AN ANALYSIS OF STOCHASTIC PROPERTIES OF ROOM AIR TEMPERATURE AND HEATING LOAD - INFLUENCES OF RANDOMNESS OF PARAMETERS

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ABSTRACT

This paper discusses the use of a building thermal analysis methodology in which the stochastic nature of the external climates and randomness of physical parameters are considered. Methods of thermal calculation which give the density function of the room air temperature and heating and/or cooling loads are proposed. Weather data is modeled by linear time series models with white noises as inputs, which take into account the auto-correlations and cross-correlations of the raw climatic data. The basic equations are the simultaneous ordinary difference (or differential) equations, that is, the state space equations. A set of moment equations are derived from these state space equations and solved to provide with the mean and standard deviation in room air temperature and heating load.

Particularly, the impact of random infiltration rate and thermophysical properties (here, thermal conductivity of the wall and heat capacity of the room) on room temperature and heating load are emphasized in this paper. Furthermore, methods of analysis that give the influence of the random variation in internal heat generation and the optimal starting time of an HVAC system in intermittent heating, are outlined. Simple example calculations are shown for illustrations.

For the analysis and design of a thermal system, this method provides a rational and convenient way of handling uncertainties caused by the degree of imprecision of construction works (workmanship) and the variation of the way in which the room is used.

1. INTRODUCTION

Environmental problems have become some of the most serious and urgent problems in our era. Energy consumption by industries and buildings is partly responsible for this problem. Thus, buildings and HVAC systems must be designed reasonably well enough so that energy can be used effectively.

Exact prediction of the heating and cooling load, proper sizing and the optimal control of the HVAC systems are essential and crucial to minimize energy consumption. Integrated Part Load Values (IPLV, ASHRAE Standard 90.1 1989) are one of the typical examples in this direction.

To correct size an air-conditioning system, information on maximum heating/cooling load is needed. Furthermore, in order to save on operating costs or reduce energy consumption, the system must be operated at or near the optimum state, thus keeping the efficiency of the air-conditioning system high. Therefore, to optimize building and air-conditioning system design, it is important to have knowledge of not only the maximum heating load but also its distribution.

The thermal performance of a building is a function of solar radiation, outdoor temperature and humidity, among other input parameters. Since external climatic conditions fluctuate randomly, it is necessary to take the randomness of the external variables into account when determining the maximum load and the load distribution. In other words, external climate must be regarded as a stochastic time series (Terai et al. 1978; Nakazawa et al. 1983; Madsen 1985; Haghighat et al. 1987; Hokoi and Matsumoto 1988; Jiang and Hong 1993).

From this point of view, a stochastic method of thermal analysis was proposed (Hokoi and Matsumoto 1988; Hokoi, et al. 1990 a, b) which gives the maximum and the distribution of heating/cooling loads under intermittent heating. Analysis considering non-linear properties of HVAC systems, and the influence of the non-Gaussianity of the external climates have also been investigated in other studies (Hokoi and Matsumoto 1993; Hokoi and Matsumoto 1995). In these analyses, physical parameters such as air infiltration rate and thermal properties were dealt with as constant.

This paper discusses the use of a building thermal analysis methodology in which the randomness of physical parameters is considered. The randomness of the physical parameters should be examined, partly because every house constructed even under the same conditions may have different parametric values depending on the degree of imprecision in workmanship, and partly because physical properties, such as thermal conductivity and thermal capacity, change over time depending on the way in which the room is used. Under these circumstances, these properties should be regarded as random and thus investigated from a standpoint of assembly average. This result makes possible thermal design that takes into account the influences borne by the degree of imprecision in construction work and the variation in the way in which the room is used (Nielsen 1995).

2. INFLUENCE OF RANDOM THERMAL PROPERTIES

In this section, the stochastic characteristics of room air temperature and heating load are investigated when the thermal conductivity of the walls and the thermal capacity of the room are random variables.

2-1 Method of Analysis

There are several methods that can deal with the random nature of parameters such as the thermal properties of the walls. In one of them, random parameters are regarded as new state variables (Soong 1973; Terai et al. 1978). As can be seen later in this paper (for example, Equation (28) in section 3-3), however, the terms with the higher moments appear in a complicated manner in moment equations. Thus, special consideration or approximation is required to resolve this problem.

