MEASUREMENT OF INFILTRATION RATES FROM DAILY CYCLE OF AMBIENT CO_2

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ABSTRACT

We propose a new approach for measuring air infiltration rates in buildings. The method belongs to the class of tracer gas techniques but, unlike conventional CO_2 based methods that assume the outdoor ambient CO_2 concentration is constant, the proposed method recognizes that photosynthesis and respiration cycle of plants and processes associated with fuel combustion produce daily, quasi-periodic, variations in the ambient CO_2 concentrations. These daily variations, which are within the detection range of existing monitoring equipment, are utilized for estimating ventilation rates without the need of a source of CO_2 in the building. The new method has the advantages that no tracer gas injection is needed, and time resolved results are easily obtained.

KEYWORDS

Tracer gas techniques; Air infiltration; Atmospheric CO_2; Discrete Fourier transform

1 INTRODUCTION

There are two main methodologies available to assess infiltration rates in buildings: one based on fan pressurization techniques; and another based on tracer gas techniques. The methods based on fan pressurization techniques have the potential advantage for providing quick results, but can be costly to implement an may not accurately represent infiltration rates at the lower pressure differentials found during normal operation of the building. The methods based on tracer gas techniques can be comparatively cheap and easy to implement but rely heavily on the assumptions of uniform mixing of the tracer gas and the indoor air. Carbon dioxide (CO_2), which is naturally present in the atmosphere and is also produced by the building occupants, has been shown to satisfy the uniform mixing assumption and can be used effectively in determining ventilation rates in buildings (Barankova, Naydenov, Melikov & Sundell, 2004). Metabolic CO_2 produced by the building occupants is commonly used as the source of the tracer gas, and an estimate of infiltration rates can be obtained from the step response of a first order system, either in concentration rise or in concentration decay situations. A fundamental assumption in the conventional CO_2 tracer gas techniques, however, is that the outdoor ambient concentration is constant (Sherman, 1990; Persily, 1997).
In the present paper, the authors propose a new approach for determining infiltration rates, which does not require the presence of building occupants as the source of CO_2. The method belongs to the class of tracer gas techniques, but instead of relying on a step forcing function, it uses a naturally occurring periodic forcing function. Despite being a common assumption in

the existing methods, the outdoor ambient CO_2 concentration is not constant (e.g., see Refs. (Massen, Kies & Harpes, 2003) and (Miyaoka, Yoshikawa Inoue, Sawa, Matsueda & Taguchi, 2007)). The photosynthesis and respiration cycle of plants, and other processes associated with burning of fuels, are the cause of variations in the ambient CO_2 concentration which can be of the order of 100 ppm in amplitude, on a daily basis. This quasi-periodic variation is taken into advantage for estimating infiltration rates without the need of a source of CO_2 in the building. This method is particularly suited for measurement during extended periods of no occupancy, e.g., during the pre-commissioning test phase of a new or a renovated building, when the building is ready for operation but has not yet been occupied. The application of the new method is demonstrated with real data obtained in a residential building.

2 THEORETICAL FORMULATION

Consider a single zone with volume V, such that air is exchanged with the outdoor environment, through one or more of its boundaries, at a volume flow rate q cubic meters per hour. Assuming complete mixing, and in the absence of filtering mechanisms, deposition and absorption processes, the time evolution of the CO_2 concentration within the enclosure, $C_{\text{int}}(t)$, is well described by the mass balance equation (Etheridge & Sandberg, 1996, p. 277)

$$VC'_{\text{int}}(t) + q[C_{\text{int}}(t) - C_{\text{ext}}] = g(t), \tag{1}$$

where the prime denotes differentiation with respect to time, C_{ext} is the outdoor CO_2 concentration, $g(t)$ is the rate of CO_2 generation within the enclosure, and q is the volume flow rate of new air entering the enclosure or, equivalently, the volume flow rate of old air leaving the enclosure.

In writing (1), it was implicitly assumed that the outdoor CO_2 concentration, C_{ext}, does not vary in time. Allowing for $C_{\text{ext}}(t)$ to be now an explicit function of time, setting the rate of generation of CO_2 to zero and writing $\lambda = q/V$, (1) becomes

$$C_{\text{int}}{}'(t) + \lambda[C_{\text{int}}(t) - C_{\text{ext}}(t)] = 0. \tag{2}$$

Dividing both sides of the equation by λ and moving $C_{\text{ext}}(t)$ to the right-hand side gives

$$\frac{1}{\lambda} C_{\text{int}}{}'(t) + C_{\text{int}}(t) = C_{\text{ext}}(t), \tag{3}$$

which is a mathematical representation of a first order Linear Time Invariant (LTI) system with the input $C_{\text{ext}}(t)$, output $C_{\text{int}}(t)$, and time constant $\tau = 1/\lambda$.

