General approach for evaluation of measurement uncertainties

Benoit SAVANIER

Outline

1. *Introduction*
2. *From Mesurand to Measurement result*
3. *Road map for evaluation of measurement uncertainty*
4. *Simple measurement model (GUM approach) and Simple measurement model (Supp. 1 GUM approach)*
5. *Complex measurement model (Supp. 1 GUM approach)*

1 - INTRODUCTION

1 - Introduction 1/2 Bibliography

- *JCGM 100:2008. Evaluation of measurement data - Guide to the expression of uncertainty in measurement available on bipm website known as the : GUM*
- *JCGM 101:2008. Evaluation of measurement data - Supplement 1 to the Guide to the expression of uncertainty in measurement - Propagation of distributions using a Monte Carlo method known as : Supplement 1 or Supp.1*
- *JCGM 200:2012 : International Vocabulary of Metrology - Basic and General Concepts and Associated Terms referred as : VIM 3 for third ed.*

1 - Introduction 2/2
Why uncertainty assessment ?

- *Evaluation of measurement data - The role of measurement uncertainty in conformity assessment (JCGM 106:2012) becomes more and more important in industrial field.*

- *To declare conformity : Tolerances & measurement & uncertainty on measurement*

  Exemple : Demonstrate your instrumentation compatibility with a testing standard requirements

- *Being able to write for exemple that $P_{measured}$ = 200 bar ± 10 bar is the goal of uncertainty assessment*

2 - FROM MESURAND TO MEASUREMENT RESULT

2 - From Mesurand to Measurement Result 1/7

Full VIM definition = definition + notes.

Don't try to read all now !!

Measurand (VIM 3 §2.3) :

Quantity intended to be measured

NOTE 1 The specification of a measurand requires knowledge of the kind of quantity, description of the state of the phenomenon, body, or substance carrying the quantity, including any relevant component, and the chemical entities involved.

NOTE 2 In the second edition of the VIM and in IEC 60050-300:2001, the measurand is defined as the "particular quantity subject to measurement".

NOTE 3 The measurement, including the measuring system and the conditions under which the measurement is carried out, might change the phenomenon, body, or substance such that the quantity being measured may differ from the measurand as defined. In this case, adequate correction is necessary.

EXAMPLE 1 The potential difference between the terminals of a battery may decrease when using a voltmeter with a significant internal conductance to perform the measurement. The open-circuit potential difference can be calculated from the internal resistances of the battery and the voltmeter.

EXAMPLE 2 The length of a steel rod in equilibrium with the ambient Celsius temperature of 23 °C will be different from the length at the specified temperature of 20 °C, which is the measurand. In this case, a correction is necessary.

NOTE 4 In chemistry, "analyte", or the name of a substance or compound, are terms sometimes used for 'measurand'. This usage is erroneous because these terms do not refer to quantities.

2 - From Mesurand to Measurement Result 2/7
Focus on Mesurand note 1

- ***NOTE 1 The specification of a measurand requires knowledge of the kind of quantity, description of the state of the phenomenon, body, or substance carrying the quantity, including any relevant component, and the chemical entities involved.***

- *Comments :*
  - *Poor knowledge available in some cases*
  - *Ishikawa or cause effect diagram are helpful in analysis*

2 - From Mesurand to Measurement Result 3/7
Focus on Mesurand note 3

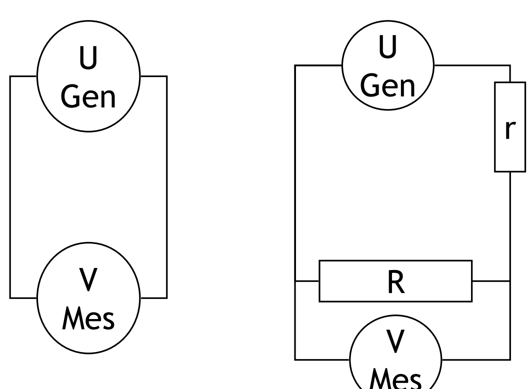
- ***NOTE 3 The measurement, including the measuring system and the conditions under which the measurement is carried out, might change the phenomenon, body, or substance such that the quantity being measured may differ from the measurand as defined. In this case, adequate correction is necessary.***