In this study, the following method based on a linear system was adopted for its easiness and simplicity. Let $f(\mathbf{x}, \mathbf{a})$ be the joint density function of the state vector **x** and parameter vector **a**, wherein the parameter vector **a** and the external climates are assumed random. The state vector **x** represents, for example, a room air temperature and wall temperatures, while the parameter vector **a** represents the thermal conductivity of the walls and heat capacity of the room. Thus, the probability density function of **x, f(x)** can be calculated as follows, when the external climates changes randomly and the parameter **a** is also random.

The following relationship,

$$f(\mathbf{x}, \mathbf{a}) = f(\mathbf{x} \mid \mathbf{a})\ f(\mathbf{a}) \qquad (1)$$

is valid between $f(\mathbf{x}, \mathbf{a})$ and $f(\mathbf{x} \mid \mathbf{a})$, where $f(\mathbf{x} \mid \mathbf{a})$ is the conditional density function of **x** given **a**.

Thus,

$$f(\mathbf{x}) = \int f(\mathbf{x}, \mathbf{a})d\mathbf{a} = \int f(\mathbf{x} * \mathbf{a})f(\mathbf{a})d\mathbf{a} \qquad (2)$$

In Equation (2), the probability density function of parameter **a**, f(**a**), is given. Furthermore, since f(**x * a**) is the density function of **x** under the condition that **a** is given or constant, it can be easily obtained by the linear calculation described in the next section 2-2. Since this probability distribution is Gaussian, f(**x**) can be computed without difficulty by making use of Equation (2).

2-2 Stochastic Method of Thermal Analysis

Since the procedures are described in detail by Hokoi et al. (1990a), the results are described briefly for the present purpose in what follows.

We shall examine the means and variances of the cooling load and room air temperature caused by randomly varying solar radiation and outdoor temperature. Since the thermal system treated here is linear and the stochastic distributions of the random components of the solar radiation and the outdoor temperature are normal (Hokoi et al. 1990b), the distribution of the cooling load and room air temperature are normal distributions as well (Kwakernaak and Sivan 1972). Thus, it is sufficient to have their means and variances.

Solar Radiation and Outdoor TemperatureRadiation and Outdoor Temperatureand Outdoor Temperature

We proposed a statistical method to generate synthetic weather data, outdoor temperature and solar radiation, taking into account the correlation (Hokoi et al. 1990b). The solar radiation, J(j), and the outdoor temperature, $T_o(j)$, are given by the following equations.

$$J(j) = \sigma_j(j)JN(j) + J_p(j) \qquad (3)$$

$$T_o(j) = T_oN(j) + T_{op}(j) \qquad (4)$$

wherein;

$J_p(j)$, $T_{op}(j)$ = deterministic components of solar radiation and outdoor temperature, respectively (period 24 h)

JN(j), $T_oN(j)$ = random components of solar radiation and outdoor temperature, respectively

$\sigma_j(j)$ = diurnal standard deviation of solar radiation (deterministic function with period 24 h) j is the discrete time and the time increment Δt is 1 h. Equations (3) and >4) mean that the solar radiation and the outdoor temperature are composed of the deterministic components $J_p(j)$, $T_{op}(j)$, and the random components JN(j) and T_o N(j).

The random components JN(j) and T_o N(j) could be generally expressed by the ARMA and ARMAX models (ARMA model with exogenous term) (Box and Jenkins 1976; Hittle and Pederson 1981; Hokoi et al. 1990b; Yoshida and Terai 1992). For summer in Tokyo (1962), these equations are:

$$JN(j) = Z^j = a_1Z^{j-1} + a_2Z^{j-2} + e^j + b_1e^{j-1} + b_2e^{j-2} + b_3e^{j-3} \qquad (5)$$

$$T_oN(j) = Y^j = c_1Y^{j-1} + c_2Y^{j-2} + d_0\sigma_J(j)Z^j + d_1\sigma_J(j-1)Z^{j-1} + d_2\sigma_J(j-2)Z^{j-2} + eN^j + g_1eN^{j-1} + g_2eN^{j-2} \qquad (6)$$

wherein, e^j and eN^j are mutually independent, discrete white noises with 0 mean and standard deviations σ_e and σ_eN, respectively. The derivation of Equations (5) and (6) and the values of the coefficients are given in Hokoi et al. (1990 b).

The ARMA (3,3) model of solar radiation and the ARMAX (2,3,2) model of outdoor temperature were confirmed to be satisfactory for characterizing actual weather data. Thus, these models, instead of actual weather data, can be used as inputs for energy analysis.