Assuming that the input to the system is the outdoor concentration time series, represented by a periodic sinusoidal function, with an amplitude A_{ext} and an angular velocity $\omega = 2\pi f$

$$C_{\text{ext}}(t) = A_{\text{ext}} \sin(\omega t)\,, \tag{4}$$

the output, after a long enough time such that transients have died away, will be another sinusoidal function attenuated in amplitude and phase lagged and will have the form

$$C_{\text{int}}(t) = A_{\text{int}} \sin(\omega t + \phi_{\text{int}})\,, \tag{5}$$

where ϕ_{int} = atan $(\omega\tau)$ is the phase lag angle. The ratio of amplitudes $A_{\text{int}}/A_{\text{ext}}$, the time constant τ, and the frequency of excitation ω are related by the well-known expression for first order, linear time invariant (LTI) systems (Holman, 1994, pp. 23)

$$\frac{A_{\text{int}}}{A_{\text{ext}}} = \frac{1}{\sqrt{1+(\omega\tau)^2}}. \tag{6}$$

So, in the case of sinusoidal excitation, both the time delay between input / output and the amplitude attenuation depend on the system time constant, that for our situation has the physical interpretation of the mean age of air inside the compartment. By rearranging (6), the air infiltration rate, which is the inverse of the mean age of air ($\lambda = 1/\tau$) can be calculated from

$$\lambda = \frac{\omega}{\sqrt{\left(\frac{1}{A_F}\right)^2 - 1}}, \tag{7}$$

where $A_F = A_{\text{int}}/A_{\text{ext}}$. To estimate A_{ext} and A_{int}, and associated uncertainty bounds, we make use of the Discrete Fourier Transform (DFT):

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N}, \quad k = 0, \dots, N-1 \tag{8}$$

where the complex coefficient X_k carries information on the amplitude and relative phase of the k^{th} sinusoidal component of the sample time series x_n. The frequency of the sinusoidal component is $f_k = kf_s/N$, where f_s is the sampling frequency, and its amplitude is

$$A_k = \frac{2|X_k|}{N}. \tag{9}$$

Assuming that most of the signal's energy in x_n is concentrated in a single frequency component, in this case the frequency corresponding to a 24 h period, a model for x_n is

$$\hat{x}_n = \frac{1}{N}\left[X_1 e^{i2\pi n/N} + X_{N-1} e^{i2\pi n(N-1)/N}\right] + \varepsilon_n, \tag{10}$$

where N is the number of sample in a 24 h period and ε_n accounts for noise, i.e., it is the time series of the residuals obtained by subtracting the deterministic part of the model in (10) from the measured data x_n. Letting $p = \text{var}(\varepsilon_n)$ be the power in the residuals in a complete period, an estimate of the standard uncertainty associated with the determination of the Fourier coefficient X_k is (Thibos, 2003)

$$u(X_k) \approx \frac{F_{2,2R,0.318}}{R} p, \tag{11}$$

where $R = (N-3)/2$ and $F_{a,b,\alpha}$ is the F-distribution with a numerator degrees of freedom, b denominator degrees of freedom, and α is the critical value such that $1-\alpha$ corresponds to the level of confidence. For a sampling rate of one sample every 30 min, the number of samples in 24 h is $N = 48$, so that $R = 22.5$ and, for a level of confidence of 68.2%, $F_{2,2R,0.318} = 1.1754$. In practice, (11) has the meaning of a radius defining a circle in the complex plane, centred in X_k (Figure 1). With the specified level of confidence of 68.2%, we can say that the true value of the Fourier coefficient is inside this circle.

