Effective flow area estimation method using a gas

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Abstract

The objective of this paper is to present a new method for estimating effective flow areas not only in the external wall of a house but also in the internal walls between rooms using only one type of tracer gas. The discharge coefficient of each wall and the pressure in each room—which are unknown variables—are determined using nonlinear simultaneous equations, which consist of balance equations for the air mass and tracer-gas concentration in the rooms. To verify the validity of this method, we performed a numerical experiment. We defined the values of the discharge coefficient of each wall, and then simulated changes in the temperature and tracer-gas concentration. We then estimated the discharge coefficient of each wall from the simulated temperature and concentration values using the method. Because it was confirmed that the estimated values were equivalent to the defined values, the validity of this method was verified.

Keywords: Effective flow area, Air leakage area, Discharge coefficient, Estimation, Newton’s method, numerical experiment.

Introduction

In houses that exhibit a high level of airtightness, it is necessary to provide appropriate ventilation in order to ensure high indoor air quality. Methods to observe the flow of air through rooms using tracer gas have previously been reported [1][2]; however, in these methods, the measured air flow is valid only if the external wind velocity and temperature are the same at the time of measurement, with the same volume of air being supplied or discharged by a machine. If the air velocity, temperature, and supply/discharge volume differ, the air flow will be different too.

To quantify air flow rates under any conditions, it is necessary to determine the effective flow areas (air leakage areas) of the walls that have air leakage areas around doors or in walls. Practical method exists for estimating effective flow area on the basis of the relation between pressure difference and air flow volume [3]. However, this method has mostly been used to estimate effective flow areas in the external walls of a house and not those in the internal walls between rooms. The objective of this paper is to present a method that can estimate effective flow areas not only in the external walls of the house but also in the internal walls between rooms using only one type of tracer gas. In this paper, we present the theoretical basis of the estimation method and the results of a numerical experiment performed to validate the method.

Balance equations for air mass and tracer-gas concentration

In this paper, we present a method to estimate the effective flow areas of walls whose position, width, and height are known. For example, it is known that air leakage areas exist above and underneath a closed door, in addition to the left and right sides of the door. In order to obtain the effective flow areas of the doorway, we consider the doorway as a “wall” and calculate the effective flow area with respect to its total area.

In houses having multiple rooms, if heat generation and the supply of tracer gas are carried out in an arbitrary room, the balance equations for the air mass and tracer-gas concentration can be expressed as in Equations 1 and 2 below, respectively.

$$\left\{ V_n \frac{d\rho_n}{dt} \right\} = -[I]\{w_m\} \qquad \text{(Eq. 1)}$$

$$\left\{ V_n \frac{d(\rho C)_n}{dt} \right\} = -[I]\{x_m\} + \{S_n\} \qquad \text{(Eq. 2)}$$

Here, {} denotes a vector; [], a matrix; the subscript n, the number of the room; and m, the number of the wall.

V : volume of room (m^3),

ρ : air density (kg/m^3),

C : tracer-gas concentration (kg $_{(gas)}$/kg$_{(air)}$),

w : net mass flow rate of air through a wall that has air leakage areas (kg$_{(air)}$/s),

x : net mass flow rate of tracer gas through a wall that has air leakage areas (kg$_{(gas)}$/s),

S : supply rate of tracer gas (kg$_{(gas)}$/s),

$[I]$: incidence matrix that represents the connections between rooms [4].

