in reference 6.

This paper will discuss the isothermal and non-linear models, the interzonal flow experiments conducted to validate the models, and compare the predicted and experimental velocity profiles, mass flow rates and heat transfer rates.

INTERZONAL FLOW THEORY

The temperature distribution in each zone and in the opening is a critical factor in determining the buoyancy driven interzonal flow. It is difficult, however, to map out the entire temperature field from available data. Furthermore, the one-dimensional models cannot properly account for the three-dimensional temperature distribution. But the use of a discharge coefficient in one-dimensional models makes it possible to bring the predicted and measured values within reasonable agreement. Hence the use of empirically determined discharge coefficients for different experimental configurations may enable one to estimate the interzonal flow for similar configurations by using one-dimensional models.

The theoretical formulation will consider the general two zone system, as shown in Figure 1. A

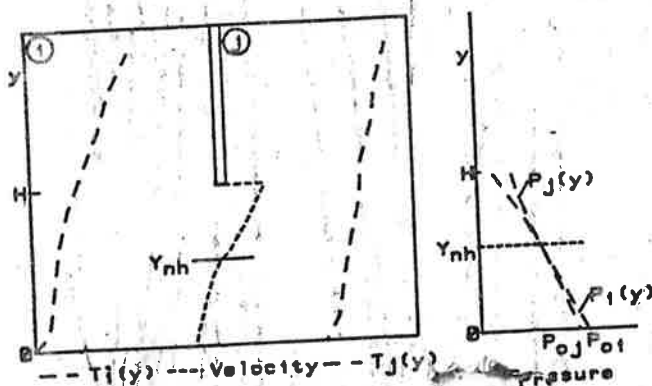


Fig. 1 Two Zone Configuration.

rectangular opening of height H and width W connects the zones i and j. The flow through the aperture is driven by the difference in the zone pressure distributions. If we assume that the pressure in each zone is hydrostatic, and that the flow is inviscid, quasi-steady and symmetrical to the mid-height of the aperture. Then the local velocity of air moving through the aperture obtained by the application of the Bernoulli equation is expressed as:

$$V_{ij}(y) = C_m \sqrt{2g\beta(T_i - T_j)(y - Y_{nh})} \quad (1)$$

Equation (1) is for the air flowing from zone i to zone j above the neutral height. Integrating equation (1) from the neutral height to the top of the aperture results in the mass flow rate as:

$$\dot{m}_{ij} = \frac{C_m W}{3} \sqrt{2g\beta H^3 (T_i - T_j)} \quad (2)$$

and the net heat transfer as:

$$\dot{Q}_{net} = \frac{C_m W}{3} \sqrt{2g\beta H^3 (T_i - T_j)} = \frac{\dot{m}_{ij}}{c_m} c_p (T_i - T_j) \quad (3)$$

When the neutral height is not assumed to be at the mid-height of the doorway, the unknown base pressure difference, $P_{o1} - P_{oj}$ (shown in Figure 1), appears in the equation of the velocity. From reference 5 the velocity of the flow from zone i to zone j may be expressed as:

$$V_{ij}(y) = C_m \sqrt{\frac{2}{\rho} (P_{o1} - P_{oj}) + 2g\beta(T_i - T_j)y} \quad (4)$$

The unknown base pressure difference can be determined by applying continuity in the doorway as follows:

$$\dot{m} = C_m W \int_0^H V(y) dy = 0 \quad (5)$$

$$\text{or } \dot{m}_{ij} = \dot{m}_{ji} \quad (6)$$

$$\text{where } \dot{m}_{ij} = \rho_j W C_m \int_{Y_{nh}}^H \sqrt{\frac{2}{\rho} (P_{o1} - P_{oj}) + 2g\beta(T_i - T_j)y} dy \quad (7)$$

$$\text{and } \dot{m}_{ji} = \rho_i W C_m \int_0^{Y_{nh}} \sqrt{\frac{2}{\rho} (P_{oj} - P_{oi}) + 2g\beta(T_j - T_i)y} dy \quad (8)$$

The "ij" and "ji" convention has been adopted to keep the quantity under the radical in equations (7) and (8) positive.