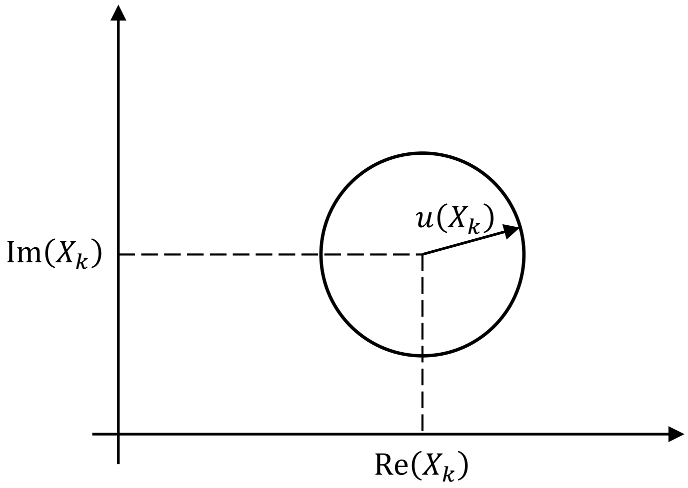

Figure 1 - Graphical representation of the standard uncertainty associated with the determination of the Fourier coefficient X_k, estimated from the confidence bound (11).

The standard uncertainty associated with the determination of λ can then be estimated from

$$u(\lambda) \approx \sqrt{\left(\frac{\partial \lambda}{\partial A_F}\right)^2 u(A_F)^2}, \tag{12}$$

where

$$\frac{\partial \lambda}{\partial A_F} = \frac{\omega A_F}{\left(1 - {A_F}^2\right)^{\frac{3}{2}}}, \tag{13}$$

and $u(A_F)^2 \approx \mathrm{var}(A_{\mathrm{int}}/A_{\mathrm{ext}})$ is estimated from the first order Taylor expansion approximation to the variance of the ratio by

$$\mathrm{var}\left(\frac{A_{\mathrm{int}}}{A_{\mathrm{ext}}}\right) \approx \frac{\mathrm{E}(A_{\mathrm{int}})^2}{\mathrm{E}(A_{\mathrm{ext}})^2}\left[\frac{\mathrm{var}(A_{\mathrm{int}})}{\mathrm{E}(A_{\mathrm{int}})^2} - 2\frac{\mathrm{cov}(A_{\mathrm{int}}, A_{\mathrm{ext}})}{\mathrm{E}(A_{\mathrm{int}})\mathrm{E}(A_{\mathrm{ext}})} + \frac{\mathrm{var}(A_{\mathrm{ext}})}{\mathrm{E}(A_{\mathrm{ext}})^2}\right], \tag{14}$$

where $\mathrm{E}(\chi)$ denotes the expected value of χ and $\mathrm{cov}(\chi, \gamma)$ denotes the covariance between χ and γ. Since the covariance between the output and the input amplitudes of a first order LTI system is always positive, we can safely neglect the covariance term in (14) to obtain an upper bound for the variance of the ratio:

$$\mathrm{var}\left(\frac{A_{\mathrm{int}}}{A_{\mathrm{ext}}}\right) \leq \frac{\mathrm{E}(A_{\mathrm{int}})^2}{\mathrm{E}(A_{\mathrm{ext}})^2}\left[\frac{\mathrm{var}(A_{\mathrm{int}})}{\mathrm{E}(A_{\mathrm{int}})^2} + \frac{\mathrm{var}(A_{\mathrm{ext}})}{\mathrm{E}(A_{\mathrm{ext}})^2}\right], \tag{15}$$

Finally, we identify the variance of the amplitude with the square of the standard uncertainty obtained from (11).

3 MATERIALS AND METHODS

Two Extech SD800 devices were used to record CO_2 concentration, at a rate of one sample every 30 min, in a two bedroom flat, built in the 1990s, located in a rural village near Oliveira do Bairro, Portugal. The flat has an interior floor area of approximately 88 m^2 and height 2.5 m,

and is at the first floor above ground of a three level building. The exterior device was placed in the east-facing balcony, shielded from direct solar radiation, whereas the interior device was placed in the living-room, leading to the same balcony. Figure 2 shows a floor plan of the flat, with the approximate locations of the measuring devices.
All windows and exterior doors were left fully closed, while all interior doors were left fully open. There were no occupants or other sources of CO2 inside the flat, and there was no heating, cooling or mechanical ventilation in operation during the entire measurement period.

