Temperature and pressure corrections for power-law coefficients of airflow through ventilation system components and leaks

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ABSTRACT

The characterization of power-law coefficients of the airflow through ventilation system components and ductwork or building leaks should include corrections on the airflow rate measurement because of two phenomena: a) the temperature and pressure conditions at the flow measurement device may not be the same as those seen by the test object; b) the temperature and pressure conditions experienced by the object may differ from reference conditions. This paper gives the analytical expression of these corrections depending on the air viscosity, air density and flow exponent. Corrections may be significant in possible test conditions and therefore should be applied systematically when performing such measurements.

KEYWORDS

Airflow measurement, ventilation, building, airtightness, leak

NOMENCLATURE

A	Cross-sectional area (m^2)
C	Airflow rate coefficient (power-law coefficient) ($m^3\ s^{-1}\ Pa^{-n}$)
C_d	Discharge coefficient (-)
k	Airflow rate coefficient (power-law coefficient) ($m^{2-3/n}\ s^{-2+1/n}$)
L	Length (m)
n	Flow exponent (-)
p	Relative pressure (Pa)
p_{atm}	Absolute pressure (Pa)
q_v	Volumetric airflow rate ($m^3\ s^{-1}$)

Greek symbols

α, β, γ	Real numbers (-)
Δp	Pressure difference (Pa)
μ	Dynamic viscosity of the air ($kg\ m^{-1}\ s^{-1}$)
ρ	Air density ($kg\ m^{-3}$)

Subscripts and superscripts

i	Pertaining to inside of ductwork or envelope where test object is located
e	Pertaining to outside of ductwork or envelope where test object is located
max	Maximum value
$meas$	At airflow measurement device

object	Pertaining to test object
test	Pertaining to conditions near the test object
0	Pertaining to reference temperature and pressure conditions

Physical constants
$T_0 = 293{,}15$ K
$p_{atm;0} = 101325$ Pa
$\rho_0 = 1.204$ kg m^{-3}

1 INTRODUCTION

Ventilation system components and leaks in ductwork or buildings are commonly characterized by a power-law with parameters that may be determined experimentally by creating artificially a series of pressure differences across the components or leaks. There may be differences either:

- between the temperature observed at the flow measurement device and the temperature experienced by the tested object;
- between the temperature experienced by the tested object and the temperature in reference conditions.

Therefore, to be able to compare test results, one shall apply corrections to account for both effects described above. There are a number of standards that propose such corrections, but there are differences between those.

The objective of this paper is to give the common background for these corrections. The analyses shown here apply when mass conservation between the test object and the flow measurement device applies.

2 BACKGROUND

We may divide the types of methods used to characterize ventilation system components or leaks in 4 major categories:

a. Envelope or ductwork depressurisation tests
b. Envelope or ductwork pressurisation tests
c. Ventilation system component tests with measurement device downstream
d. Ventilation system component tests with measurement device upstream

Common application of these methods is represented in Figure 1. This figure shows that the conditions seen by the test object are different depending on the method. Note however that all methods implicitly assume that there is no significant temperature and absolute pressure change across the test object, i.e., that the object can be characterized at the temperature and absolute pressure conditions of the air entering the object.