State Equations (Discretization)

The equations for heat flow in the wall, room air temperature and heat input used in this paper are described in Appendix 1 as Equations (A-3) to (A-7). These equations are discretized with respect to coordinates and time by explicit finite difference method (refer to Equation (16)). The discretized equation is as follows (Hokoi et al. 1990a).

$\mathbf{T}^{j+1} = [a]^j\mathbf{T}^j + [b]^j\mathbf{f}^j$ (7)

Wherein :

$\mathbf{T}^j = [T_{w1}{}^j\text{Y}, T_{wi}{}^j\text{Y}, T_{wN}{}^j, T_R{}^j, Q^j]^T$

$\mathbf{f}^j = [T_o{}^j, J^j, T_s{}^j]^T$

$[a]^j$, $[b]^j$ = matrices with (N+2) x (N+2), (N+2) x 3 elements, respectively.

Q = heat input by air-conditioning system

T_R = room air temperature

T_S = target temperature

T_{wi} = wall temperature of node point i

N = number of node points

Making use of the vector notation,

$\mathbf{X}^j = [Z^j, Z^{j-1}, Z^{j-2}, e^{j-1}, e^{j-2}, e^{j-3}, Y^j, Y^{j-1}, eN^{j-1}, eN^{j-2}, T_{w1}{}^j\text{Y}, T_{wi}{}^j\text{Y}, T_{wN}{}^j, T_R{}^j, Q^j]^T$

$= [x_i{}^j]^T$ (i = 1, Y, N+12) (8)

wherein the superscript T denotes transposition, Equations (3) through (7) are rewritten as follows.

$\mathbf{X}^{j+1} = [A]^jX^j + [B]^jF^j + [C]^je^j$

Wherein:

$F^j = [T_{op}{}^j, J_p{}^j, T_S{}^j]^T$

$e^j = [e^j, eN^j]^T$

$[A]^j$, $[B]^j$, $[C]^j$ = matrix with M x M, M x 3, M x 2 elements, respectively (M=N+12).

The vectors X^j, F^j, and e^j represent the state vector, the deterministic (average) components of the outdoor temperature and solar radiation, and the random components of them, respectively.

Derivation of Moment Equationsof Moment EquationsMoment Equations

Means

By averaging Equation (9), we get:

$E[\mathbf{X}^{j+1}] = [A]^jE[X^j] + [B]^jF^j$ (10)

where, E[C] represents the expectation operation.

Variance B Covariance functionsB Covariance functionsB Covariance functions

According to Kwakernaak and Sivan (1972), the variance function of $\mathbf{X}^j$ is:

$R^{j+1} = [A]^jR^j[A]^{jT} + [C]^jV^j[C]^{jT}$ (11)

where, R^j and V^j are variance matrices of $\mathbf{X}^j$ and the white noise vector $\mathbf{e}^j$, respectively. It is sufficient to solve Equations (10) and (11) under the appropriate initial condition. These equations can be solved by simple mathematical operations. By making use of the result, the conditional density function f(**x*a**) can be obtained.

2-3 Example Calculation

In order to illustrate the applicability of this method, a simple room construction is used as an example. It is not intended to be typical of real buildings but rather to provide examples showing the effectiveness of the calculation procedure.

(1) Data used in calculation

The characteristics of the building calculated are as follows.

Room dimensions = 2.5 x 8.0 x 5.0 [m;], V = 100 [m;], n = 1/3600 [1/s], Q_0=0[W], S_w =

140 [m5], wall thickness l_w = 0.12[m], a_w = 0.69 x 10^{-7} [m5/s],

S_g = 5 [m5], glass thickness l_g = 0.003[m], λ_g = 0.744 [W/mCK], K_g = 6.47 [W/m5CK],

A_s = 0.6[-], τ_g = 0.85 [-], α_o = 23.3 [W/m5CK], α_i = 9.3 [W/m5CK].

This room has a south-facing glazed window. The occupancy period is from 09:00 to 18:00 and the air-conditioning is started at 08:00. The target temperature is set at 26EC. Figures 1 and 2 show the mean and standard deviation of the external temperature and solar radiation.

The density function of the thermal conductivity λ_w is assumed to have a mean and a standard deviation of 1.63 W/mK and 0.35 W/mK, while that of the heat capacity $c\gamma$ 6290J/m;K and 1260 J/m;K, respectively. The mean value of the thermal conductivity, 1.63 W/mK, is frequently used as the standard value for concrete in thermal design. The minimum value of the heat capacity, 1260 J/m;K, is that of the air, while the mean value of 6290 J/m;K is adopted to represent the heat capacity equivalent to the wooden furniture, with a volume of 1 m;, in the room.