A non-linear temperature distribution can be included by dividing the area between the base and top of the door into N horizontal sections (not necessarily of equal area) and assuming an isothermal temperature in each section. When this is done, equations (6), (7), and (8) become:

$$\dot{m} = \sum_{k=1}^N \dot{m}_{ijk} - \sum_{k=0}^L \dot{m}_{jik} = 0 \quad (9)$$

where section L contains the neutral plane and where

$$\dot{m}_{ijk} = \rho_{ik} W C_m \int_{L_k}^{U_k} \sqrt{\frac{2}{\rho_{ik}} (P_{oi} - P_{oj}) + 2g\beta(T_{ik} - T_{jk})y} dy \quad (10)$$

and

$$\dot{m}_{jik} = \rho_{jk} W C_m \int_{L_k}^{U_k} \sqrt{\frac{2}{\rho_{jk}} (P_{oj} - P_{oi}) + 2g\beta(T_{jk} - T_{ik})y} dy \quad (11)$$

The neutral plane is located at the height where the zone pressure difference and velocity is

equal to zero as shown in Figure 1. By setting equation (4) equal to zero, the location of the neutral height above the floor is found as:

$$Y_{nh} = \frac{P_{oi} - P_{oj}}{V_L \beta g (T_{jL} - T_{iL})} \quad (12)$$

Assuming that the zone temperature distribution is known, substitution of equations (10), (11), and (12) into equation (9) will result in an expression for the only unknown, the base pressure difference. The base pressure difference can be determined by the secant method described in reference 5.

The net heat transfer rate from zone j to zone i is determined by integrating the enthalpy across the doorway as:

$$\dot{Q}_{net} = -C_p \int_0^H V(y) T(y) dy \quad (13)$$

For the velocity and temperature profiles shown in Figure 1, equation (13) simplifies to:

$$\dot{Q}_{net} = -C_p \sum_{k=L}^N \dot{m}_{ijk} T_{ik} - C_p \sum_{k=0}^L \dot{m}_{jik} T_{jk} \quad (14)$$

where the mass flow rates, \dot{m}_{ijk} and \dot{m}_{jik} , are defined by equations (10) and (11). Equation (14) assumes the temperature of the air in the doorway is equal to the upstream temperature at the same height. *figs*

DESCRIPTION OF EXPERIMENT

The two zone full scale natural convection experiments were conducted at the NBS Passive Solar Test Facility. The building is a rectangular one story, slab on grade, frame structure, with the long axis running from east to west. It is divided into four identical rooms of equal floor area (30.1m²) and volume (88.4m³). Figure 2 shows

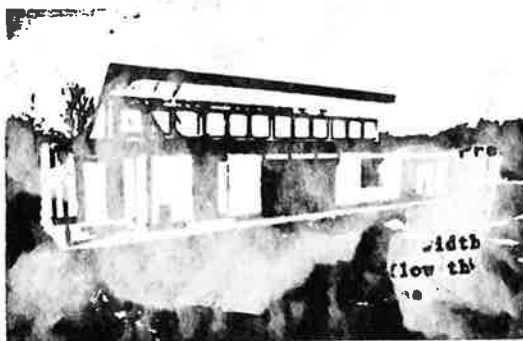


Fig. 2 NBS Passive Solar Test Building

the south facing exterior of the test building, and Figure 3 shows the floor plan. A complete description of the building is available in reference 7. The experiment was performed in cell #2

and cell #3. Doorway velocity and temperature distributions were measured by eight hot wire anemometers and thermocouples located along the vertical centerline of the doorway. Zone air temperatures were measured using two vertical arrays of thermocouples. Neither cell was internally exposed to solar radiation.