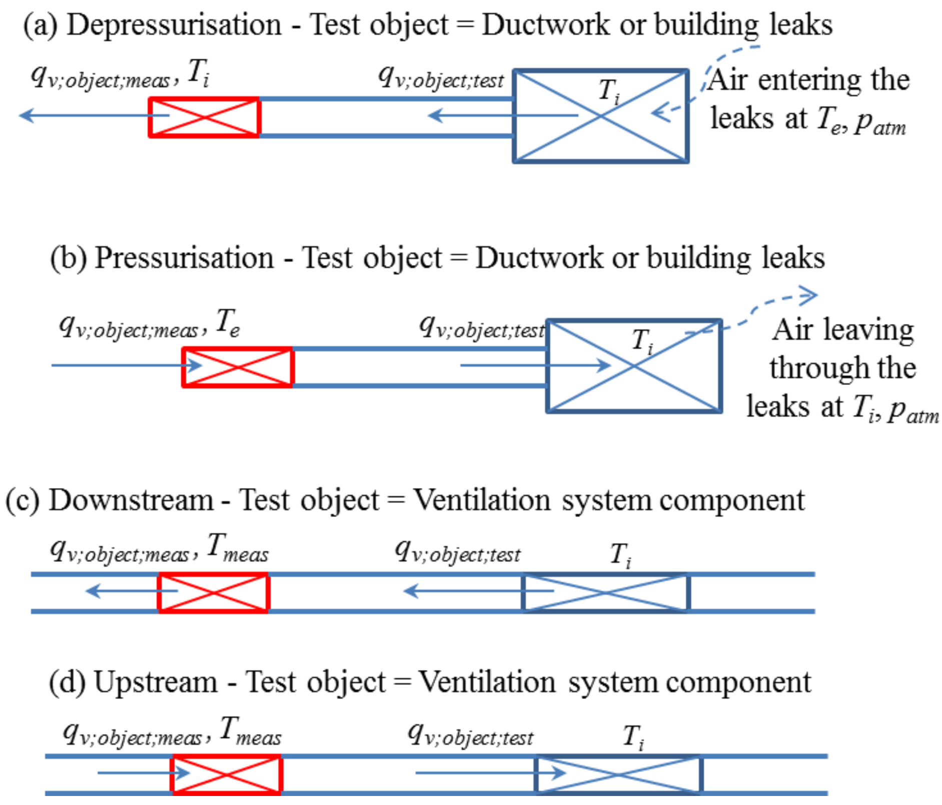

Figure 1: Common application of methods used to characterize ventilation system components or leaks

3 PRESSURE-FLOW RELATIONSHIP

The relationship between flow and pressure through a ventilation system component or through leak(s) is commonly expressed as follows:

$$q_{v;object;test} = C_{object;test} \cdot \Delta p^{n} \qquad (1)$$

Where

$q_{v;object;test}$	is the airflow rate through the test object ($m^3\ s^{-1}$)
$C_{object;test}$	is the flow coefficient that characterizes the test object ($m^3\ s^{-1}\ Pa^{-n}$)
Δp	is the pressure across the test object (Pa)
n	is the flow exponent ($0.5 \leq n \leq 1.0$)

The coefficient C depends on the only physical characteristics of the system, which are:

- The dynamic viscosity of the air, μ ($kg\ m^{-1}\ s^{-1}$)
- The air density, ρ ($kg\ m^{-3}$)
- The characteristic length of the system, L (m)

Therefore, C is a combination of terms of the form: $\mu^{\alpha} \cdot \rho^{\beta} \cdot L^{\gamma}$ where α, β, γ are unknown exponents. Dimensional homogeneity allows us to write the following system[1]:

[1] Note that $1\ Pa = 1\ N\ m^{-2} = 1\ kg\ m^{-1}\ s^{-2}$

Dimension	C		μ	ρ	L
Length (m)	$3+n$	=	$-\alpha$	-3β	γ
Time (s)	$2\,n-1$	=	$-\alpha$		
Mass (kg)	$-n$	=	α	β	

(2)

Therefore:

$$\begin{aligned} \alpha &= 1-2n \\ \beta &= n-1 \\ \gamma &= 2n+1 \end{aligned} \quad (3)$$

Leading to:

$$C \propto \mu^{1-2n}\,\rho^{n-1}\,L^{2n+1} \quad (4)$$

Note that equation (4) is consistent with the correction proposed by Walker (1998) and ASTM E 779-2010.

When n = 0.5, we find:

$$C \propto \rho^{-0.5}\,L^{2} \quad (5)$$

In this particular case (n = 0.5), proportionality is consistent with the commonly used relationship for an inertial flow through a perfect orifice:

$$q_v = C_d \cdot A \cdot \left(\frac{2\,\Delta p}{\rho}\right)^{0.5} \quad (6)$$

Another common expression of the flow-pressure relationship is:

$$\Delta p = k \cdot \rho \cdot q_v^{1/n} \quad (7)$$

Similarly, we obtain:

$$k \propto \mu^{2-\frac{1}{n}}\,\rho^{\frac{1}{n}-2} \quad (8)$$

4 MASS CONSERVATION THROUGH THE TEST OBJECT

The temperature and pressure conditions around the test object are not necessarily the same as the ones near the airflow measurement device. Therefore, mass conservation gives:

$$q_{v;object;test} = \frac{\rho_{meas}}{\rho_{test}} q_{v;object;meas} \quad (9)$$

5 FLOW COEFFICIENT CORRECTION

To allow comparison between tests, we are interested in the flow through the test object under reference conditions ("0") for the same pressure difference, Δp. Therefore:

$$q_{v;object;test} = C_{object;test} \cdot \Delta p^n \tag{10}$$

and

$$q_{v;object;0} = C_{object;0} \cdot \Delta p^n \tag{11}$$

If n is unknown, $C_{object;test}$ and n may be determined together with a regression analysis based on equation (10). (Note that this implicitly assumes that all test points are obtained under similar temperature and pressure conditions.) Then, using equation (4) allows one to obtain the airflow rate coefficient at reference temperature and pressure conditions:

$$\frac{C_{object;0}}{C_{object;test}} = \left(\frac{\mu_{test}}{\mu_0}\right)^{2n-1} \left(\frac{\rho_{test}}{\rho_0}\right)^{1-n} \tag{12}$$