The net mass flow rates of the air and tracer gas through a wall that has air leakage areas can be expressed as Equations 3 and 4 below, respectively. We assume that the air flow rate is proportional to the square root of pressure difference across an opening as long as there is no vertical distribution of pressure difference.

$$\{w_m\} = \{f_w(\alpha_m, b_m, h_m, \rho, p_m)\} \quad \text{(Eq. 3)}$$

$$\{x_m\} = \{f_x(\alpha_m, b_m, h_m, \rho, C, p_m)\} \quad \text{(Eq. 4)}$$

where:

α : discharge coefficient of wall that has air leakage areas (-), $\left(= \frac{Q_0}{bh}\left(\frac{\rho}{2}\right)^{0.5} (p_r)^{a-0.5} \right)$ [3],

Q_0 : air leakage coefficient ($m^3/(s\ Pa^a)$) [3],

a : air flow exponent (-) [3],

p_r : reference pressure difference (Pa) [3], (in Japan, 9.8 Pa),

b : width of wall (m),

h : height of wall (m),

p : pressure difference across wall (Pa).

The pressure difference across the wall, as shown in the equation below, is expressed in terms of the pressure in each room and the incidence matrix.

$$\{p_m\} = [I]^T \{P_n\} \qquad \text{(Eq. 5)}$$

where:

P : pressure in room (Pa).

We consider that the air density and tracer-gas concentration in each room are obtained through measurement. Therefore, in the Equations 1 and 2 with the Equations 3, 4 and 5, the discharge coefficient $\{\alpha_m\}$ of each wall and the pressure $\{P_n\}$ in each room are the unknown variables. The effective flow area of each wall is expressed as $\{\alpha_m b_m h_m\}$, and the width and height of the wall are considered as known. Therefore, if $\{\alpha_m\}$ of each wall is known, we can obtain the effective flow area of each wall.

Estimation method using Newton's method

Since the discharge coefficient of the wall and the room pressure have a nonlinear relationship, as apparent in Equations 1 and 2 with the Equations 3, 4 and 5, we use Newton's numerical method to solve the system of nonlinear equations. A flow chart of the method used to estimate the discharge coefficients is shown in Figure 1.

To solve the simultaneous equations, it is necessary for the number of equations to be equal to the number of unknown variables.

For a certain concentration and temperature state, it is possible to derive as many balance equations for the air mass and tracer-gas concentration as the number of rooms. The number of equations that can be derived is given by the following formula:

[number of temperature and tracer-gas concentration states (K) × number of rooms (N) × 2].

The pressure in each room is different for different temperature and tracer-gas concentration states, but the discharge coefficient of each wall does not change with the temperature and concentration state. Therefore, the number of unknown variables is represented by the following formula:

[number of temperature and tracer-gas concentration states (K) × number of rooms (N) + number of walls that have air leakage areas (M)].

In order to make the number of equations equal to the number of unknown variables, we choose KN + M equations out of 2KN balance equations for the air mass and tracer-gas concentration and set up ${}_{2KN}C_{KN+M}$ systems of nonlinear equations (step I). We assign a number to each system of nonlinear equations from y = 1 to ${}_{2KN}C_{KN+M}$ and start solving from y = 1.

Hypothetical values are assumed for the discharge coefficients $\{\hat{\alpha}_m\}$ of each wall (m: 1 through M) and the pressure $\{\hat{P}_n\}_k$ in each room (n: 1 through N) for each temperature and concentration state (k: 1 through K), which are the unknown variables (step II).

The adjustment of the discharge coefficient of each wall and the pressure in each room is repeated according to the method described below until the errors $\{\Delta W_n\}$ (Equation 6) and $\{\Delta X_n\}$ (Equation 7) in mass balance and tracer-gas concentration, respectively, were below the allowable limits (step III through VII).

$$\{\Delta W_n\}_k = -[I]\{w_m + \Delta w_m\}_k - \left\{V_n \frac{d\rho_n}{dt}\right\}_k \qquad \text{(Eq. 6)}$$

$$\{\Delta X_n\}_k = -[I]\{x_m + \Delta x_m\}_k + \{S_n\}_k - \left\{V_n \frac{d(\rho C)_n}{dt}\right\}_k \qquad \text{(Eq. 7)}$$

where:

$$\{w_m + \Delta w_m\}_k = \{f_w(\hat{\alpha}_m, p_m + \Delta p_m)\}_k, \quad \text{(Eq. 8)}$$