Before data collection was initiated, a removable door was placed in the doorway connecting cells #2 and #3. Cell #2 was heated by baseboard heaters while cell #3 was cooled using a fan coil unit. When the average zone temperature difference between cell #2 and cell #3 was approximately 12°C, the fan coil units were turned off in both zones, the door between the zones was removed. The status of the baseboard heaters in cell #2 for different tests is summarized in Table 1. Data were then collected every two minutes for a period of seven or more hours. The maximum uncertainty is estimated to be 0.5°C in the temperature measurement and 0.025 m/s in the air speed measurements.

DISCUSSION OF RESULTS

Four interzonal test were performed. These tests are the February 15 a.m., February 15 p.m., September 21, and October 3 test. In the February 15 p.m. test, September 21 test, and October 3 test, auxiliary heat was supplied by the west electric baseboard heaters in cell #2 shown in Figure 3. In the September 21 test, the radiation

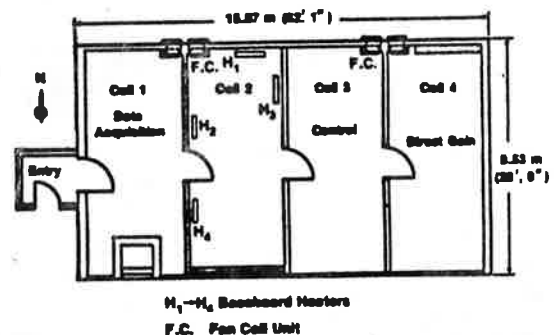


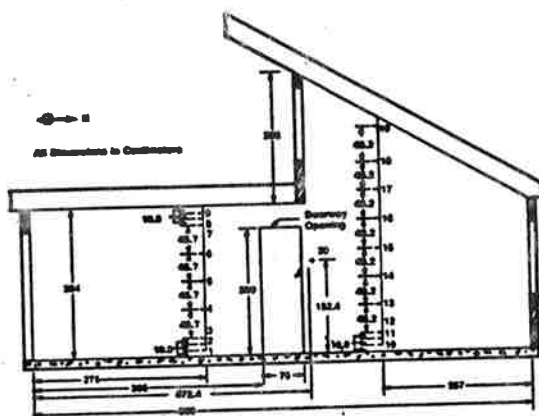
Fig. 3 Floor Plan of the NBS Passive Solar Test Building

shields were removed from the cell #2 air thermocouples to see if they were affecting the data. No effect was evident in the results. Radiation shields were not used in cell #3. In the September 21 and October 3 tests, the South air thermocouple arrays, shown in Figure 4, were moved to the east-west center plane of the doorway and at a distance of one meter from the doorway. This type of thermocouple positioning allows predicted results of both zone temperature distributions in and out of the flow stream to be compared. The predicted and experimental results of these tests will be compared in the following. The general configuration of each test is summarized in Table 1.

In Figure 5, the data taken one hour after the

Table 1. Test Configurations

Test	Date	Sensors in cell #2	Radiation shields in cell #2	Location of South T.C. array
1	Feb. 15 am	no	yes	Fig. 4
2	Feb. 15 pm	yes	yes	Fig. 4
3	Sep. 31	yes	no	Near door
4	Oct. 3	yes	no	Near door



Thermocouple Strings are on the North to South Centerline

Fig. 4 Schematic East View of Test Cells Showing Location of Air Temperature Sensors

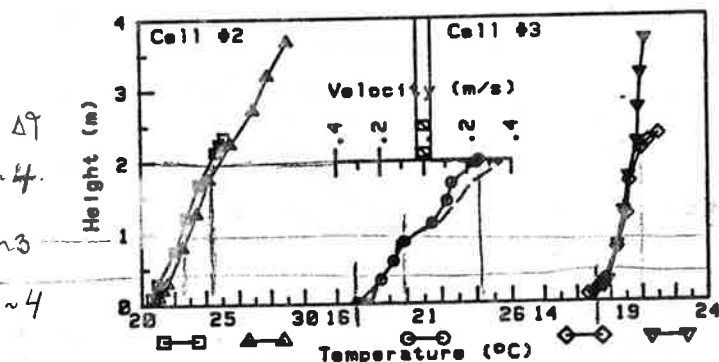


Fig. 5 Temperature and Velocity Distribution One Hour After Start of No Auxiliary Heated Test, February 15