Results and discussion

Figures 3 and 4 show typical examples of the mean and standard deviation in cooling load and room air temperature in the case of λ_w = .63 W/mK and $c\gamma$ = 1260 J/m;K (without furniture). The room air temperature in the occupancy period is kept nearly constant at a set point temperature of 26EC. The standard deviation in heat input is about 30% of the mean value, and it can be said that the variance has a great influence on determining the capacity of the air-conditioning system. With this information (the values of the mean and standard deviation), one can design an air-conditioning system with different levels of confidence.

Figures 5 and 7 show the probability density functions of the room air temperature and cooling load at 12:00, corresponding to various values of thermal conductivity and capacity, respectively.

When the value of the thermal conductivity is varied from 0.6 W/mK to 2.4 W/mK and the heat capacity is fixed at 1260 J/m;K (Figure 5, upper one), the mean cooling increases from 4.60 kW to 7.73 kW, while the standard deviation varies from 1.45 kW to 2.51 kW. This increase in average cooling load is causes by the increase in heat flow through the external walls, since room air temperature is kept almost constant at 26EC. Needless to say, the scattering of the load indicated by the standard deviation becomes larger with thermal conductivity.

Figure 6 is the density function of the cooling load, considering the probability density of the thermal conductivity, that is, the result obtained by integrating the results shown in Figure 5 based on Equation (2). The cooling load varies in a wide range from 1 kW to 12 kW with a mean value of about 6.40 kW.

The mean value of the room temperature is about 26.3EC (Figure 5, lower one), slightly higher than the set point value. This is caused by integral control, where the room temperature becomes higher than the set point value for a while after the start of the air-conditioning as shown in Figure 3. The standard deviation in room air temperature is very small.

Figure 7 shows the result when the heat capacity is varied from 1260 J/m;K to 12600 J/m;K. There is very little variation in average cooling load, ranging from 6.59 kW to 6.66 kW. The standard deviation remains almost constant at 2.14 kW, thus the influence of heat capacity on load is negligible. This is because the effect of thermal capacity or thermal storage almost disappears at time 12:00 since the air-conditioning starts at 8:00. In fact, the mean value of the cooling load at 9:00 in the morning ranges from 1.45 kW to 1.59 kW, about a 10% change, which indicates the rather large influence of the heat capacity. The density function of the cooling load, taking into account the probability density of the heat capacity, is omitted because it is almost the same as Figure 7.

Thus far, a poorly insulated simple room construction has been used as an example in order to illustrate the applicability of the proposed method and to show clearly the influence of the random variation of thermal conductivity. Several example calculations, wherein more realistic room models with insulation are dealt with, are reported elsewhere (Hokoi et al. 1990 a). Figure 8 shows the results from that paper, where the room is insulated by a 30mm-thick fiberglass inside or outside of the exterior walls. It can be seen that both of the means and standard deviations of the cooling load are very small when compared to those in the case without the insulation. Outer insulation operates well to decrease the standard deviation in the room temperature during the unoccupied period. The influence of the randomness of the thermal conductivity on the cooling load may be negligible in the case with the insulation, although the ratio of the random component to the deterministic component is not small.

3. INFLUENCE OF RANDOM INFILTRATION RATE

In this section, the method of analysis giving the stochastic nature of the heating load and room temperature are shown when the infiltration rate changes randomly (Haghighat et al. 1987). Here, formulation for a continuous system is given.

3-1 Fundamental Equations

(1) Outdoor Temperature

Input outdoor temperature, $T_o(t)$, is assumed to change as follows (Hokoi and Matsumoto 1988).

$$T_o(t) = a_0 + 3\ (a_i \cos\omega_i t + b_i \sin\omega_i t) + \sigma(t)Z(t) = T_P(t) + \sigma(t)Z(t) \qquad (12)$$

$$Z(t)+d_1Z(t)+d_2Z(t) = g_1(t) \qquad (13)$$

Wherein the upper @ means time derivative and :

$\sigma(t)$ = deterministic cyclic function with a period of 24h, t=time [s],

ω_i = angular velocity [1/s], a_0, a_i, b_i = constants.

The intensity of the white noise g_1 is D_1. Equations (12) and (13) mean that the outdoor temperature is composed of the deterministic component $T_P(t)$ and the random component $(Z(t))$ which is excited by the white noise $g_1(t)$. These equations can be regarded as continuous versions of Equations (3) to (6).