$$\{x_m + \Delta x_m\}_k = \{f_x(\hat{\alpha}_m, p_m + \Delta p_m)\}_k, \quad \text{(Eq. 9)}$$

$$\{p_m + \Delta p_m\}_k = [I]^T \{\hat{P}_n\}_k . \quad \text{(Eq. 10)}$$

By considering that $\{\hat{\alpha}_m\}$ and $\{\hat{P}_n\}_k$ differ from the true values $\{\alpha_m\}$ and $\{P_m\}_k$ by $\{\Delta\alpha_m\}$ and $\{\Delta P_m\}_k$, respectively, we can write the following expressions.

$$\{\hat{\alpha}_m\} = \{\alpha_m + \Delta\alpha_m\} \quad \text{(Eq. 11)}$$

$$\{\hat{P}_n\}_k = \{P_n + \Delta P_n\}_k \quad \text{(Eq. 12)}$$

From Equations 5, 10 and 12, the relation between the error in the room pressure and that in the pressure difference across each wall can be expressed as

$$\{\Delta p_m\}_k = [I]^T \{\Delta P_n\}_k . \quad \text{(Eq. 13)}$$

After substituting Equation 11 into Equations 8 and 9 and performing the first-order Taylor expansion around α_m and p_m, Equations 6 and 7 can be expressed as the following equations using the relation expressed as Equation 13.

$$\{\Delta W_n\}_k = -[I]\{f_{w,k}(\alpha_m, p_m)\} - [I]\left[\frac{\partial f_{w,k}}{\partial \alpha_m}\right]\{\Delta\alpha_m\} - [I]\left[\frac{\partial f_{w,k}}{\partial p_{m,k}}\right][I]^T\{\Delta P_n\}_k - \left\{V_n \frac{d\rho_n}{dt}\right\}_k \quad \text{(Eq. 14)}$$

$$\{\Delta X_n\}_k = -[I]\{f_{x,k}(\alpha_m, p_m)\} - [I]\left[\frac{\partial f_{x,k}}{\partial \alpha_m}\right]\{\Delta\alpha_m\}_k - [I]\left[\frac{\partial f_{x,k}}{\partial p_{m,k}}\right][I]^T\{\Delta P_n\}_k + \{S_n\}_k - \left\{V_n \frac{d(\rho C)}{dt}\right\}_k \quad \text{(Eq. 15)}$$

Here, $\left[\dfrac{\partial f_{w,k}}{\partial \alpha_m}\right]$, $\left[\dfrac{\partial f_{w,k}}{\partial p_{m,k}}\right]$, $\left[\dfrac{\partial f_{x,k}}{\partial \alpha_m}\right]$ and $\left[\dfrac{\partial f_{x,k}}{\partial p_{m,k}}\right]$ are diagonal matrices of size N.

Using the relations expressed as Equation 1 through 4, Equations 14 and 15 can be expressed as following equations.

$$\{\Delta W_n\}_k = -[I]\left[\frac{\partial f_{w,k}}{\partial \alpha_m}\right]\{\Delta \alpha_m\} - [I]\left[\frac{\partial f_{w,k}}{\partial p_{m,k}}\right][I]^T\{\Delta P_n\}_k \qquad \text{(Eq. 16)}$$

$$\{\Delta X_n\}_k = -[I]\left[\frac{\partial f_{x,k}}{\partial \alpha_m}\right]\{\Delta \alpha_m\}_k - [I]\left[\frac{\partial f_{x,k}}{\partial p_{m,k}}\right][I]^T\{\Delta P_n\}_k \qquad \text{(Eq. 17)}$$

By setting up Equations 16 and 17 for K temperature and concentration states and N rooms and by rearranging the terms, we can obtain the following equation.