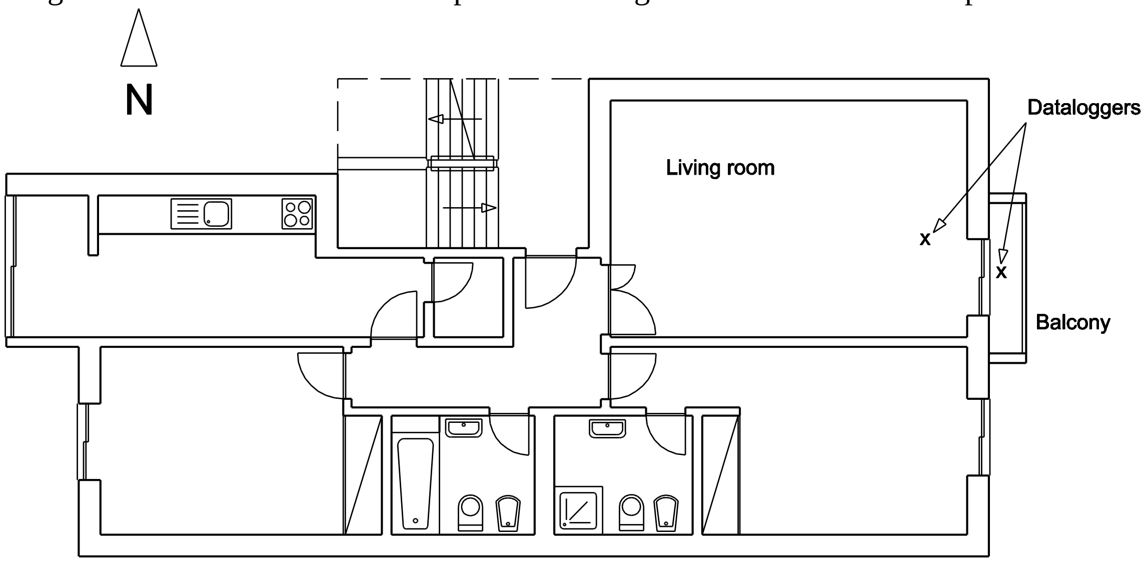

Figure 2 - Sketch of the residential flat were the measurement campaign took place. The locations of the interior and exterior measuring devices are shown with crosses, in the living room and in the balcony. All windows and exterior doors were left closed during the measurement period

4 RESULTS AND DISCUSSION

Continuous CO_2 concentration time series were obtained between 15:49 on August 26 and 15:19 on August 28, 2013. Figure 3 shows the raw data obtained from both the exterior and the interior devices. The time series are shown with an artificial vertical offset for better visualization. The shaded areas indicate the time periods from 20:00 to 07:00, which correspond approximately to the night periods.
The daily quasi-periodic nature of the atmospheric CO_2 is well visible in Figure 3. During the night periods, plants enter the respiration phase and produce CO_2, which reaches a maximum concentration around the early hours of the morning, before the sunrise. In the morning, as the sun rises above the horizon, plants enter the photosynthesis phase and the exterior CO_2 concentration starts decreasing, until it reaches a minimum around midday. As the sun sets in the evening, the exterior concentration starts to increase again. The interior time series clearly follows the trend of the exterior time series, with a lower amplitude and a phase lag. This behaviour is characteristic of the input-output relation of a first order system.
Both exterior and interior time series where segmented in overlapping periods of 24 h, with 48 samples each, i.e. the first segment starts at sample 1 and ends at sample 48; the second segment starts at sample 2 and ends at sample 49, and so on. The DFT was then applied to each segment, from which the amplitude of the second Fourier component (corresponding to the 24 h period) was calculated. The infiltration rate corresponding to each 24 h segment was then calculated from (7). The infiltration rate, as a function of time, is shown as a solid line in Figure 4, where each point on the line represents an average infiltration rate for the last 24 h. The dashed lines represent upper and lower expanded uncertainty estimates (coverage factor $k = 2$). The infiltration rate is seen to vary randomly about a mean value of 0.19, which is in agreement with typical values for infiltration rates in Portuguese residential buildings built in the last 30

years (Gomes, Gameiro da Silva & Simões, 2013). The expanded uncertainty varies from 10% to 20% of the estimated infiltration rate.

