EXHAUST EFFECTIVENESS BASED ON RESIDUAL LIFETIME OF CONTAMINANT IN A VENTILATED SPACE

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Abstract

Exhaust effectiveness indicates how effectively contaminated air can be removed from a space, whereas air change effectiveness indicates how effectively distribute fresh air into the space. It is intended to describe the exhaust effectiveness based on the residual-life-time of contaminant in the context of logical extension of supply effectiveness based on LMA. It is proved theoretically that the room-mean values of LMA and LMR are identical, even though their local distributions are different from each other depending on inlet/outlet configurations. It is necessary to define ventilation effectiveness clearly so as to distinguish supply and exhaust effectiveness in describing local distributions in a room. Overall ventilation effectiveness, however, need not be distinguished, since both of the room averages of supply and exhaust indices are identical. They are dependent on the airflow pattern only, but not on the contaminant distribution in the space.

Keywords: Ventilation Effectiveness, Supply Effectiveness, Exhaust Effectiveness, Local Mean Age(LMA), Local Mean Residual-life-time(LMR), Tracer Gas experiment

Nomenclature

α_p : Local supply index
ε_p : Local exhaust index
$<\alpha>$: Room supply effectiveness
$<\varepsilon>$: Room exhaust effectiveness
C : Concentration of tracer gas
C_{ex}^{p} : Concentration at exhaust with a tracer source at P
C_{ex}^{sup} : Concentration at exhaust with a tracer source at supply outlet
C_{p}^{sup} : Concentration at P with a tracer source at supply outlet
$\bar{C}_p$: Concentration at P with a uniformly distributed tracer source in the space
$<C^p>$: Room average concentration with a tracer source at P

θ_p : Local mean age at P
φ_p : Local mean residual-life-time at P
$< \theta >$: Room average of LMA
$< \varphi >$: Room average of LMR
$\dot{M}$: Contaminant generation rate
$\dot{m}$: Contaminant generation rate per unit volume
Q : Volumetric airflow rate
t : Time
τ_n : Nominal time constant (=V/Q)
V : Room volume

Introduction

Ventilation effectiveness has been defined in various ways by many investigators. The term, "ventilation efficiency" was first used by Yaglou and Witheridge [1]. They defined it as the ratio of the carbon dioxide concentration in a room to that in the extract duct. The ventilation was considered to be effective when the exhaust air contains a large amount of the contaminant before it spreads out into the room. This definition has been a corner-stone for various definitions of ventilation efficiencies since.

Meanwhile, mathematical concepts of age and residence time were introduced to investigate the mixing characteristics in reactors by chemical engineers such as Danckwerts [2] and Spalding [3]. They mentioned the similarity between mixing of gases in reactors and the mixing of air in ventilated rooms. It was Sandberg [4] who first applied the concept of age of air to ventilation studies. He summarized various definitions of ventilation efficiency including relative efficiency, absolute efficiency, steady state efficiency, transient efficiency, etc. The sooner the supply air reaches a point in the room, the better the air change effectiveness at that point. Nowadays, the concept has been widely accepted by my organizations throughout the world including ASHRAE and AIVC [5-6].

In the ASHRAE Handbook [5], it is stated that ventilation effectiveness is a description of an air distribution system's ability to remove internally generated pollutants from a building, zone, or space. Whereas air change effectiveness is a description of system's ability to deliver ventilation air to a building, zone, and space. Namely, it is possible to understand the ventilation effectiveness indicates exhaust effectiveness and the air change effectiveness supply effectiveness. There are definitions of air change effectiveness based on the concept of age of air, but no definitions for ventilation effectiveness in the standard.

It is necessary to define clearly what to mean and how to obtain ventilation effectiveness. It is the objective of the present paper to provide one to one analogy of exhaust effectiveness with supply effectiveness using the concept of the residual life time of contaminant generated in a space. The meanings of local and overall values of exhaust effectiveness need be understood appropriately in conjunction with the previous definitions of ventilation effectiveness.

Definitions of ventilation effectiveness

Age and residual-life-time

The age of air is the length of time required for the supply air to reach an internal point of interest. As air can reach the point through various paths, the mean value of the ages at the point is called local mean age (LMA). Likewise, the length of time required for the air or contaminant located at an internal point to reach the exhaust is called the residual-life-time. The mean value through various paths is local mean residual-life-time (LMR). Local mean age represents the freshness of supply air, so that it can be used as local supply index at the point. Local mean residual-life-time represents the quickness of removal of the contaminant generated at the point, and can be used to represent local exhaust index. The LMA and LMR respectively represent the local supply and exhaust effectiveness at the point of the room. It should be stressed they depend on the room airflow pattern only but should not be dependent on the source distribution of a contaminant in the space, unless the contaminant concentration alters the airflow characteristics of the room.