$$\begin{Bmatrix} \{\Delta W_n\}_{k=1} \\ \vdots \\ \{\Delta W_n\}_{k=K} \\ \{\Delta X_n\}_{k=1} \\ \vdots \\ \{\Delta X_n\}_{k\sim K} \end{Bmatrix} = \begin{bmatrix} [A_{w,k=1}] & [B_{w,k=1}] & [0] & [0] & \cdots & [0] \\ [A_{w,k=2}] & [0] & [B_{w,k=2}] & [0] & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ [A_{w,k=K}] & [0] & [0] & [0] & \cdots & [B_{w,k=K}] \\ [A_{x,k=1}] & [B_{x,k=1}] & [0] & [0] & \cdots & [0] \\ [A_{x,k=2}] & [0] & [B_{x,k=2}] & [0] & \cdots & [0] \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ [A_{x,k=K}] & [0] & [0] & [0] & \cdots & [B_{x,k=K}] \end{bmatrix} \begin{Bmatrix} \{\Delta \alpha_m\} \\ \{\Delta P_n\}_{k=1} \\ \vdots \\ \{\Delta P_n\}_{k=K} \end{Bmatrix} \qquad \text{(Eq. 18)}$$

where:

$$[A_{w,k}] = -[I]\left[\frac{\partial f_{w,k}}{\partial \alpha_m}\right], \quad [B_{w,k}] = -[I]\left[\frac{\partial f_{w,k}}{\partial p_{m,k}}\right][I]^T,$$

$$[A_{x,k}] = -[I]\left[\frac{\partial f_{x,k}}{\partial \alpha_m}\right], \qquad [B_{x,k}] = -[I]\left[\frac{\partial f_{x,k}}{\partial p_{m,k}}\right][I]^T .$$

The vector on the left side of Equation 18 represents the errors in the mass balance and tracer-gas concentration balance for N rooms at K different temperature and concentration states. The size of the vector is 2KN. The size of the matrix on the right side is 2KN × (KN + M). In Equation 18, if we eliminate all elements except those related to the KN + M balance equations for the air mass and tracer-gas concentration (selected in step I), the size of the vector on the left size becomes KN + M, while that on the right side becomes (KN + M) × (KN + M). This can be expressed as follows.

$$\{V_{\Delta W,\Delta X}\} = [D]\{V_{\Delta\alpha,\Delta P}\} \qquad \text{(Eq. 19)}$$

$\{V_{\Delta\alpha,\Delta P}\}$ can be obtained by multiplying both sides of Equation 19 by the inverse of matrix $[D]$.

$$\{V_{\Delta\alpha,\Delta P}\} = [D]^{-1}\{V_{\Delta W,\Delta X}\} \qquad \text{(Eq. 20)}$$

Using $\{\Delta\alpha_m\}$ and $\{\Delta P_n\}_k$ obtained using Equation 20, $\{\hat{\alpha}_m\}$ and $\{\hat{P}_n\}_k$ can be adjusted using Equation 21 and 22.

$$\{\hat{\alpha}_{m,new}\} = \{\hat{\alpha}_m\} - r_\alpha\{\Delta\alpha_m\} \qquad \text{(Eq. 21)}$$

$$\{\hat{P}_{n,new}\}_k = \{\hat{P}_n\}_k - r_P\{\Delta P_n\}_k \qquad \text{(Eq. 22)}$$

Here, r_α and r_P are constants used to adjust the discharge coefficient and pressure (set to 0.6 in the present research).

Steps II through VIII are repeated until the completion of calculations for all systems of nonlinear equations chosen in step I. The calculated discharge coefficient of each wall may vary with the specific system of nonlinear equations considered. For each wall, the discharge coefficient is estimated as the average of the obtained values.

