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Technical Note

Summary   The computation of interstitial condensation risk, in particular the rate of condensation, has increased in importance in assessing the acceptability of building details, especially those incorporating insulation or vapour-resisting materials. This Note details a model which incorporates effects currently omitted from the present methods; in particular, the effect of the presence of the saturated water film, the absorption of water by the materials and the latent heat flow resulting from the condensation or drying out process. It is shown that in some simple example cases these effects can considerably alter the conclusions that might be drawn concerning the acceptability or otherwise of a combination of materials and environmental conditions.

Interstitial condensation in building structures: Revised model for identifying problem material combinations

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List of symbols

L	Latent heat of vaporisation of water $(\text{J kg}^{-1}) = 2.5 \times 10^6 \, \text{J kg}^{-1}$
m	Mass of condensed or evaporated water (kg m^{-2})
t	Elapsed time (s)
λ_w	Thermal conductivity of water saturated material $(\text{W m}^{-1}\text{K}^{-1})$ (assumed to be $0.58 \, \text{W m}^{-1}\text{K}^{-1}$)
b	Condensed water film thickness (m)
T_s	Temperature of condensation face of water film (°C)
T_f	Temperature of cold face of water film (°C)
T_i	Internal temperature of building (°C)
T_o'	External temperature of building (°C)
p_i	Internal water vapour pressure (Pa)
p_s	Water vapour pressure (saturated) at water film interface S (Pa)
P_f	Water vapour pressure (saturated) at water film interface F (Pa)
P_o	External water vapour pressure (Pa)
P_{sat}	Saturated water vapour pressure (Pa)
r_i	Thermal resistance between an interface and the building interior $(\text{m}^2\text{K W}^{-1})$
r_o	Thermal resistance between an interface and the building exterior $(\text{m}^2\text{K W}^{-1})$
R_i	Water vapour resistance between an interface and the building interior (N s kg^{-1})
R_o	Water vapour resistance between an interface and the building exterior (N s kg^{-1})
S	$= L\rho\sigma/\lambda_w$
ρ	Water film density (kg m^{-3}) (assumed to be $1000 \, \text{kg m}^{-3}$)
a	Physical constant $= L/R$ (kg K)
c	Constant of integration in Clausius–Clapeyron equation
θ	Absolute temperature (K)
σ	Material porosity fraction
μ	Diffusion resistance factor = vapour permeability ratio $\delta_{\text{air}}/\delta_{\text{material}}$
A, B, C, E, F, G	Grouped constants for any interface defined in the text at equations 10–15
$\Gamma, \Phi, \psi, \Sigma$	Grouped constants defined in terms of A, B, C, E, F, G in the text at equations 18–21

1 Introduction

Over a number of years a variety of methods have developed for estimating the likelihood and extent of interstitial condensation in building details[1, 2]; however, they all rely to some degree on certain assumptions and in some cases can produce inconsistent results, for instance by violating the conservation of heat flow in the steady state. The result of this is that sometimes problems may arise where in theory they should not, and conversely problems do not arise where in theory they could. Generally the severity of condensation problems is assessed by the rate of deposition of water. The fact that a region may under some circumstances be below the local dew point may be of little significance; it is the water deposition rate that is of major concern, and any model should be capable of assessing this parameter accurately. The current version of *BS 5250*[2] utilises the concept of condensation occurring at a single dew-point temperature of an interface, although there is no thermodynamic reason why any condensation should occur when a vapour is in contact with its liquid at the dew point. That is, we have no reason to expect a vapour to condense at the dew point or conversely to expect a liquid to evaporate at dew-point temperatures, since the dew point is by definition the temperature at which they are in equilibrium. A temperature difference across the liquid is required for condensation to continue, and this difference, although small, is a determining factor in the rate of condensation. This Note outlines a simple theoretical model which includes this point and derives the condensation rates and condensation criterion necessary for deposition at any interface. The model also ensures that the latent heat component of the heat released in condensation is

included in the heat flow balance equations and is thus consistent with the established classical model of condensation in heat exchangers[3, 4]. To clarify this point this Note derives the condensation rate at the interface in a building structure such that all the rate limiting steps are satisfied.