Supply and exhaust effectiveness

A complete mixing condition is considered to be a reference condition on which we can define ventilation effectiveness. The nominal air change rate is the volumetric airflow rate divided by the room volume. The inverse of the air change rate is the nominal time constant.

$$\tau_n = V/Q \qquad \text{(Eq. 1)}$$

The local supply and exhaust indices are defined as the ratios of local mean age and the local mean residual-life-time compared to the nominal time constant, respectively. These local indices can be over 100% and as large as infinity.

$$\alpha_p = \tau_n/\theta_p \qquad \text{(Eq. 2)}$$

$$\varepsilon_p = \tau_n/\varphi_p \qquad \text{(Eq. 3)}$$

where the subscript P is the location of interest. Notice that the LMA at the exhaust means the total residence time of supply air in the space, and it is the same with the LMR at the supply. It can be noticed that these values are the same with the nominal time constant.

$$\theta_{ex} = \varphi_{sup} = \tau_n \qquad \text{(Eq. 4)}$$

Therefore, the local supply index can be understood as the ratio of the LMA at the exhaust to that at the point, and the local exhaust index as the ratio of the LMR at the supply to that at the point. The overall room effectiveness can also be defined similarly.

$$<\alpha> = \tau_n/<\theta> = \theta_{ex}/<\theta> \qquad \text{(Eq. 5)}$$

$$<\varepsilon> = \tau_n/<\varphi> = \varphi_{sup}/<\varphi> \qquad \text{(Eq. 6)}$$

where < > means the spatial average over the entire space. It will be proved later in this article that the room averages of LMA and LMR are identical. Therefore, the overall room supply effectiveness and exhaust effectiveness should be the same. We may not need terminology to distinguish between the overall values. We may simply call it overall ventilation effectiveness. Supply effectiveness and exhaust effectiveness are meaningful only for local values. The corresponding concepts are summarized in Table 1.

Table 1 Corresponding concepts for supply and exhaust effectiveness

Supply Effectiveness	Exhaust Effectiveness
Age of Air θ_p = Local Mean Age at P $<\theta>$= Room Average of LMA	Residual Life Time of Air φ_p = Local Mean Residual-life-time at P $<\varphi>$= Room Average of LMR
Local Supply Index $\alpha_p = \tau_n/\theta_p$ $= \theta_{ex}/\theta_p$ = LMA at exhaust/LMA at P	Local Exhaust Index $\varepsilon_p = \tau_n/\varphi_p$ $= \varphi_{sup}/\varphi_p$ = LMR at supply/LMR at P
Overall Room Supply Effectiveness $<\alpha>= \tau_n/<\theta>$	Overall Room Exhaust Effectiveness $<\varepsilon>= \tau_n/<\varphi>$

Measurement methods

Local supply index

In order to obtain LMA or LMR numerically or experimentally, transient concentration responses should be monitored after a tracer gas is injected. There are three commonly used ways of injecting a tracer gas, i.e. step-up, step-down, and pulse methods. The equations to calculate these values from concentration responses are different from one injection method to another, but the results should be the same. Here we will use a step-up method for most of the discussion.

The local mean age can be obtained by integrating the area above the concentration curve divided by the steady state concentration after a step-up tracer injection as follows:

$$\theta_p = \int_0^\infty \left(1 - \frac{C_P{}^{\sup}(t)}{C_P{}^{\sup}(\infty)} \right) dt \qquad \text{(Eq. 7)}$$

where the subscripts indicate the monitoring point, and the superscripts indicate the injection point. Note the steady state concentration, $C_P^{sup}(\infty)$, is uniform regardless of the location P, since a tracer is injected at a supply.

Another method to obtain the LMA distribution is to calculate the steady concentration distribution with uniformly distributed constant volumetric sources in the space. The equivalency of these methods has been shown numerically by Han [9]

$$\theta_p = \frac{\overline{C}_P(\infty)}{\dot{m}} \qquad \text{(Eq. 8)}$$

where $\dot{m}$ is the tracer generation rate per unit volume. In the equations above, C does not have a superscript but an over-bar, which means a uniform tracer injection throughout the entire space. The LMA at exhaust can be expressed similarly by replacing the subscript 'P' with 'ex'. Therefore, the local supply index, which is the ratio of the LMA's at exhaust and at P is calculated by the ratio of the steady concentrations with over-bars at those points. This method is especially useful for numerical calculations, since the distribution of LMA can be obtained from a single steady state concentration calculation rather than from lengthy transient calculations.

$$\alpha_p = \frac{\overline{C}_{ex}(\infty)}{\overline{C}_p(\infty)} \qquad \text{(Eq. 9)}$$