Examining the validity of the estimation method through a numerical experiment

The validity of the proposed method for estimating effective flow areas is verified by means of a numerical experiment. Figure 2 shows an outline of the numerical experiment. The shape and discharge coefficients of each wall of the building model used in the experiment, the volume of the tracer gas supplied, and the generated heat are provided as inputs to an air flow network calculating program [4] in order to simulate the temperature and tracer-gas concentration in each room. The temperature and tracer-gas concentration values obtained in the simulation are input into a program that uses the effective flow area estimating method and determines the discharge coefficient of each wall. The discharge coefficient values provided as input to the air flow network calculating program and those estimated using the present estimation method are compared for each wall.

Figure 3 shows a building model considered in a numerical experiment. The building contains four rooms, and eight walls that have air leakage areas. The discharge coefficients of the walls provided as inputs to the air flow network calculating program [4] are shown in parenthesis of Figure 3. Table 1 shows the three conditions considered for heat generation and the supply of tracer gas.

Figures 4 and 5 show changes in the temperature and tracer-gas concentration for the three conditions. The flat parts of the curves shown in the Figures are considered to represent steady states and the values are listed in Table 2.

The values of the steady-state temperature and tracer-gas concentration corresponding to the three conditions are input into the program that uses the effective flow area estimation method proposed here, and the discharge coefficient of each wall is obtained. Figure 6 shows the estimated discharge coefficient of each wall. The estimated value is obtained as the average of the discharge coefficients of the wall, which depend on the specific system of nonlinear equations considered. The same figure shows the minimum and maximum values of the discharge coefficient calculated by considering different systems of nonlinear equations. The average, minimum, and maximum values of each wall almost coincide with the true value that is shown in parenthesis of Figure 3 (the value provided as input to the air flow network calculating program). The maximum difference between the maximum or minimum value and the true value is 7.2%, and the maximum difference between the average and true value is 2.9%.

The total number of combinations of the system of nonlinear equations was 10,626. Out of these systems, 6,653 systems yielded solutions. Of the remaining, 49 were linearly dependent, while 3,924 did not converge and did not yield solutions. If a method can be found to select systems of nonlinear equations that yield solutions, there will be no need to solve all systems. Future studies should focus on the development of such a method.

Summary and conclusions

We present a method for estimating effective flow areas not only in the external wall of a house but also in the internal walls between rooms using only one type of tracer gas. The discharge coefficient of each wall that has air leakage areas and the pressure in each room—which are the unknown variables—are determined using nonlinear simultaneous equations, which consist of balance equations for the air mass and tracer-gas concentration in the rooms. The nonlinear simultaneous equations are solved using Newton's method.

To verify the validity of this method, we performed a numerical experiment. We defined the values of the discharge coefficient of each wall, and then simulated changes in the temperature and tracer-gas concentration. We then estimated the discharge coefficient of each wall from the simulated temperature and concentration values using the method. It was confirmed that the estimated discharge coefficient of each wall was equivalent to the defined value. Thus, the validity of this method was verified.