(2) Wall and room temperature and heat input

For simplicity reasons, the room consisted of only a single layer wall, and a window is dealt with without any loss of generality. Details are given in Appendix 1.

(3) Infiltration rate n

Although infiltration rate is determined by the external wind velocity, temperature difference between the room and the outdoor air, etc.; it is modelled as follows.

$$n(t) = n_0(t)[1 + \sigma_n(t)n'(t)] \qquad (14)$$

Wherein $n_0(t)$ is the average infiltration rate, and $\sigma_n(t)$ is the normalized standard deviation. Both functions are deterministic. The $n'(t)$ denotes a random component of the infiltration rate.

Equation (14) is used in this study on the assumption that the infiltration is mainly determined by wind velocity, and that the wind velocity is usually not correlated with other climatic conditions.

The random component $n'(t)$ is assumed to be given by the following equation.

$$n^1(t) = a'n'(t) + g_2(t) \qquad (15)$$

Wherein $g_2(t)$ is a Gaussian white noise independent of $g_1(t)$ with intensity D_2.

3-2 State Space Expression

(1) Discretization of walls

By discretization the heat equations (A-3) to (A-5) with respect to position, the following equations are obtained.

$$T_{w1} = 2s[T_{w2} - T_{w1} - T_{wi} + P_0 (T_0 - T_{w1})]$$
$$T_{wi} = s(T_{wi+1} - 2T_{wi} + T_{wi-1}) \quad (i=2, ..., N-1) \qquad (16)$$
$$T_{wN} = 2s[T_{wN-1} - T_{wN} - P_i(T_{wN} - T_R)]$$

Wherein,

$$s = a/(\Lambda x)^2,\ P_0 = \alpha_0\Lambda x/\lambda_w,\ P_i = \alpha_i\Lambda x/\lambda_w$$

In what follows, the results when the wall is discretized into three masses are shown for illustration. By setting,

$$X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T = [Z, Z, T_{w1}, T_{w2}, T_{w3}, T_R, Q, n']^T \qquad (17)$$

Equations (12), (13), (A-6), (A-7) and (14) -(16) are expressed as follows.

$$\begin{array}{lll} x_1 & x_2 & 00 \\ g_1 & & \\ x_2 & -d_1x_2 - d_2x_1 & 10 \\ g_2 & & \\ x_3 & 2s\{x_4 - x_3 + P_0[T_P(t) + \sigma(t)x_1 - x_3]\} & 00 \\ x_4 = & s(x_5 - 2x_4 + x_3) \qquad + & 00 \\ x_5 & 2s[x_4 - x_5 - P_i (x_5 - x_6)] & 00 \\ x_6 & [s_w \alpha_i (x_5 - x_6) + S_g K_g (T_P + \sigma x_1 - x_6) + x_7]/(c\gamma V) + n_0 \sigma_n x_8 T_P + \sigma x_1 - x_6) & 00 \\ x_7 & A_1G_1(t)(T_s - x_6) - A_2G_2(t)x_7 & 00 \\ x_8 & a'x_8 & 01 \end{array} \qquad (18)$$

Wherein,

$$\overline{S_g K_g} = S_g K_g + c\gamma V n_0 \qquad (19)$$

Thus, the Equation (18) is expressed as the following Ito's Equation (Soong 1973).

$$d\mathbf{X}(t) = f[\mathbf{X}(t), t]dt + \mathbf{G} @ d\mathbf{B}(t) \qquad (20)$$

Wherein $f[\mathbf{X}(t), t]$ is the first term, and G and $d\mathbf{B}(t)$ are the matrix and the vector in the second term of Equation (18), respectively. This equation corresponds to Equation (9) in discrete-time formulation.

3-3 Derivation of Moment Equations

(1) Average (Mean) $E[x_i(t)]$

By averaging Equation (20) (Equation [18], the equations for the average can be obtained.