- ***Comments :***
  - *Exemple : analog output of a pressure transmitter*
  - *Exemple : resistance value of PRT PT100 thermometer*
  - *Correction exemple : Self-heating effect, Fluid column effect in pressure measurement*

2 - From Mesurand to Measurement Result 4/7
Focus on Mesurand example 1 (continued)

EXAMPLE 1 Potential difference measurement between the terminals of a battery

Mesurand without the conductance effect



Mesurand with the conductance effect

2 - From Mesurand to Measurement Result 5/7
Focus on Mesurand example 2

EXAMPLE 2 Length measurement of an object at different temperatures

$$L_\theta = L_{20°C} \cdot (1 + \alpha \cdot (\theta - 20))$$

α : linear thermal expansion coefficient

Comments :
Temperature measurement is needed to correct the value of mesurand.
Knowledge is need about linear thermal expansion coeff.
Knowledge is need about lenght measurement

2 - From Mesurand to Measurement Result 6/7

Don't try to read all now !!

- **Measurement result :**

Set of quantity values being attributed to a measurand together with any other available relevant information

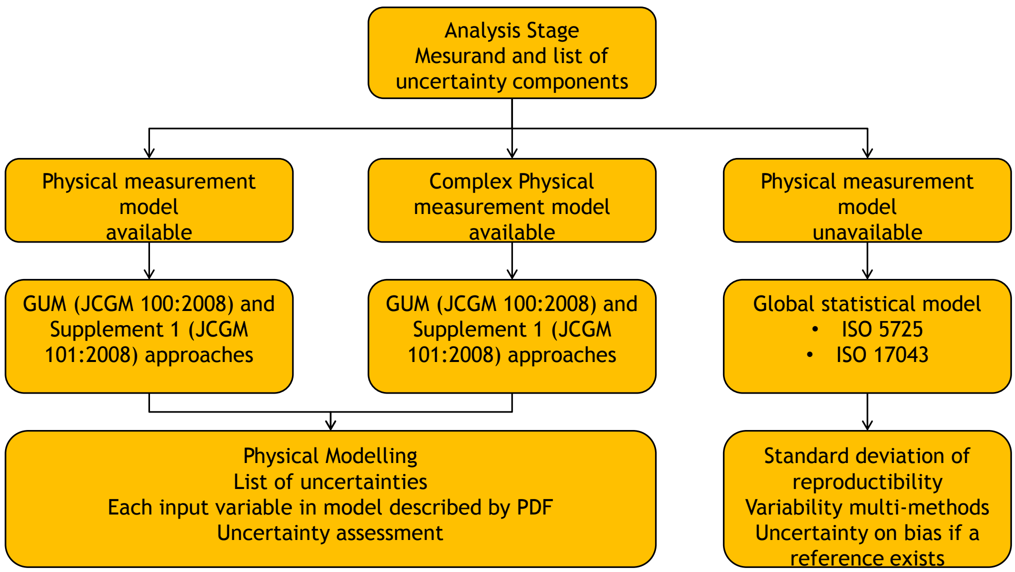
- *NOTE 1 A measurement result generally contains "relevant information" about the set of quantity values, such that some may be more representative of the measurand than others. This may be expressed in the form of a probability density function (PDF)*
- *NOTE 2 A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty. If the measurement uncertainty is considered to be negligible for some purpose, the measurement result may be expressed as a single measured quantity value. In many fields, this is the common way of expressing a measurement result.*
- *NOTE 3 In the traditional literature and in the previous edition of the VIM, measurement result was defined as a value attributed to a measurand and explained to mean an indication, or an uncorrected result, or a corrected result, according to the context.*

2 - From Mesurand to Measurement Result 7/7
Summary

- *To define a mesurand implies to be able to write down a model, i.e. an equation between the measurand and input or influence quantities*
- *Measurement could be seen as the attribution of a set of values to the mesurand*
- *Measurement result = measured value + measurement uncertainty*
  Exemple : T=20,00°C ± 0,10°C (k=2)