2 Condensation model

To keep the description as simple as possible and to emphasise the physical elements, we consider the simplest possible situation with a one-dimensional assembly of materials. We have a region of vapour flow up to an interface of materials where condensation is postulated to be occurring, a region of material saturated with liquid water, assumed to be thin relative to the thickness of the structure layers, and a region where the vapour is assumed to be flowing to the external environment[5]. This is shown in Figure 1.

Consider the situation at an instant of time t when the liquid condensation layer has grown to a thickness b, taken as small relative to the building structure thickness. The precise position of the liquid layer relative to the interface is dependent on the capillary effects and porosity of the materials on each side of the interface. Subject to some simple assumptions this can readily be taken into account. In some materials the film may not be able to grow beyond a certain thickness before flowing under gravity; also, a uniform film may not always form but occur in droplets or patches. In these cases the theory given here would predict an upper limit to the possible rates[6, 7]. Thus there is a flow of vapour up to the liquid layer facing the higher pressure side. This vapour condenses and releases its latent heat, which is transferred across the film. On the low-pressure side the liquid evaporates, taking up latent heat from the film. Applying the requirements of heat and mass balance at the respective interfaces the following set of equations may be derived. For thermal equilibrium at surface S:

$$L\frac{dm_s}{dt} - \frac{\lambda_w}{b}(T_s - T_f) + \frac{T_i - T_s}{r_i} = 0 \qquad (1)$$

For thermal equilibrium at surface F:

$$L\frac{dm_f}{dt} - \frac{\lambda_w}{b}(T_s - T_f) + \frac{T_f - T_o}{r_o} = 0 \qquad (2)$$

Also the net condensation rate and film thickness rate of increase are related by:

$$\frac{dm}{dt} = \rho\sigma\frac{db}{dt} = \frac{dm_s}{dt} - \frac{dm_f}{dt} \qquad (3)$$

If equation 2 is subtracted from equation 1 and using equation 3 then we have the simplified form of equation 1 where the grouped parameters are as defined below (equations 10–15):

$$AT_s + BT_f = \frac{db}{dt} + C \qquad (4)$$

Also the mass flow into and out of the condensation film gives:

$$\frac{P_i - P_s}{R_i} - \rho\sigma\frac{db}{dt} - \frac{P_f - P_o}{R_o} = 0 \qquad (5)$$

where the parameter σ is to allow for the water absorption porosity of the material at the position where the

liquid film is forming. It will be shown below that this parameter plays a part in determining the long-term rate of water deposition in condensation zones. Although it is not accounted for at all in the conventional procedure, practical experience shows that condensation is more difficult when porous materials are involved and *vice versa*.

In general the saturation vapour pressure and absolute temperature can also be related by integrating the Clausius–Clapeyron equation for an ideal gas to give[8]:

$$\log(p_{sat}) = -\frac{a}{\theta} + c \qquad (6)$$

However, if the temperature range is very small, which is the case here as we are only considering the temperature difference across the liquid film, which in general will only amount to fractions of a degree, we may satisfactorily approximate a linear relation between the saturation vapour pressure and Celsius temperature as:

$$p_{sat} = \alpha T + \beta \qquad (7)$$

where the parameters α and β are chosen to suit the temperature band under consideration. For example between $0°C$ and $3°C$ $\alpha = 488\,Pa\,K^{-1}$ and $\beta = 6100\,Pa$ and this has an error of less than 0.6% over this range. Thus rearranging, using equation 7 in equation 5 gives:

$$ET_s + FT_f = -\frac{db}{dt} + G \qquad (8)$$

Solving equation 4 and equation 8 for T_s and T_f gives the following expressions. We note that these temperatures change slowly with time as the liquid layer grows in thickness.

$$T_s = \frac{GB - FC}{EB - AF} - \frac{db}{dt}\frac{F + B}{EB - AF} \qquad (9)$$

$$T_f = \frac{GA - EC}{AF - WB} - \frac{db}{dt}\frac{E + A}{AF - EB} \qquad (10)$$