Some of them, for example, are as follows.

$$X_6 = [S_w \alpha_i (X_5 - X_6) + \overline{S_g K_g}(T_P + \sigma X_1 - X_6) + X_7]/(c\gamma V) + n_0 \sigma_n (T_P X_8 + \sigma E[x_1 x_8] - E[x_6 x_8])$$

$$X_7 = A_1G_1(t)(T_s - X_6) - A_2G_2(t)X_7 \qquad (21)$$

Wherein,

$$X_i = E[x_i] \qquad (22)$$

(2) Variance and covariance functions $E[x_k x_1]$

For variable $\mathbf{X}(t)$, which satisfies Equation (20), an arbitrary function $h[\mathbf{X}(t), t]$ satisfies the following relation (Soong 1973).

$$E[h] = 3\ E[f_j\ M\ h/Mx_j] + 3\ E(GDG^T)_{ij}\ M^2h/M\ x_i\ M\ x_j] + E[M\ h/M\ t] \qquad (23)$$

Wherein f_j is the j-th component of $\mathbf{f}$, and D is the variance matrix of $d\mathbf{B}(t)$. By setting $h=x_kx_i$, the following relations are obtained (Hokoi and Matsumoto 1988).

$$\begin{cases} \delta h/\delta x_j = \delta_{kj}\, x_1 + \delta_{lj}\, x_k & (24) \\ \delta^2 h/\delta x_i \, M \, x_j = \delta_{kj}\, \delta_{li} + \delta_{lj}\, \delta_{ki} & (25) \\ \delta h/\delta t = 0 & (26) \end{cases}$$

Wherein δ_{ij} is the Kronecker's delta, that is, $\delta_{ij} = 1$ when $i = j$, and 0 otherwise. By inserting these relations into Equation (23), we obtain the following equation.

$$E[x_k x_l] = \Sigma E[f_j\,(\delta_{kj}\, x_1 + \delta_{lj}\, x_k)] + \Sigma\,[(GDG^{T)}{}_{ij}(\delta_{kj}\, \delta_{li} + \delta_{lj}\, \delta_{ki})] \quad (27)$$

The examples of this equation are as follows :

$$E[x_2^2] = -\,2d_1E[x_2^2] - 2d_2E[x_1x_2] + D_1$$

$$E[x_6^2] = 2\cdot\{S_w\, \alpha_i\, (E[x_5x_6]-E[x_6^2])+\overline{S_g\, K_g}\,(T_PE[x_6]+\sigma\, E[x_1x_6]-E[x_6^2])+E[x_6x_7]\}/(c\gamma V)$$
$$+ 2n_0\, \sigma_n\, (T_PE[x_6x_8] + \sigma\, E[x_1x_6x_8] - E[x_6^2x_8]) \quad (28)$$

By solving Equations (21) and (28) under appropriate initial conditions, the mean and the variances can be obtained. As for several terms in Equation (28) with higher moments than the second order, the approximate method shown in the reference (Hokoi and Matsumoto 1993) can be used.

3-4 Numerical Example

(1) Numerical method and computational conditions

To illustrate this method, a simple example where the random component can be expressed as a white noise, that is :

$$n(t) = n_0(t)[1 + \sigma_n\,()\, g_2\,(t)] \quad (29)$$

was calculated. Other numerics used are as follows.

$d_1=1.6$, $d_2=0.05$, $D_1=0.46$, $n_0=1/3600[1/s]$, $\sigma_n(t)=1$,

$\lambda_w=1.63[W/m\cdot K]$, $c\gamma=1260[J/m^3\cdot K]$, $A_1=3000$, $A_2=100$, $\Delta x=0.06[m]$

The target temperature was set at 22°C. Other constants were set at the same values given in section 2-3. As the numerical method, the Runge-Kutta-Gill method was adopted. The time increment, Δt, was set as 0.01[h].

(2) Results and discussion

Room air temperature and heating load are shown in Figures 9 and 10 for the cases of D_2=0.0 and 1.0, where D_2 is an intensity of g_2. The solid and broken lines denote the mean and mean ± standard deviations, respectively.

While the standard deviation in room temperature during the occupancy period is very small in the case of the deterministic rate (Figure 9, D_2 = 0), it changes significantly, to the contrary, when the infiltration rate is random. This is because heat supply cannot suppress the sudden change caused by the instantantenous change in infiltration rate.

4. RANDOM INTERNAL HEAT GENERATION

Thus far, the internal heat generation Q_0 has been regarded as deterministic. However, this quantity fluctuates due to the changes in the number of occupants and the use of lighting and office automated equipment. This has to be dealt with as changing deterministically on the average while also changing stochastically depending on the time of work and the way in which work is done. In order to make this clear, measured data in an actual situation must be obtained and examined. Here, it is assumed to be expressed in the same manner as the external climate, as follows.

$$Q_0(t) = Q_{0m}\,(t) + \sigma_Q(t)w(t) \quad (30)$$

Wherein the mean $Q_{0m}(t)$ denotes the number of occupants which changes deterministically depending on time of work, and $\sigma_Q(t)$ is its standard deviation. Heat generated from illumination and electric appliances is assumed to be proportional to the number of the occupants. If some of them are not proportional but constant, they can be included into $Q_{0m}(t)$.