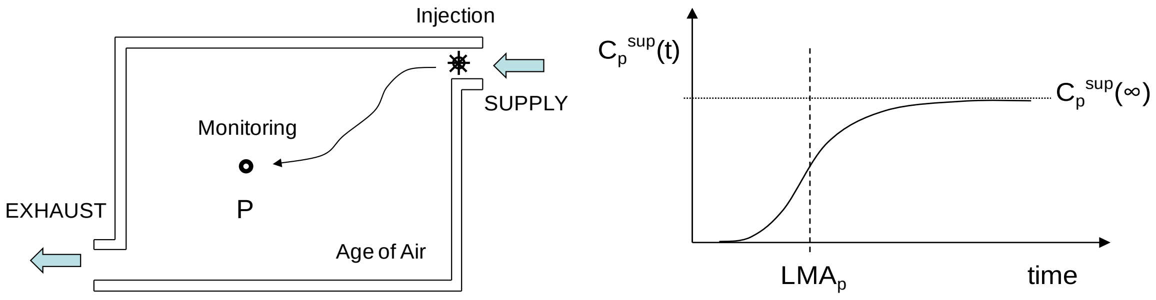

Fig. 1 Transient step-up method used to determine local mean age

Local exhaust index

Similarly, the local mean residual-life-time can be obtained from its definition as is shown in Fig. 2. The tracer should be injected at P and the transient concentration should be monitored at the exhaust.

$$\phi_p = \int_0^{\infty} \left(1 - \frac{C_{ex}{}^{P}(t)}{C_{ex}{}^{P}(\infty)} \right) dt \qquad \text{(Eq. 10)}$$

where the subscripts indicate the monitoring point and the superscripts indicate the injection point. The steady state concentration at the exhaust $C_{ex}^{p}(\infty)$ should be equal to $\dot{M}/Q$, as the mass should be balanced in the room. The equation can be derived as follows.

$$\phi_p = \frac{1}{\dot{M}} \int_0^{\infty} \dot{M} - QC_{ex}^{p}(t)\,dt = \frac{< C^{p}(\infty) > V}{\dot{M}} = \frac{< C^{p}(\infty) > \tau_n}{C_{ex}^{p}(\infty)} \qquad \text{(Eq. 11)}$$

where $\dot{M}$ is the contaminant generation rate at P. The first term in the integral is the total generation rate and the second term is the rate of contaminant leaving the room through the extract duct. The sum of the difference up to a steady state results in the amount of contaminant left in the room, which is called as the internal hold-up. It is the product of the average concentration of the room times the room volume. [4]

Therefore, the local exhaust index can be obtained either from the definition of the LMR ratio or by the ratio of the room average concentration to the exhaust concentration when a source is located at P.

$$\varepsilon_p = \frac{C_{ex}{}^{p}(\infty)}{< C^{p}(\infty) >} \qquad \text{(Eq. 12)}$$

It looks quite similar with the classical definition of ventilation efficiency introduced by Yaglou and Witheridge [1]. They defined ventilation efficiency also as the concentration ratio, but for a given contaminant source, and considered it as an overall efficiency of the room, not as a local efficiency at the source location. It should be stressed that the ratio shown in the equation is not the room exhaust index but the local exhaust index in case a source is located at P. A similar definition has been also proposed as the removal effectiveness by several authors [7-8]. Although there have been some studies on the measurements of LMA [10], the distributions of LMR have rarely been measured experimentally [11].

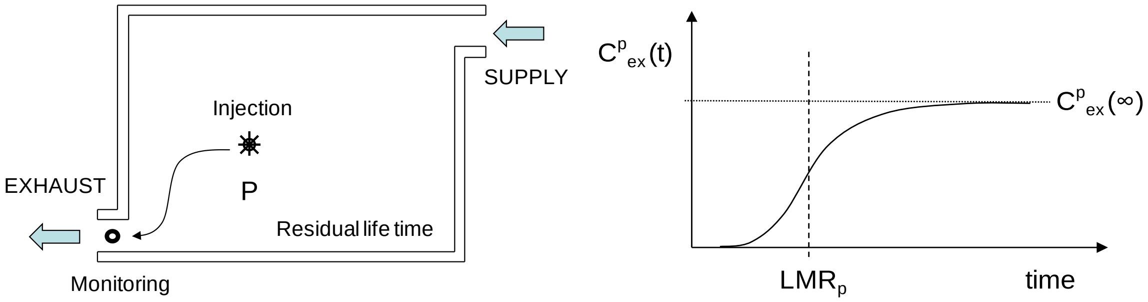

Fig. 2 Transient step-up method used to determine local mean residual-life-time.