Where the grouped values defined for any interface A, B, C, G, E, F are calculated from:

$$A = \frac{1}{r_i L\rho\sigma} \qquad (11)$$

$$B = \frac{1}{r_o L\rho\sigma} \qquad (12)$$

$$C = \frac{T_o}{r_o L\rho\sigma} + \frac{T_i}{r_i L\rho\sigma} \qquad (13)$$

$$E = \frac{\alpha}{R_i \rho\sigma} \qquad (14)$$

$$F = \frac{\alpha}{R_o \rho\sigma} \qquad (15)$$

$$G = \frac{P_i - \beta}{R_i \rho\sigma} + \frac{P_o - \beta}{R_o \rho\sigma} \qquad (16)$$

If we substitute the expressions 9–10 for T_s and T_f into equation 4 then after rearranging we may derive the following expression relating b to the condensation film thickness and time:

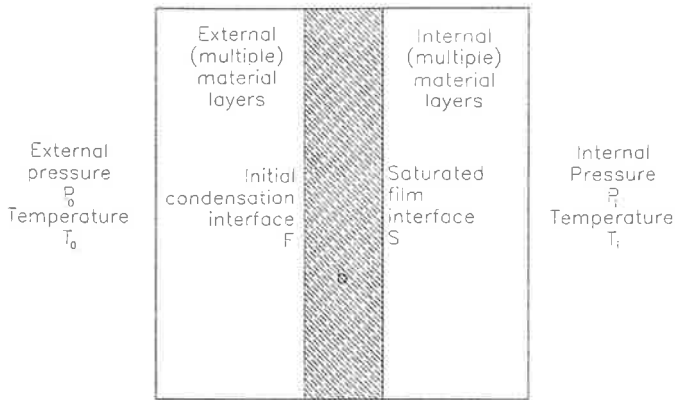$$t = \int \frac{\Gamma b + \Phi}{\Sigma b + \psi}\,db \qquad (17)$$

Figure 1 Condensation layer

Integrating this expresssion gives:

$$t = \left(\frac{\Phi}{\Sigma} - \frac{\Gamma\psi}{\Sigma^2} \right) \log_e \left(\frac{\psi + \Sigma b}{\psi} \right) + \frac{\Gamma b}{\Sigma} \qquad (18)$$

Again to simplify the appearance of the expressions we define the following grouped constants for the interface:

$$\Gamma = S \frac{(E+A)(F+B)}{EB - AF} \qquad (19)$$

$$\Sigma = S \left(\frac{(GB - FC)(E+A)}{EB - AF} - \frac{(P_i - \beta)}{\alpha} E - T_i A \right) \qquad (20)$$

$$\Phi = \frac{(A+B+E+F)}{EB - AF} \qquad (21)$$

$$\psi = \frac{G(A+B) - C(E+F)}{EB - AF} \qquad (22)$$

From this we see that the condensation rate and liquid film growth rate are given by the difference between a linear and a logarithmic form, exhibiting the characteristics of a decreasing condensation effect, if condensation conditions persist for long enough, that is if $\psi/\Sigma \approx b$. Also from equation 18 it may be seen that the film has a limiting thickness which it will take an infinite time to attain. Although in many cases this will be of no practical significance it occurs when:

$$b = -\frac{\psi}{\Sigma} \qquad (23)$$

This requirement can only be valid when Σ is less than zero. If $\Sigma b/\psi \ll 1$ which is the case when condensation begins and b is very small then we may further simplify the expression, by expanding the logarithmic term to the form:

$$\log_e \left(\frac{\psi + \Sigma b}{\psi} \right) \approx \frac{\Sigma b}{\psi} \qquad (24)$$

and obtain the following expression for the film thickness at any time t:

$$b = \frac{\psi}{\Phi} t \qquad (25)$$

Thus from equations 3 and 25 the condensation rate becomes:

$$\frac{dm}{dt} = \rho\sigma \frac{G(A+B) - C(E+F)}{A+B+E+F} \qquad (26)$$

The expression 26 for the initial condensation rate thus has a particularly simple form for any interface.

These relations are different in form and value from those derived by the conventional procedure. Examining these relations we may derive the conditions to be satisfied for condensation to be possible. Since we require the expression for b to satisfy $b > 0$, then as the denominator in equation 24 will always be positive, the requirement becomes that the numerator should also be positive, which is only possible if:

$$G(A+B) > C(F+E) \qquad (27)$$

Expansion of inequality 27 gives:

$$\frac{(P_i - \beta)R_o + (P_o - \beta)R_i}{R_i + R_o} > \alpha \frac{T_i r_o + T_o r_i}{r_i + r_o} \qquad (28)$$

which is equivalent to requiring the temperature at the condensation interface to be at or below the dew-point temperature as in the conventional procedure. It may be seen that the same equations and the given solutions may also be applied to the drying-out case, that is when b is decreasing in magnitude and the rate of growth of b is negative. In that case the condensation rate becomes the drying-out rate.