start of the test are shown for the no auxiliary heat experiment conducted during the morning of February 15th. The data are displayed on an east-west cross section of the building with temperature scales located at the bottom of the figure and enclosure height shown on the left side. In the center of the figure, the doorway is shown with the velocity scale located at the top of the door. The velocity distribution is represented

~ Linear $v(y)$
agrees with Linear $T(y)$

by the dashed line and the doorway temperature distribution by the line marked by circles. The non-linear zone temperature distributions of the thermocouple arrays, shown in Figure 4, are displayed in both cell #2 and cell #3. The zone temperature distribution in cell #2 is seen to be more stratified and in cell #3 less stratified. The doorway temperature distribution is fairly linear.

Figure 6 shows the data taken one hour after the start of the test with auxiliary heat on in cell #2. In this Figure, the zone temperature distribution above the doorway in cell #2 is destratified. The temperature distribution in zone #3 is similar to the one in Figure 5 except that in this test the south thermocouple arrays were moved to a distance of one meter from the door. This

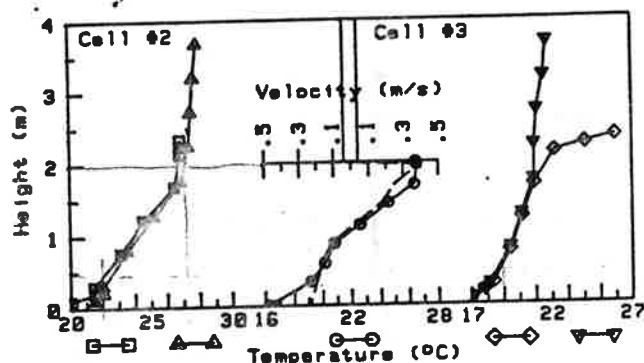


Fig. 6 Temperature and Velocity Distributions One Hour After Start of Auxiliary Heated Test, October 3

closer positioning caused the top south zone temperatures to be higher relative to the ones displayed in Figure 5.

Referring to Figure 5 and 6, the path of the warm air flowing into cell #3 apparently rises to the ceiling just above the doorway as indicated by the large temperature increase at the top of the shorter thermocouple array. The velocity profile is observed to be non-symmetric about the neutral height. The location of the neutral height is approximately 5.0 cm above the mid-height of the door determined by linear interpolation of velocity data. The shape of the profile below the neutral height is nearly parabolic as predicted by the isothermal model with a maximum velocity of 0.22 m/s. Above the neutral height the velocity profile shape is more linear with a maximum velocity of 0.37 m/s near the top of the door. This non-symmetric velocity distribution in the doorway is apparently caused by the non-symmetric boundary constraints at the top and bottom of the opening presented by this experimental set-up. The curving of the warm air streamlines around the top edge of the opening causes the flow to accelerate near the top; while the interaction of the cool air streamlines with the floor (no slip condition) causes the flow to retard near the floor [8].

$$v(z) = C_1 \sqrt{g/T} \sqrt{2z \Delta T}$$

$$C_1 = 0.6$$

$$z = +0.95$$

$$\Delta T = 27 - 21$$

$$U = .37 \text{ m/s}$$

$$-1.05$$

$$20 - 17$$

$$.27 \text{ m/s}$$

$$.22$$

$$\Delta T = 2 \text{ not } 3$$

The experimental and predicted velocity profiles are compared in Figure 7. The predicted values for the isothermal model were obtained by using a value of $C_m=0.47$ in equation (1), and for the non-linear model by using a value of $C_m=0.41$ in equation (4). These values of C_m were obtained by curve fitting the mass flow rate data as discussed in the next paragraph. It should be noted that the value of C_m suggested in reference 2 and 3 is 0.611. As seen in the figure, very good agreement exists between both predicted and experimental velocity profiles. Since the position of the neutral height was variable in the non-linear model, it is located at 1.05m which is very near the experimental neutral height. The major deviation occurs near the top of the door where both models under predict the actual velocity. This higher experimental velocity apparently occurs because of the bending of the