Acknowledgments

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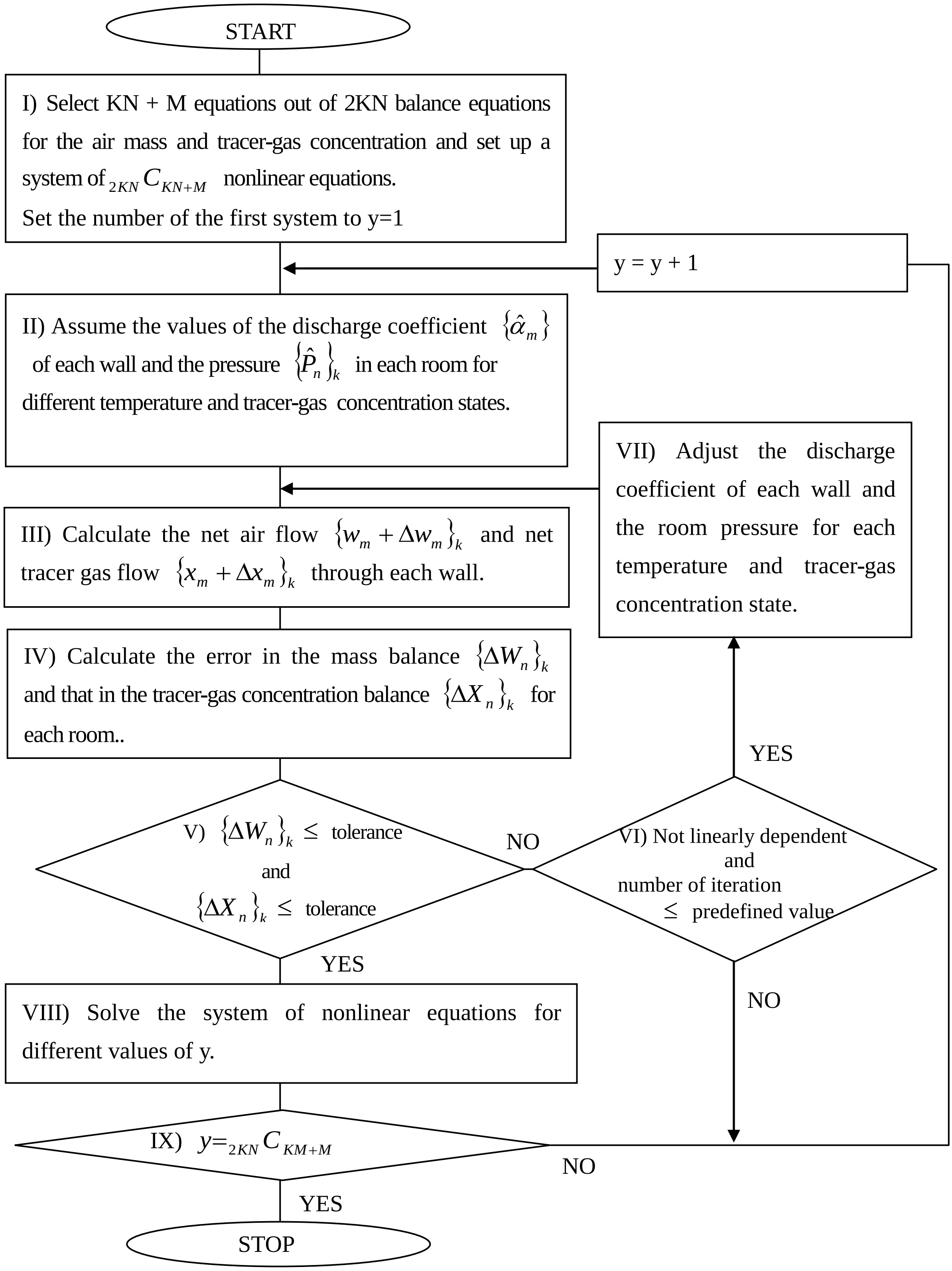