3 - ROAD MAP FOR EVALUATION OF MEASUREMENT UNCERTAINTY

3 - Road map for evaluation of measurement uncertainty 1/1



4 - SIMPLE MEASUREMENT MODEL GUM AND SUPP.1

4 - Simple measurement model (GUM and Supp. 1)

STEP	ITEMS
1	Identify inputs and output of physical model and write down measurement model Exemple : temp. effect in lenght measurement
2	Attribute standard uncertainty to each input quantity Exemple : uncertainty on temp. Measurement needs to correct lenght measurement
3	Apply GUM uncertainty assessment to model (GUM) Or Use a software to simulate measurement result (Supp.1)
4	Calculate final uncertainty
5	Analyse final uncertainty
6	Is the result satisfying ?
7	If response = no modify model then redo from step 1 until response = yes

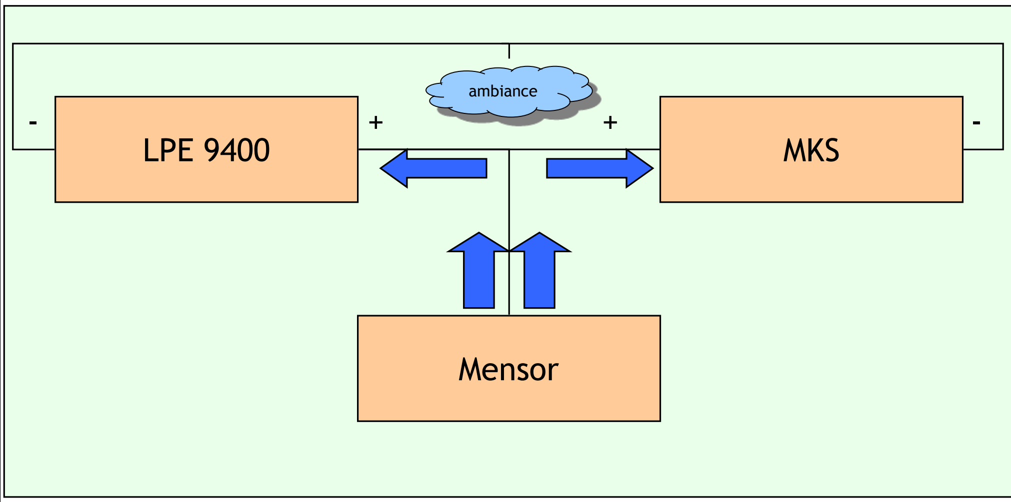
4 - Simple measurement model (GUM and Supp. 1)
Step 1 Identify inputs and output of physical model

Unit under Test UUT

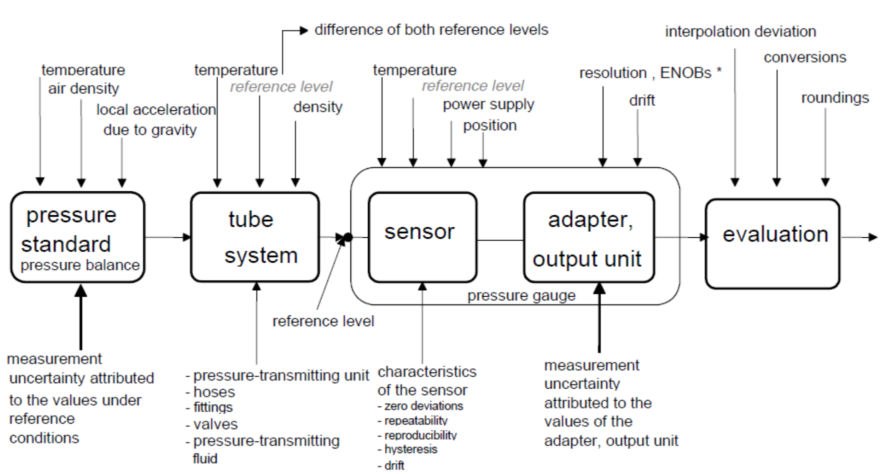
Regulation

Reference Measurement Standard

4 - Simple measurement model (GUM and Supp. 1)
Step 1 Identify inputs and output of physical model