This also yields:

$$q_{v;object;0} = \frac{C_{object;0}}{C_{object;test}} \frac{\rho_{meas}}{\rho_{test}} q_{v;object;meas} \tag{13}$$

and therefore:

$$q_{v;object;0} = \left(\frac{\mu_{test}}{\mu_0}\right)^{2n-1} \left(\frac{\rho_{test}}{\rho_0}\right)^{1-n} \frac{\rho_{meas}}{\rho_{test}} q_{v;object;meas} \tag{14}$$

Note that when $n = 0.5$, the air viscosity vanishes from equation (14).

6 APPLICATION TO PRESSURISATION TESTS WITH UNKNOWN FLOW EXPONENT

This paragraph is relevant to tests performed at multiple pressure stations to obtain both the airflow rate coefficient and the flow exponent. In this case, we proceed with the following steps:

- Apply mass conservation correction equation (9) to obtain the air leakage flow rate through the test object for various pressure differences;
- Extract $C_{object;test}$ and n from the values of $q_{v;object;test}$ obtained;
- Apply equation (12) to characterize the envelope or ductwork airtightness in reference conditions.

In depressurisation mode, the air entering the measurement device is inside air (so at temperature T_i) while the air passing through the leaks is at temperature T_e (see Figure 1). Therefore:

$$q_{v;object;test} = \frac{\rho_i}{\rho_e} q_{v;meas} \tag{15}$$

$$C_{object;0} = \left(\frac{\mu_e}{\mu_0}\right)^{2n-1} \left(\frac{\rho_e}{\rho_0}\right)^{1-n} C_{object;test} \tag{16}$$

In pressurisation mode, we obtain:

$$q_{v;object;test} = \frac{\rho_e}{\rho_i} q_{v;meas} \tag{17}$$

$$C_{object;0} = \left(\frac{\mu_i}{\mu_0}\right)^{2n-1} \left(\frac{\rho_i}{\rho_0}\right)^{1-n} C_{object;test} \tag{18}$$

These equations are identical to those used in ASTM E 779-2010. They differ from ISO 9972 where viscosity dependence is neglected.

7 APPLICATION TO PRESSURISATION TESTS WITH DEFAULT FLOW EXPONENT

This paragraph is relevant either:

- When the measurement is performed at only one pressure station;
- When the leakage airflow rate(s) at one or several pressure stations is (are) compared to a threshold value.

In both cases, the exponent is set to a default value and the underlying criterion is:

$$q_{v;object;0} \leq C_{max} \cdot \Delta p^n \tag{19}$$

Therefore, we will need to derive $q_{v;object;0}$ to check this criterion.

In pressurisation mode:

$$q_{v;object;0} = \left(\frac{\mu_i}{\mu_0}\right)^{2n-1} \left(\frac{\rho_i}{\rho_0}\right)^{1-n} \frac{\rho_e}{\rho_i} q_{v;object;meas} \tag{20}$$

In depressurisation mode:

$$q_{v;object;0} = \left(\frac{\mu_e}{\mu_0}\right)^{2n-1} \left(\frac{\rho_e}{\rho_0}\right)^{1-n} \frac{\rho_i}{\rho_e} q_{v;object;meas} \tag{21}$$

8 APPLICATION TO CHARACTERIZATION OF VENTILATION SYSTEM COMPONENTS

Equation (9) is used to obtain $q_{v;object;test}$. However, unlike in envelope or ductwork pressurisation tests, the temperature of the air flowing through the measurement device is specific to the test apparatus. It may not be assumed in all cases that $T_{meas} = T_i$ or $T_{meas} = T_e$ as we did previously: this may or may not be true depending on where the air flowing through the measurement device comes from.