The random component w(t) may be approximately expressed, for example, by the following ARMA model.

$$w(t) = w^j = a_1'w^{j-1} + a_2'w^{1-2} + ... + e''^j + e''^{j-2} + ... \quad (31)$$

By making use of these equations, the same method of analysis as that in section 2 can be used.

5. RANDOM AIR-CONDITIONING STARTING AND STOPPING TIMES

In sections 2 and 3, the room was air-conditioned during a predetermined time, that is, from 8:00 tot 18:00. However, longer pre-heating or pre-cooling may be required if the outdoor temperature during the previous night was very low or high. In such case, HVAC systems should be operated depending on, for example, the external temperature. These problems may also be dealt with by assuming the starting and stopping times as random variables. That is, it may be sufficient to regard the parameters, which prescribe the starting and stopping times in the step functions $G_1(t)$ and $G_2(t)$ for controlling heat input (Equation (A-7)), as random. The methods of analysis described below, however, seem much better and simpler.

5-1 Air-conditioning Stopping Time

It seems reasonable to adopt as a stopping time for the air-conditioning, the time when the room is unoccupied or the internal heat generation explained in section 4 becomes 0 or very close to 0. This can be realized by multiplying a certain function of Q_0 (typically a step function), $G_3(Q_0, t)$, by the heat generation Q(t).

5-2 Air-conditioning Starting Time

The starting time for the air-conditioning has usually been determined by office hours and HVAC system management, in Japan. However, as shown in the reference (Hokoi and Matsomoto 1991), the optimal strategy of preheating, taking thermal storage into account, is to lower the capacity of the HVAC system and operating cost, at the same time, by making use of cheap nighttime electricity. Therefore, this problem should be formulated as how to optimize preheating time, and thus should include optimal control of the HVAC systems under random external climates and fluctuating internal heat generation.

The solution of the optimal control theory indicates that the room must be air-conditioned all day long although the heat input may be very little at some time such as midnight (Hokoi and Matsumoto 1991). Therefore, the starting time will be determined as a compromize between the optimal solution and other factors such as high nighttime labor cost.

6. CONCLUSIONS

This paper discusses the use of a building thermal analysis methodology in which the stochastic nature of the external climates and randomness of physical parameters are considered.

The impact of the random infiltration rate and thermophysical properties (here, the thermal conductivity of the wall and the heat capacity of the room) on room temperature and cooling load are investigated in sections 2 and 3.

Furthermore in sections 4 and 5, the methods of analysis that give the influence of the random variation of the internal heat generation and the optimal starting and stopping time of an HVAC system in an intermittent heating, are outlined.

The probability density function of temperature and heating load, thus the maximum load also, can be obtained by making use of these results. Therefore, it is possible to correctly size an air-conditioning system with a certain degree of confidence, which should be decided by designers or owners. For the analysis and design of a thermal system, this method provides a rational and convenient way of handling uncertainties caused by the degree of imprecision in construction work (workmanship) and the variation in the way in which the room is used.

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NOMENCLATURE

a_w = thermal diffusivity of the wall [m^2/s]
A_1, A_2 = constants
A_s = absorptivity of the exterior surface of the wall to the solar radiation [-]
$[a]^j$, $[b]^j$ = matrices with (N+2) x (N+2), (N+2) x 3 elements, respectively
$[A]^j$, $[B]^j$, $[C]^j$ = matrices with M x M, M x 3, M x 2 elements, respectively (M=N +12)
$c\gamma$ = volumetric heat capacity of the room [J/m^3@K]
D_1, D_2 = intensities of white noises $g_1(t)$, $g_2(t)$, respectively
e^j, e'^j = mutually independent discrete white noises with 0 mean and standard deviations σ_e and σ_e', respectively
E[@] = expectation operation
$\mathbf{f}^j$: $[T_o^j, J^j, T_s^j]^T$
$J_p(j)$ = deterministic component of the solar radiation [W/m^2]
$J'(j)$ = random component of the solar radiation [W/m^2]
j = discrete time
K_g = overall heat transfer coefficient of the window [W/m^2@K]
l_g = thickness of glass [m]
l_w = thickness of wall [m]
n = air change rate [1/s]
N = number of node points
$P_o = \alpha_o \Lambda x/\lambda_w$, $P_i = \alpha_i \Delta x/\lambda_w$
Q_0 = internal heat source generated by human body, illumination, etc. [W]
Q = heat input by the air-conditioning system [W]
R^j = variance matrix of X^j
$s = a_w/(\Lambda x)^2$
S_w = area of wall [m^2]
S_g = area of window [m^2]
t = time (s)
Λt = time increment (= 1 h)
$T_{op}(j)$ = deterministic component of the outdoor temperature [EC]
$T_o'(j)$ = random component of the outdoor temperature [EC]
T_R = room air temperature [EC]
T_s = target temperature [EC]
T_w = wall temperature [EC]
T_{wi} = wall temperature of i th node point [EC]
V = room volume [m^3]
V[@] = variance
V^j = variance matrix of white noise vector e^j
x = coordinate [m]
Δx = mesh increment [m]