The procedure in evaluating a structure according to this model is firstly to compute the individual elements' thermal and vapour resistances and then to sum them for each interface and apply the criterion 28. Any interface satisfying criterion 28 can then be examined further as a potential condensation interface with the rate computed initially from equation 26, or if conditions are to persist for long periods of time equation 18. The latter equation would have to be solved numerically to find an explicit value for b for substitution into the following relation combining equations 3 and 17 to give for the rate at time t:

$$\frac{dm}{dt} = \rho\sigma \frac{\Sigma b + \psi}{\Gamma b + \phi} \qquad (29)$$

More simply the time to reach any particular water film thickness, say 1, 2, 3 mm etc. may be computed directly from equation 25 or for longer times equation 18, and then the condensation rate may be computed directly from equation 26 or 29 depending on the timescale, or the following alternate form of equation 26 derived from equations 3 and 25 may be used:

$$\frac{dm}{dt} = \rho\sigma \frac{\psi}{\Phi} \qquad (30)$$

It may be noted that when the interface is at the dew point then the initial rate will be zero, as with the predictions of the conventional calculation. Examination of equation 29 and the definitions of the grouped material constants Σ, Γ, ϕ and ψ in terms of A, B, C, E, F, G and the external and internal conditions shows that there is a symmetry between the thermal properties (scaled by the latent heat of vaporisation L) and vapour transport properties (scaled by $1/\alpha$ defined in equation 7) for a structure determining the possible condensation rate. It may be verified that for a great many building materials the values of the constants A, B and E, F are of comparable magnitude. This theory is demonstrated in some cases as follows.

Table 1 Parameters for Example 1

Material no.	Thermal conductivity (W m⁻¹ K⁻¹)	Thickness (m)	Diffusion resistance factor μ	Water absorption porosity
1 (Inside)	0.16	0.012	10	0.4
2	0.18	0.105	20	0.3
3	0.026	0.05	2000	0.9
4 (Outside)	1.2	0.012	100	0.2

Environmental conditions	Location	
	Internal	External
Temperature (°C)	24	0
Relative humidity (%)	70	70

Table 2 Parameters for Example 2

Material no.	Thermal conductivity (W m⁻¹ K⁻¹)	Thickness (m)	Diffusion resistance factor μ	Water absorption porosity
1 (Inside)	0.16	0.012	10	0.4
2	0.15	0.15	1	0.3
3 (Outside)	0.13	0.1	10	0.2

Environmental conditions	Location	
	Internal	External
Temperature (°C)	24	−5
Relative humidity (%)	70	70

3 Example 1: Low-permeability calculation

Consider the assembly of materials described by the parameters given in Table 1. The values are those commonly found in building materials and plastic foamed and fibrous insulations. The water absorption porosity is identified with the simple pore volume ratio of the material, although for strongly hydrophobic or hydrophilic materials this assumption may be seriously wrong. We see that condensation is possible at the interface between materials 3 and 4. Further the initial rate is calculated as $0.3185 \times 10^{-7}\,\mathrm{kg\,m^{-2}\,s^{-1}}$ and this rate will remain steady for a considerable period of time, as $\Sigma b \ll \psi$ and $\Gamma b \ll \phi$ for the range of possible values of b. In these circumstances this structure is thus relatively safe for short-term exposure but decidedly not suitable for long-term exposure to the postulated conditions, for during a winter period of these conditions around $300\,\mathrm{g}$ of water would collect in each square metre and the saturated film would grow to a thickness of several mm. The algorithms could easily be computerised.

The same problem can be examined using the standard *BS 5250* procedure. The result $(0.718 \times 10^{-9}\,\mathrm{kg\,m^{-2}\,s^{-1}})$ is markedly different and the conclusions on the suitability of the assembly for the postulated conditions are consequently different. The British Standard[6] procedure gives a rate an order of magnitude less than the procedure defined here. This result is not by any means general; in many cases the two procedures will give very similar results. In general where the rate is determined principally by mass diffusion processes the procedures will give similar results. Where, as here, heat and mass transfer are both important the results will often be very different.