Fig. 1 Flow chart of the method used to estimate discharge coefficients

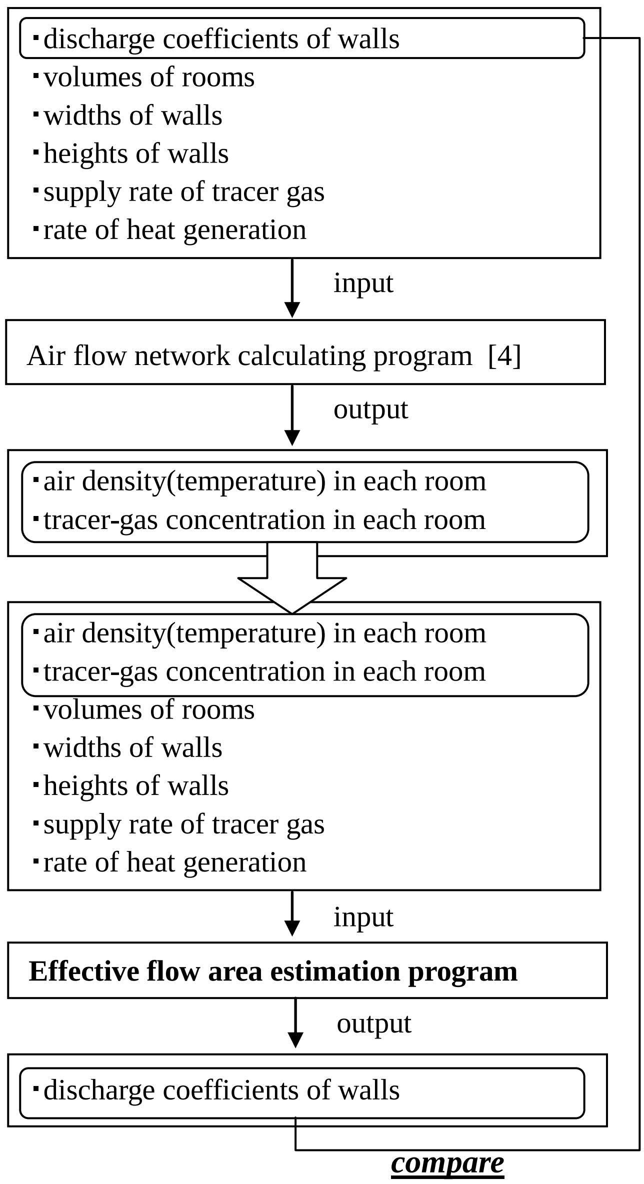


Fig. 2 Outline of numerical experiment

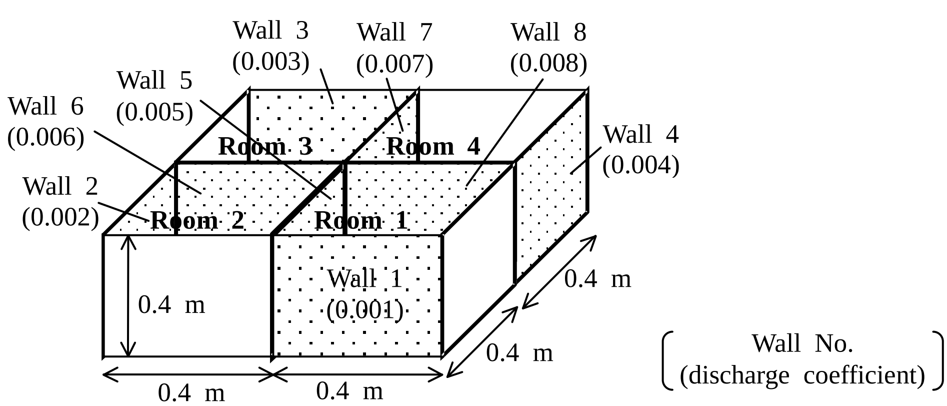
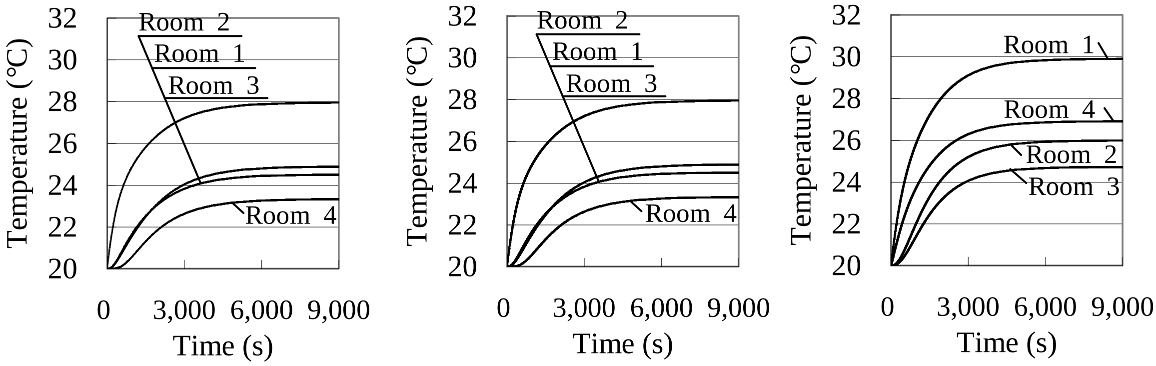
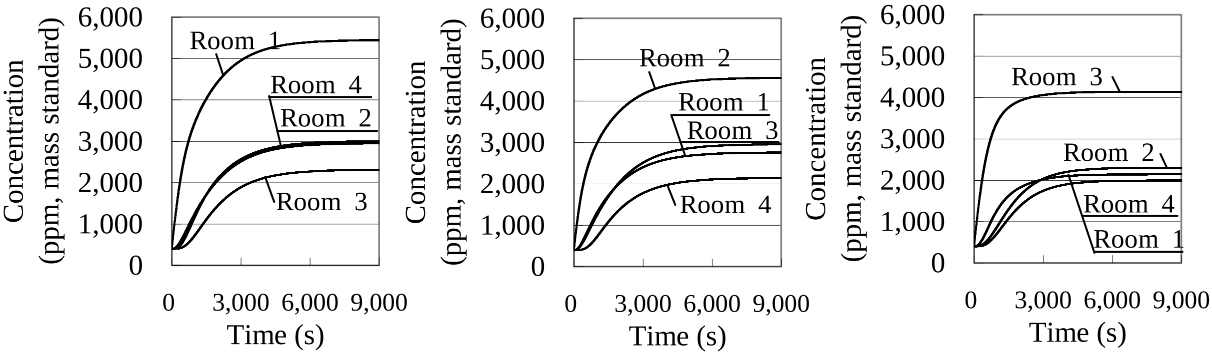