4 - Simple measurement model (GUM and Supp. 1)
Step 1 Identify inputs and output of physical model
Cause effect Diagram - Ishikawa (DKD R6-1)



4 - Simple measurement model (GUM and Supp. 1) Step 1 Identify inputs and output of physical model

- ***Write down the model***
  - *In this case we are calibrating, this lead to error determination*

$$Ej = P_{UUT} - P_{ref}$$

  - P_{UUT} *: pressure reading on the unit under test*
  - P_{ref} *: corrected pressure reading on the calibration standard*

4 - Simple measurement model (GUM and Supp. 1) Step 1 Identify inputs and output of physical model

- ***Write down the model (Hardest Part) :***

$$P_{ref} = P_{read} + C_{calib.} + C_{Drift\ beet.cal.} + C_{corr.error} + C_{Stab.} + C_{resolution} + C_{fluid\ column} + C_{Temp.} + C_{zero\ drift}$$

$$P_{uut} = C_{res.UUT} + C_{repeatability.UUT}$$

4 - Simple measurement model (GUM and Supp. 1) Step 2 Attribute standard uncertainty to each input quantity

- *Standard uncertainty is evaluated on basis of a pdf*
- *Choice is dictated by data origin and metrologic choice*
- *GUM and Suppl.1 presented the subject in depth*
- *Two main pdf are used : the gaussian and the rectangular pdf*

4 - Simple measurement model (GUM and Supp. 1) Step 2 Attribute standard uncertainty to each input quantity

Available information	Assigned PDF and illustration (not to scale)
Lower and upper limits a, b	Rectangular: $\mathrm{R}(a, b)$

X has expectation and variance

$$E(X) = \frac{a+b}{2}, \qquad V(X) = \frac{(b-a)^2}{12}.$$

Available information	Assigned PDF and illustration (not to scale)
Best estimate x and associated standard uncertainty $u(x)$	Gaussian: $\mathrm{N}(x, u^2(x))$

X has expectation and variance

$$E(X) = x, \qquad V(X) = u^2(x).$$

4 - Simple measurement model (GUM and Supp. 1) Step 2 Attribute standard uncertainty to each input quantity

- *Exemple :*
  - *Calibration certificate gives correction associated with an expanded uncertainty*
  - *It is corresponding to a gaussian law (GUM)*
  - *Expanded uncertainty divide by k=2 gives standard uncertainty*
  - *Mean is corresponding with Correction*

- ***For each term we will have to attribute standard uncertainty***

4 - Simple measurement model (GUM and Supp. 1) Step 3 - Apply GUM uncertainty assessment to model

- ***With a model***

$$P_{ref} = P_{read} + C_{calib.} + C_{Drift\,beet.cal.} + C_{corr.error} + C_{Stab.} + C_{resolution} + C_{fluid\,column} + C_{Temp.} + C_{zero\,drift}$$

- ***Applying GUM (§4 and §5)***

$$Y = f(X_1, X_2, ..., X_N) \qquad u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

For uncorrelated quantities

- ***Combined uncertainty is obtained throught application of right formulae***

4 - Simple measurement model (GUM and Supp. 1) Step 3 - Apply GUM uncertainty assessment to model

- ***For P_{ref}***

$$u^2(P_{ref}) = u^2(C_{calib.}) + u^2(C_{drift}) + u^2(C_{Stability})$$
$$+ u^2(C_{resolution}) + u^2(C_{fluid\ column}) + u^2(C_{Temp.effect}) + u^2(C_{Zero})$$

- ***For P_{uut}***

$$u^2(P_{UUT}) = u^2(C_{repeatab.}) + u^2(C_{resolution})$$

- ***NOTE : here are two examples of simple model uncertainty evaluation (P_{ref} & P_{uut})***

CETIAT ensemble, innover et valider

4 - Simple measurement model (GUM and Supp. 1) Step 3 - Apply GUM uncertainty assessment to model