As for the airflow rate or the airflow coefficient, because the test conditions seen by the object are those inside the ducts in which the object is placed, we may write:

$$q_{v;object;0} = \left(\frac{\mu_i}{\mu_0}\right)^{2n-1} \left(\frac{\rho_i}{\rho_0}\right)^{1-n} \frac{\rho_{meas}}{\rho_i} q_{v;object;meas} \tag{22}$$

9 TEMPERATURE AND PRESSURE DEPENDENCE OF AIR VISCOSITY AND AIR DENSITY

To apply those corrections, it is useful to know the how the air viscosity and air density vary with temperature and absolute pressure. When the humidity can be neglected, the following correlations may be used:

$$\rho = 1.204 \cdot \frac{p_{atm}}{p_{atm;0}} \frac{T_0}{T} \tag{23}$$

and

$$\mu = \left(17{,}1 + 0{,}048(T - 273{,}15)\right) \cdot 10^{-6} \tag{24}$$

Giving $\mu_0 = 18.06$ 10^{-6} kg m^{-1} s^{-1}

Of course, depending on the accuracy needed and test conditions, other correlations may be used, such as those proposed in ASTM E779 (2010) and CR 14738 (2002).

10 NUMERICAL APPLICATION

To see the magnitude of the changes induced by those corrections, we have calculated the airflow rate correction defined as follows in various test conditions:

For equations (20) and (21): Airflow correction $= \frac{q_{v;measured} - q_{v;0}}{q_{v;0}}$ (25)

For equation (22): Airflow correction $= \frac{q_{v;object} - q_{v;0}}{q_{v;0}}$ (26)

The results show relatively high deviations in possible test conditions (see Table 1 and Table 2).

				Airflow correction assuming $\rho_{meas} = \rho_i$	
p_{atm} (Pa)	ϑ_i (°C)	ρ_i (kg m^{-3})	μ_i (kg m^{-1} s^{-1})	Eq. (22), *n* = 0,5	Eq. (22), *n* = 0,65
101325	0	1,29	$17{,}1 \cdot 10^{-6}$	-3,5%	-0,8%
101325	5	1,27	$17{,}3 \cdot 10^{-6}$	-2,6%	-0,6%
101325	10	1,25	$17{,}6 \cdot 10^{-6}$	-1,7%	-0,4%
101325	15	1,22	$17{,}8 \cdot 10^{-6}$	-0,9%	-0,2%
101325	20	1,20	$18{,}1 \cdot 10^{-6}$	0,0%	0,0%
101325	25	1,18	$18{,}3 \cdot 10^{-6}$	0,8%	0,2%
101325	30	1,16	$18{,}5 \cdot 10^{-6}$	1,7%	0,4%
90000	0	1,15	$17{,}1 \cdot 10^{-6}$	2,4%	3,4%
90000	5	1,13	$17{,}3 \cdot 10^{-6}$	3,4%	3,6%
90000	10	1,11	$17{,}6 \cdot 10^{-6}$	4,3%	3,8%
90000	15	1,09	$17{,}8 \cdot 10^{-6}$	5,2%	4,0%
90000	20	1,07	$18{,}1 \cdot 10^{-6}$	6,1%	4,2%
90000	25	1,05	$18{,}3 \cdot 10^{-6}$	7,0%	4,4%
90000	30	1,03	$18{,}5 \cdot 10^{-6}$	7,9%	4,6%

Table 1: Airflow correction (in %) derived from equation (22).

		Airflow correction assuming ϑ_e = 10 °C			
p_{atm} (Pa)	ϑ_i (°C)	Eq. (20), *n* = 0,5	Eq. (20), *n* = 0,65	Eq. (21), *n* = 0,5	Eq. (21), *n* = 0,65
101325	0	0,1%	2,8%	-5,2%	-3,9%
101325	5	-0,8%	1,2%	-3,5%	-2,2%
101325	10	-1,7%	-0,4%	-1,7%	-0,4%
101325	15	-2,6%	-1,9%	0,0%	1,4%
101325	20	-3,4%	-3,4%	1,8%	3,1%
101325	25	-4,2%	-4,8%	3,5%	4,9%
101325	30	-5,0%	-6,2%	5,2%	6,6%
90000	0	6,2%	7,2%	0,6%	0,1%
90000	5	5,2%	5,5%	2,4%	2,0%
90000	10	4,3%	3,8%	4,3%	3,8%
90000	15	3,4%	2,2%	6,1%	5,6%
90000	20	2,5%	0,7%	8,0%	7,5%
90000	25	1,6%	-0,8%	9,8%	9,3%
90000	30	0,8%	-2,3%	11,6%	11,1%

Table 2: Airflow correction derived from equations (20) and (21).

11 CONCLUSION

Our analyses show that because the airflow rate coefficient depends on air viscosity, air density and flow exponent, the corrections that have to be applied to airflow rates through ventilation system components and leaks to characterize these objects depend on these variables as well. However, when the flow is inertial (*n* = 0.5), these corrections do not depend on air viscosity. Still, the corrections may be significant, which calls for including these effects systematically when such measurements are performed.

12 REFERENCES

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