Overall room ventilation effectiveness

The room average effectiveness should be the spatial average of local values over the entire space. The spatial average of LMA can be obtained in two different ways as was mentioned earlier; transient and steady ways. The steady method indicates it can be obtained from the spatial average of the steady state concentration under the steady condition with uniform generation from the Eq. 9.

$$< \theta > = \frac{< \overline{C}(\infty) >}{\dot{m}} \qquad \text{(Eq. 13)}$$

Therefore, the overall supply effectiveness is the ratio of the exhaust concentration to the room mean concentration as is shown in Table 2.

In order to obtain overall exhaust effectiveness, LMR should be obtained at every internal point to calculate its spatial average over the entire space. Unlike the method used for the previous age measurements, however, a monitoring point should be fixed at the exhaust, and a tracer should be injected at every point in the space repeatedly. The concentration response by simultaneous tracer injections can be obtained by superimposing every injection source present over the entire space, since the concentration equation is linear. Therefore, the room average exhaust effectiveness is the ratio of the exhaust concentration to the room average concentration with a uniformly distributed source superimposed in the space, which is exactly the same as the steady method used to determine overall supply effectiveness.

This concludes the proof that the overall supply effectiveness is equal to the overall exhaust effectiveness. Table 2 summarizes the methods that can be used to obtain local supply and exhaust indices and overall room effectiveness.

$$<\varepsilon>=<\alpha> \qquad \text{(Eq. 14)}$$

Table 2 Methods to obtain local supply and exhaust indices and overall effectiveness

	Supply Effectiveness	Exhaust Effectiveness
Local index	Definition: $\alpha_p = \frac{\tau_n}{\theta_P}$	Definition: $\varepsilon_p = \frac{\tau_n}{\phi_P}$
	Method 1. By step-up generation at supply - Transient concentration at P - Steady concentration at P $\alpha_p = \frac{\tau_n}{\int_0^\infty \left(1 - \frac{C_P{}^{\sup}(t)}{C_P{}^{\sup}(\infty)}\right) dt}$ where $\tau_n = V/Q$ Method 2. By uniform generation in space - Steady concentration at P - Steady concentration at exhaust $\alpha_p = \frac{\overline{C}_{ex}(\infty)}{\overline{C}_P(\infty)}$	Method 1. By step-up generation at P - Transient concentration at exhaust - Steady concentration at exhaust $\varepsilon_p = \frac{\tau_n}{\int_0^\infty \left(1 - \frac{C_{ex}{}^P(t)}{C_{ex}{}^P(\infty)}\right) dt}$ where $\tau_n = V/Q$ Method 2. By step-up generation at P - Steady concentration at exhaust - Steady room average concentration $\varepsilon_p = \frac{C_{ex}{}^P(\infty)}{< C^P(\infty) >}$
	Definition: $< \alpha >= \frac{\tau_n}{< \theta >}$	Definition: $< \varepsilon >= \frac{\tau_n}{< \phi >}$

Overall effectiveness	Method 1. By step-up generation at supply - Transient concentration at exhaust - Steady concentration at exhaust $< \alpha >= \frac{\tau_n{}^2}{\int_0^\infty t\left(1-\frac{C_{ex}{}^{\sup}(t)}{C_{ex}{}^{\sup}(\infty)}\right)dt}$ Method 2. By uniform generation in space - Steady room average concentration - Steady concentration at exhaust $< \alpha >= \frac{\overline{C}_{ex}(\infty)}{< \overline{C}(\infty) >}$	Method 1. By simultaneous step-up generations at every P in space - Steady concentration at exhaust - Steady room average concentration $< \varepsilon >= \frac{\overline{C}_{ex}(\infty)}{< \overline{C}(\infty) >}$ Therefore, $< \varepsilon > = < \alpha >$

Conclusions

1. Ventilation effectiveness can be categorized into two concepts; supply effectiveness and exhaust effectiveness. Supply effectiveness represents the performance of supply air distribution in a space, whereas exhaust effectiveness represents the performance of contaminant removal from the space.

2. As local supply index is defined based on local mean age, local exhaust index can be defined as the local mean residual-life-time to the nominal time constant. It represents how the contaminant generated at the point can be removed effectively. It can be obtained from transient concentration measurements at the exhaust after injecting a tracer gas at the point of interest. This is equivalent to the ratio of the concentration at the exhaust to the room mean concentration. The distribution of local exhaust index is different from that of local supply index, but closely related to airflow patterns in the space.

3. Unlike local indices, the room average exhaust effectiveness is identical to the room average supply effectiveness, which can be called as ventilation effectiveness. It depends not only on supply-exhaust configurations, but also on air change rate. Experimentally, it can be obtained by taking the 0th and the first moments of the concentration curve at the exhaust after injecting a tracer at the supply. Numerically, it is more convenient to take the ratio of the steady state exhaust concentration to the room average concentration with a uniformly distributed source in the space.

Acknowledgments

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