- ***Finally***

$$u_{Ej}^2 = u_{P_{UUT}}^2 + u_{P_{ref}}^2$$

$$U_{Ej} = k.\sqrt{u_{Ej}^2}$$

$$with\ k = 2$$

- ***Note : In fact we have evaluated a complex measurement model combining two simple measurement models***
- ***Let's put some value***

4 - Simple measurement approach (GUM and Supp. 1) Step 4 - uncertainty assessment result for Pref

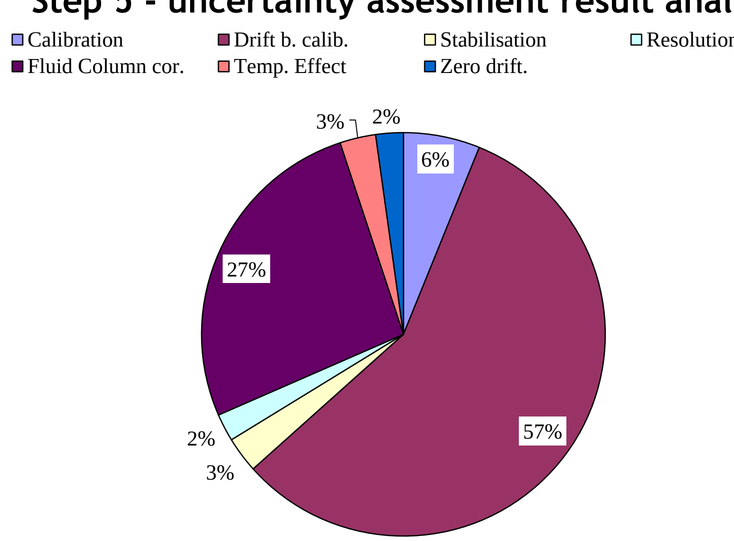
Comp.	Abs. Unc.	Rel. Unc.	Loi de distribution PDF	Sens. Coeff.	Standart mes. Uncert.	Standart mes. Uncert.	Standart mes. Uncert.	Variance	Variance	Variance
Calibration	0,0110	3,80E-05	Normale	2,00	5,75E-03	8,03E-03	5,50E-03	3,30E-05	6,44E-05	3,03E-05
Drift b. calib.	0,0300	7,50E-04	Rectangulaire	1,73	2,29E-02	7,49E-02	1,73E-02	5,27E-04	5,61E-03	3,00E-04
Correct. Model error	0,0029		Rectangulaire	1,73	1,67E-03	1,67E-03	1,67E-03			
Stabilisation	0,0130		Rectangulaire centrée	3,46	3,75E-03	3,75E-03	3,75E-03	1,41E-05	1,41E-05	1,41E-05
Resolution	0,0100		Rectangulaire centrée	3,46	2,89E-03	2,89E-03	2,89E-03	8,33E-06	8,33E-06	8,33E-06
Fluid Column cor.	0,12		Rectangulaire centrée	3,46	3,46E-02	3,46E-02	3,46E-02	1,20E-03	1,20E-03	1,20E-03
Temp. Effect		4,00E-05	Arc sinus	1,41	3,68E-04	3,76E-03		1,35E-07	1,42E-05	
Zero drift.	0,01		Rectangulaire centrée	3,46	2,89E-03	2,89E-03	2,89E-03	8,33E-06	8,33E-06	8,33E-06

Source	Autre
Grandeur	Pression
Unité	Pa
Plage bas	13,00
Plage haut	133,00

	Calcul Plage bas	Calcul Plage haut	Calcul en zéro	
Sum of variances	**1,79E-03**	**6,92E-03**	**1,56E-03**	
Combined standart mes. Uncert.	**4,2E-02**	**8,3E-02**	**4,0E-02**	
Expanded Mes. Uncert. (k=2)	**8,8E-02**	**1,7E-01**	**8,2E-02**	
Expanded Mes. Uncert. (k=2) en f(P)	**0,0819**	**Pa**	**6,81E-04**	***P**