4 Example 2: High-permeability calculation

As a further example consider now the assembly described by Table 2. Condensation is possible at the interface between materials 2 and 3. The rate is $0.3160 \times 10^{-5}\,\mathrm{kg\,m^{-2}\,s^{-1}}$ which is several orders of magnitude higher than that for the previous example. During a winter season enough water may collect to cause significant damage. In this case the British Standard procedure, although giving a rate around a quarter that computed by the current model, produces a closer result than in the previous example. The difference between the results of the methods may be eliminated or even reversed by examining different forms of construc-

tion. 'Dry-out' rates will also show similar differences to those demonstrated by these two examples.

5 Conclusion

A model incorporating heat diffusion effects and water accumulation effects can lead to condensation and drying-out rates considerably at variance with the conventional estimation procedure. The use of the equations derived in this Note may throw more light on some condensation situations in building details where the conventional procedure leads to misleading conclusions, particularly where materials of low porosity or hydrophobic properties are involved[6]. Where condensation conditions are just beginning then the rate computed from this model may be substantially different from that computed by the conventional procedure. This method of calculation would inevitably require more information on the water absorption properties of building materials, as this factor can affect the results obtained by this method. However, these properties are not currently included in the present commonly employed procedures. Information on the thermal conductivity of saturated materials is also needed.

References

1 Moisture transfer and condensation *CIBSE Guide* Section A10 (London: Chartered Institution of Building Services Engineers (1986)

2 *BS 5250: 1989* Code of practice for control of condensation in buildings (London: British Standards Institution) (1989)

3 Rogers G F C and Mayhew Y R *Engineering Thermodynamics, Work and Heat Transfer* pp 557–567 (London: Longman) (1969)

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5 Luikov A K *Heat and mass transfer in capillary porous bodies* (Oxford: Pergamon) (1966)

6 Croome D J and Sherrat A F C *Condensation in Buildings* (New York: Applied Science) (1972)

7 Sieffert K *Damp Diffusion and Buildings* (Amsterdam: Elsevier) (1970)

8 Glasstone S and Lewis D *Elements of Physical Chemistry* pp 133–135 (London: Macmillan) (1966)

Corrigenda and errata

Room heat needs in relation to comfort temperature: Example using the rad–air model[1]

In the above Technical Note by M G Davies the expression for Q_t in section 3.1 (p110) should read:

$$Q_t = \frac{56.56(T_c - T_o)}{1 + 0.034p}$$

and that for $T_{av} - T_{rs}$ should read:

$$T_{av} - T_{rs} = \frac{0.20 - 0.47p}{1 + 0.034p}(T_c - T_o)$$

Reference

1   Davies M G Room heat needs in relation to comfort temperature: Example using the rad–air model *Building Serv. Eng. Res. Technol.* 12(3) 107–110 (1991)

Air infiltration through background cracks due to temperature differences[2]

Dr L Shao wishes the following text to be appended to the above paper: 'The research work presented in this paper was sponsored by the Science and Engineering Research Council (Grant no. GR/F28397), whose support is gratefully acknowledged. Dr L Shao is with the Building Science Unit, University of Sheffield.'

Reference

2   Shao L and Howarth A T Air infiltration through background cracks due to temperature difference *Building Serv. Eng. Res. Technol.* 13(1) 25–30 (1992)

Thermal stratification in intermittently heated heavyweight buildings (Churches)[3]

Figures 10(a) and 11(a) have inadvertently been transposed. On p125, column 1, line 12, the reference to Table 7 should be to Table 11. On p128, column 2, line 2, the reference to Table 9 should be to Table 13. The caption to Table 4 (p122) should include the period of analysis: 10 November 1989 to 13 May 1990. On p123, column 2, line 2, 'units' should read 'inputs'. In the caption to Figure 5 (p125) the word 'difference' should be omitted.

Reference

3   Bemrose C R and Smith I E Thermal stratification in intermittently heated heavyweight buildings (Churches) *Building Serv. Eng. Res. Technol.* 13(3) 119–131 (1992)