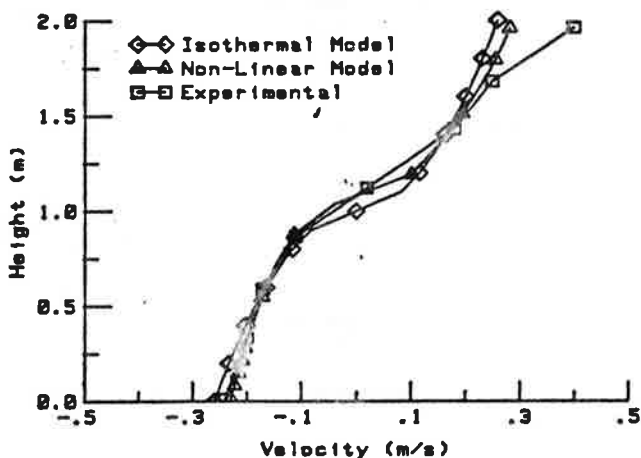


Fig. 7 Velocity Distributions One Hour After Start of Auxiliary Heated Test, October 3

streamlines around the top of the opening which is not accounted for in the models. The velocity profiles of both the heated and non-heated tests are very similar in shape.

In Figure 8, the experimental and theoretical mass flow rates predicted by the isothermal model are compared. The experimental mass flow rates were determined by summing the product of the density, velocity, and respective area at the eight measurement locations in the doorway. The average error in satisfying experimental continuity was 9% of the total flow through the door. The theoretical mass flow rates were determined from equation (2). The temperature difference in equation (2) was calculated using the temperature at one meter above the floor (doorway mid-height) of the north air thermocouple arrays. A linear least squares curve fit was used to determine the coefficient of discharge, C_d , from the experimental data, the values are shown in Table 2. The variation in the value of C_d for each test is attributed to the difficulty in measuring low air speed and inability to control the enclosure surface temperature. The overall discharge coefficient (i.e. the value obtained by using data

from all tests) for the isothermal model is 0.51.

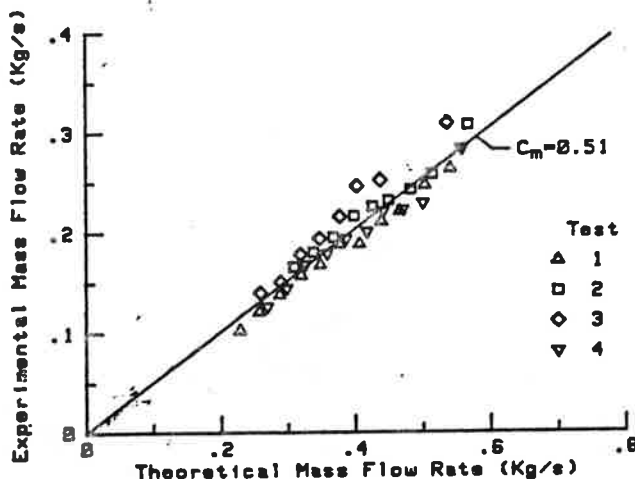


Fig. 8 Experimental and Predicted Isothermal Model Mass Flow Rates

Table 2. Values of C_m and C_d Determined from Experimental Data

Test	Isothermal model C_m	Non-linear model C_m	Isothermal model C_d	Non-linear model C_d
1 (heat off)	0.47	0.43	0.43	0.28
2 (heat on)	0.53	0.47	0.72	0.40
3 (heat on)	0.54	0.46	0.82	0.41
4 (heat on)	0.47	0.41	0.63	0.35
All tests	0.51	0.45	0.72*	0.39*