CETIAT — ensemble, innover et valider

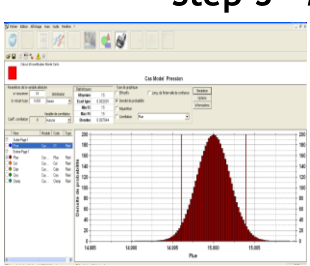
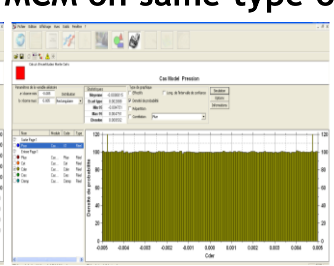
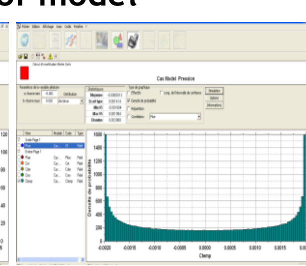
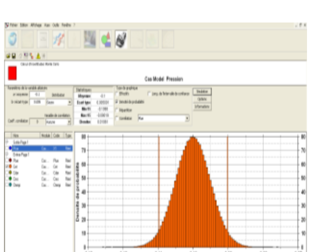
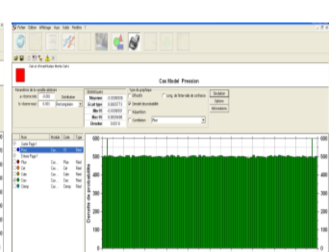
4 - Simple measurement approach (GUM and Supp. 1) Step 5 - uncertainty assessment result analysis



4 - Simple measurement approach (GUM and Supp. 1) using Monte-Carlo Simulation

- *All previous steps 1 and 2 are valid*
- *Key point: simulating measurement following attributed pdf instead of computing standard uncertainty and combining them.*
- *A software is mandatory*
- *Models are the same*

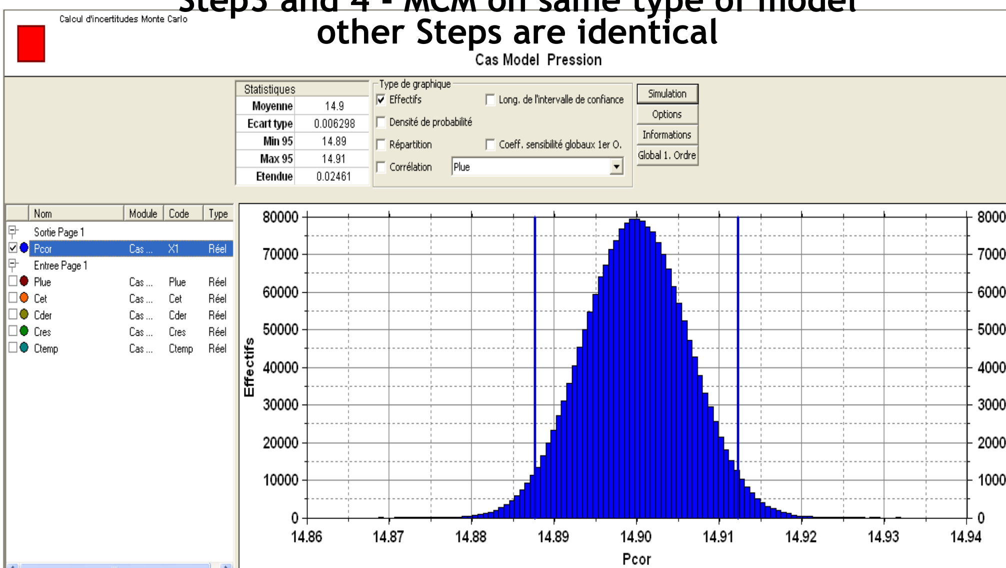
4 - Simple measurement approach (Supp1.) Step 3 - MCM on same type of model

Example on the P_{ref} model (with less terms)

Values are not readable there !