Fig. 3 Building model considered in numerical experiment



(a) Condition 1 (b) Condition 2 (c) Condition 3

Fig. 4 Temperature



(a) Condition 1 (b) Condition 2 (c) Condition 3

Fig. 5 Tracer-gas concentration

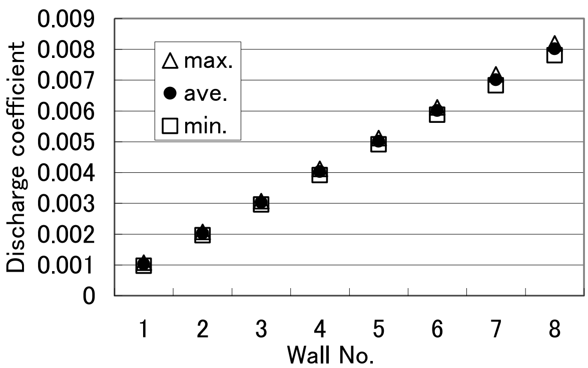


Fig. 6 Estimated discharge coefficients

Table 1 Conditions for heat generation and supply of tracer gas

	Rate of heat generation (W)				Supply rate of tracer gas (g/h)			
	Room 1	Room 2	Room 3	Room 4	Room 1	Room 2	Room 3	Room 4
Condition 1	-	0.8	-	-	1.5	-	-	-
Condition 2	-	0.8	-	-	-	1.5	-	-
Condition 3	0.8	-	-	0.4	-	-	1.5	-

Table 2 Values of the steady-state temperature and tracer-gas concentration

	Temperature (℃)				Tracer-gas concentration (ppm) (mass standard)			
	Room 1	Room 2	Room 3	Room 4	Room 1	Room 2	Room 3	Room 4
Condition 1	24.88	27.95	24.50	23.32	5,443	2,951	2,310	2,999
Condition 2	24.88	27.95	24.50	23.32	2,956	4,558	2,757	2,140
Condition 3	29.89	25.98	24.71	26.90	1,992	2,298	4,134	2,142