* only heated tests

In Figure 9, the experimental and theoretical

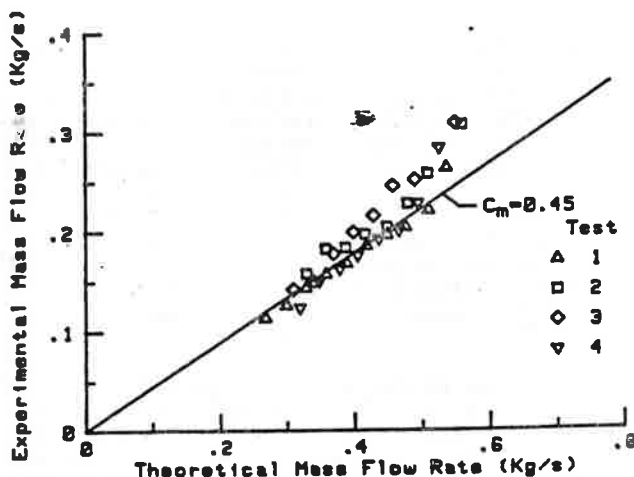


Fig. 9 Experimental and Predicted Non-linear Model Mass Flow Rates

mass flow rates predicted by the non-linear model are compared. The predicted values were determined by using the zone temperature distribution of the north thermocouple arrays in equations (9) through (12). The discharge coefficients shown in figure 9 for the non-linear model, were also determined by using a linear least squares curve fit. A comparison of figures 8 and 9 show that in general the discharge coefficients predicted by the non-linear model are about 10% lower than those predicted by the isothermal model.

A similar analysis was performed using both the north and south thermocouple arrays of the September 21st and October 3rd tests. When the south temperature distributions are used in the non-linear model, the discharge coefficient is only about 2% higher than when the north temperature distributions are used. However, when the south mid-height temperatures are used in the isothermal model, the discharge coefficients are about 5% higher than when the north mid-height temperatures were used; but the values predicted by the north temperature correlate better with experimental values.

During these two tests, the south thermocouple arrays were in the flow path and the north thermocouple arrays were in a more stagnate region. These results indicate that as long as temperature measurements in two zone configurations are made outside of the interzonal flow path (away from the doorway), the locations of the temperature measurements are not very critical. The non-linear model is less sensitive to temperature measurement location than the isothermal model.

In Figures 10 and 11, the experimental least

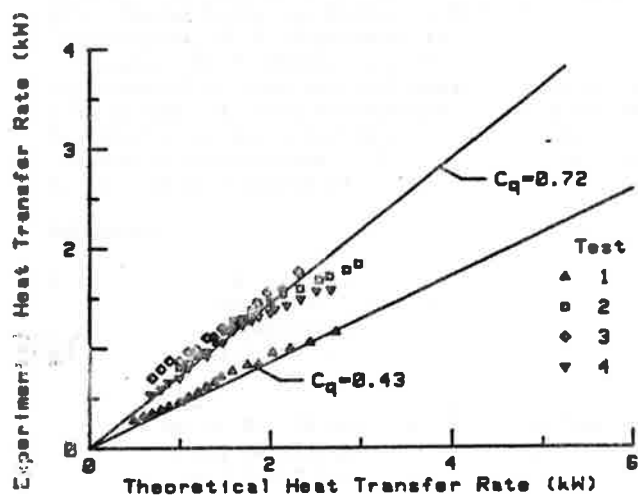


Fig. 10 Experimental and Predicted Isothermal Model Heat Transfer Rates

transfer rates are respectively compared with the values predicted by the isothermal and non-linear models. The experimental heat transfer rates were determined by summing the product of the density, velocity, temperature, specific heat, and

$$\int T_w V_w dz - \int T_c V_c dz$$

respective area of the eight measurement locations in the doorway. The predicted heat transfer rate was calculated with north zone temperature