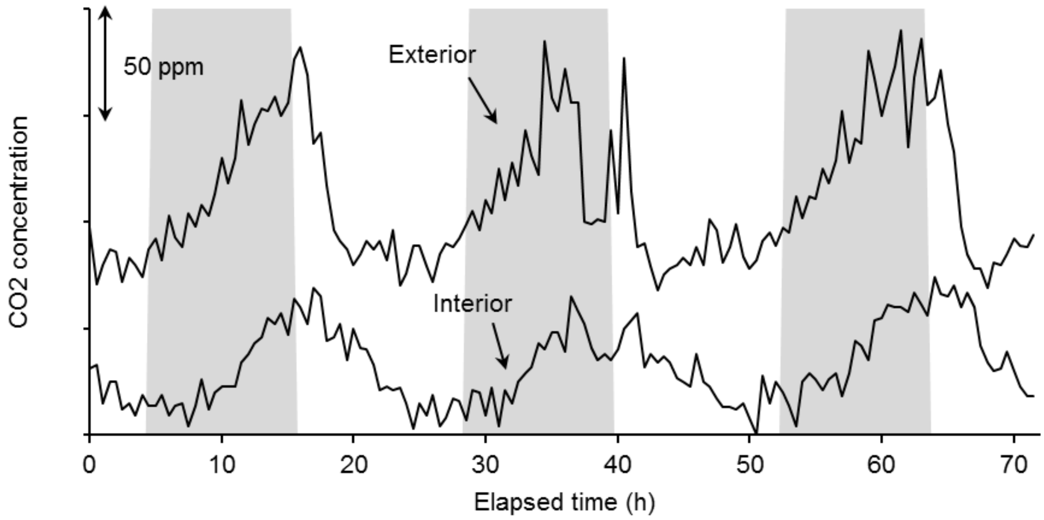

Figure 3 - Time series, recorded over 3 days, of exterior and interior CO_2 concentrations: raw data shown with an artificial vertical offset for better visualization. Ticks on the vertical axis are 50 ppm apart and the shaded areas identify night periods, between 20:00 and 07:00.

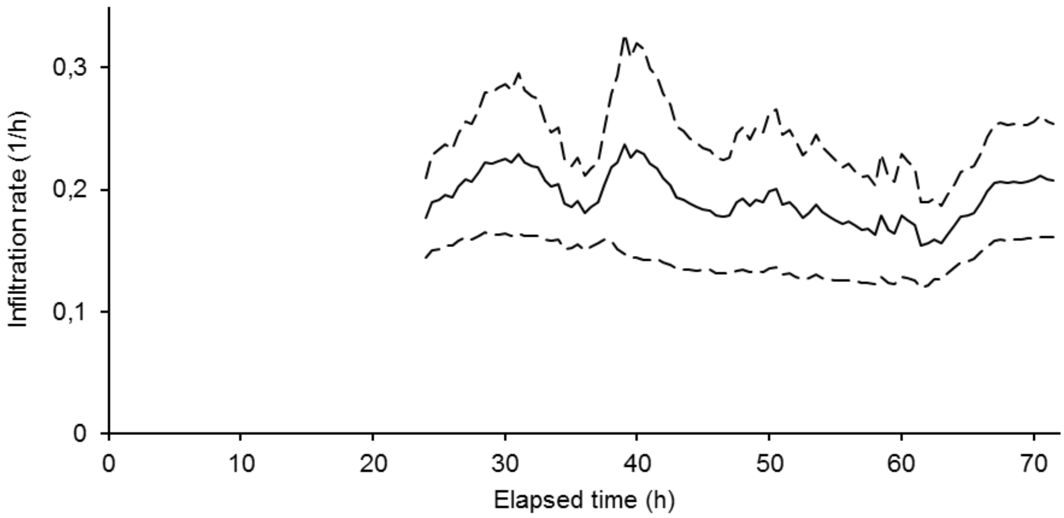

Figure 4 – Air infiltration rate results obtained from the raw data in Figure 3. Each point on the solid line represents an average infiltration rate corresponding to the previous 24 h. The dashed lines are estimates of the expanded uncertainty associated with the determination of the infiltration rate (coverage factor $k = 2$).

5 CONCLUSION

A new method to determine air infiltration rates in buildings has been proposed. The method belongs to the class of tracer gas techniques but, contrary to established methods that are based on the step response to the sudden introduction or extinction of a source of tracer gas, it uses the naturally occurring variation in atmospheric ambient CO_2 concentrations to derive 24 h moving average estimates of the infiltration rate. This novel approach has several advantages: it does not rely on the injection of a tracer gas or the use of metabolic CO_2 generated by the building occupants; it produces 24 h moving average time series of infiltration rates with the time resolution dictated only by the sampling frequency; and it may be less sensitive to mixing assumptions compared to methods which require the injection or generation of a tracer gas inside the building. The new method may provide a very useful tool for studying the dynamic behaviour of ventilation in buildings, which remains a main source of uncertainty in modelling building systems and, consequently, in the assessment of the energy balance in buildings as well as the quality of the indoor environment.

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