4 - Simple measurement approach (Supp1. GUM) Step3 and 4 - MCM on same type of model other Steps are identical



5 - COMPLEX MEASUREMENT MODEL SUPP.1

5 - COMPLEX MEASUREMENT MODEL SUPP.1
Starting Model

$$Qm = \alpha.\frac{\pi}{4}.d^2.\sqrt{2.P_e.\rho} \text{ en [kg/s]}$$

$$QmCReduitRefoul = \alpha.\frac{3{,}14159 \ .D\_Diaphragme^2}{4}.\sqrt{2.\rho.P_e}$$

$$\rho = RoAirHum(Pe + Pa, T, Td)$$

$$\beta = D\_Diaphragme/D\_Caisson. \quad \alpha = A\left[1 - \frac{P_e}{Pa}\left(B - \frac{P_e}{Pa}.C\right)\right]$$

$\beta \le 0{,}5$

A = 0,5993 + 0,1599.m - 0,9156.m^2 + 6,5675.m^3 - 9,1429.m^4

B = 0,249 + 0,0701.m + 0,243.m^2 + 0,113.m^3

C = 0,0757 + 0,058.m + 0,22.m^2 + 0,25.m^3

P_e : Delta P at orifice plate P_a : atmospheric pressure

Complex enough ?

5 - COMPLEX MEASUREMENT MODEL SUPP.1

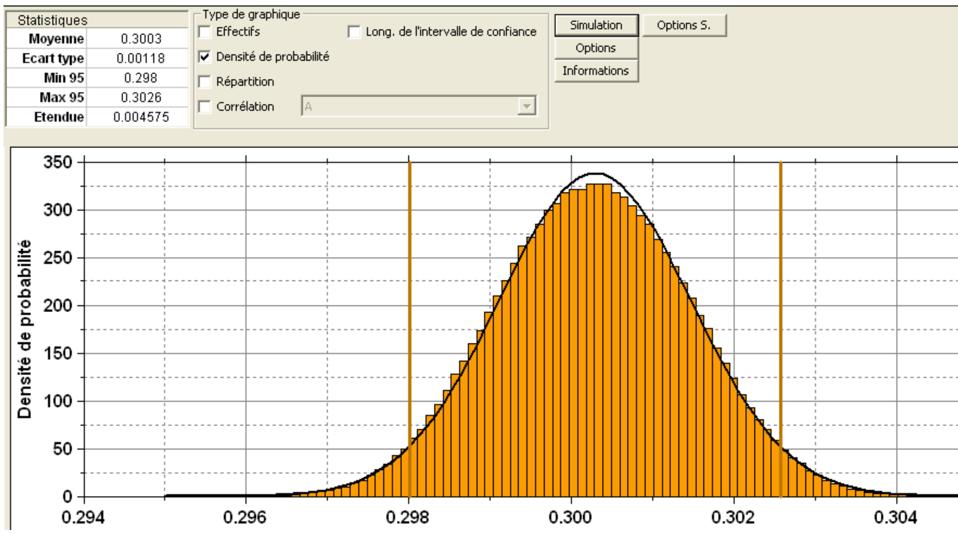
Entry Data with uncertainty resulting from simple analysis on each terms

Données d'entrée

Code	Nom	Valeur	Incertitude-type	Relative Inc.-type	Distribution
A	A	0.5975	**0.598500**	100.167	Rectangulaire
B	B	0.2485	**0.249500**	100.402	Rectangulaire
C	C	0.07565	**0.075750**	100.132	Rectangulaire
Pe	Pe	-700	**4.000000**	-0.571	Gauss
Pa	Pa	100000	**250.000000**	0.250	Gauss
D	D	0.12488	**0.125130**	100.200	Rectangulaire
RhoAir	RhoAir	1.19	**1.210000**	101.681	Rectangulaire

5 - COMPLEX MEASUREMENT MODEL SUPP.1

Result



Conclusion

- *Several assessment methods for several case of model*
- *Each method could be seen as adressing a specific case*
- *Calculation needs range from simple excel file to full dedicated software*
- *Uncertainty gives a measure of the quality of measurement*
- *Uncertainty assessment analysis contributes to improve measurement quality*

Merci de votre attention

Thank you for your attention