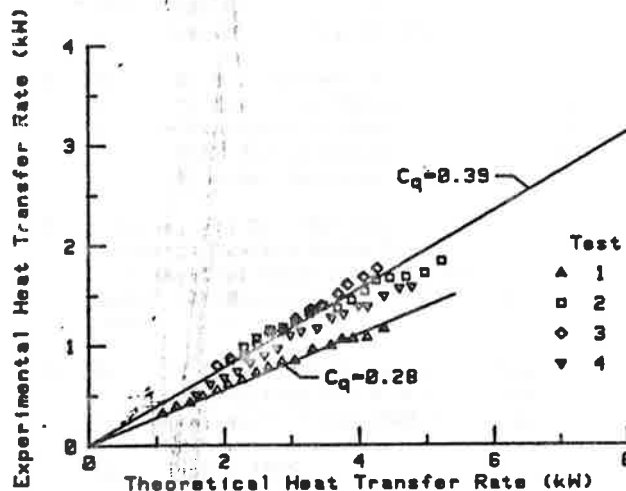


Fig. 11 Experimental and Predicted Non-linear Model Heat Transfer Rates

distributions in equation (9) and (13) for the isothermal model and non-linear model respectively. The heat transfer rate discharge coefficients, C_q , obtained by using a linear least squares curve fit are shown in Table 2.

These data show, the discharge coefficients, C_q , for the three tests with the auxiliary heaters on in cell #2 are higher than that for the test with no heat. The average discharge coefficient using north temperature distributions for the heated tests is 0.72 and 0.39 for the isothermal model and non-linear model respectively. When the south temperatures are used in the isothermal model, the discharge coefficient increased by 27% and 16% respectively for the September and October tests. However a similar comparison for the non-linear model, showed a 5% increase in the value of discharge coefficient for both of these tests.

The data of Table 2 shows that the status of auxiliary heat in cell #2 had little effect on C_m , while it has a significant effect on C_q . It was mentioned earlier while discussing figures 5 and 6, that the status of auxiliary heat in cell #2 effects the doorway and zone temperature distributions. Apparently, the addition of auxiliary heat produces a flow path that destratifies the zone temperature above the door in cell #2, causing air temperatures in the doorway above the neutral height to be higher than those during the non-heated test. These higher doorway temperatures result in the higher value of C_q for the tests with the heat on.

The effect of adding energy in cell #2 next to the wall that is opposite the doorway is similar to having south facing glazing being warmed by solar radiation. Previous research in interzonal tests conducted between sunspaces and north zones has shown that the temperature of the glazing is a

major factor in determining the sunspace temperature distribution and the interzonal flow rate and heat transfer [9]. The importance of this boundary condition on the heat transfer rate is very evident in the data. The doorway temperature profiles of the auxiliary heated tests are consistent with the temperature profiles in reference 5.

RESULTS AND CONCLUSIONS

The velocity distribution predicted by the isothermal and non-linear models are in good agreement with the data when experimentally determined discharge coefficients are used. The effect of heating the air near the wall opposite the doorway in the warm zone is that the zone flow paths are significantly altered. This change did not affect the predicted mass flow rates. However, the predicted heat transfer rate is very dependent on the condition of this surface. Linear least squares curve fits between predicted and experimental data resulted in lower discharge coefficients for the non-linear model than the isothermal model. The variation in the discharge coefficient between using north and south temperature distributions for predicting values is only slight for the non-linear model but significant for the isothermal model. Except for the variation of the discharge coefficient in the auxiliary heated test and non-heated test, the non-linear model correlates very well with the experimental data. The effects of zone surface conditions and zone configuration on the discharge coefficient needs further investigation.

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Le grand problème est de savoir où les températures sont mesurées. La mesure à un mètre de la porte est probablement pas suffisant.

En fait je ne crois pas à ces résultats. Le fait que C_d montre des variations plus fortes que C_m indique que les ΔT sont mal déterminés.