@ = time derivative

Greek Letters

α_o = exterior film coefficient [W/m^2@K]
α_i = interior film coefficient [W/m^2@K]
λ_g, λ_w = thermal conductivities of glazing and wall, respectively [W/m@K]
$g_1(t)$, $g_2(t)$ = Gaussian white noises
$\sigma_j(j)$ = diurnal standard deviation of solar radiation [W/m^2]
τ_g = transmittance of glazing to solar radiation [-]
ω_i = angular velocity [1/s]

Subscripts, Superscripts

g = glass, i = i th node point,
j = time, that is, t = jΔt,
T = transpose, w : wall.

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Appendix 1 Fundamental Equations for Thermal Analysis

(1) Outdoor Temperature

Input outdoor temperature, $T_o(t)$, is assumed to change as follows.

$T_o(t) = a_0 + 3(a_i\cos\omega_i t)+b_i\sin\omega_i t)+\sigma(t)Z(t)= T_P(t)+\sigma(t)Z(t)$ (12) (A-1)

$Z(t)+d_1Z5TE+D_2Z(t)=g_1(t)$ (13) (A-2)

(2) Wall and room

A room consisted of a single layer wall and a window is dealt with, and the floor is assumed as adiabatic.

$T_w = a_w@M^2T_w/Mx^2$ (A-3)

$\alpha_o(T_0 - T_w) = - \lambda_w @ MT_w/M x$ (A-4)

$\alpha_i(T_w - T_R) = - \lambda_w @ MT_w/M x$ (A-5)

Wherein,

T_w=wall temperature [EC], T_o=outdoor temperature [EC], x=coordinate [m]
a_w = thermal diffusivity [m^2/s], λ_w= thermal conductivity of wall [W/m@ K],
T_R = room air temperature [EC],
α_i, α_o = inner and outer coefficients of heat transfer [W.m^2 @ K].

(3) Room air temperature

The equation for the room air temperature T_R is as follows.

$c\gamma\ V\ @\ T_R = S_w\alpha_i\ (T_{wN} - T_R) + (S_gK_g + c\gamma\ Vn)(T_o - T_R) + Q$ (A-6)

Wherein;

$c\gamma$ = volumetric heat capacity of room [J/m^3@ K], V = room volume [m^3],
S_w = surface area of wall [m^2], S_g = surface area of glazing [m^2],
K_g = U value of window [W.m^2 @ K], n = infiltration rate [1/s],
Q(t) = heat supply [W].

(4) Heat input (supply)

The heat input under intermittent heating is given approximately by the following time variant equation.

$$Q = A_1 G_1(t)(T_S - T_R) - A_2 G_2(t) Q \qquad \text{(A-7)}$$

Wherein,

A_1, A_2 = constants, G_1, G_2 = unit step functions, T_S = target temperature [EC].

(5) Infiltration rate n

An infiltration rate is modelled as follows.

$$n(t) = n_0(t)[1 + \sigma_n(t) n'(t)] \qquad \text{(14)} \quad \text{(A-8)}$$

Wherein $n_0(t)$ is the average infiltration rate, and $\sigma_n(t)$ is the normalized standard deviation. The random component $n'(t)$ is assumed to be given by the following equation.

$$n'(t) = a'n'(t) + g_2(t) \qquad \text{(15)} \quad \text{(A-9)}$$

Wherein $g_2(t)$ is a Gaussian white noise independent of $g_1(t)$.

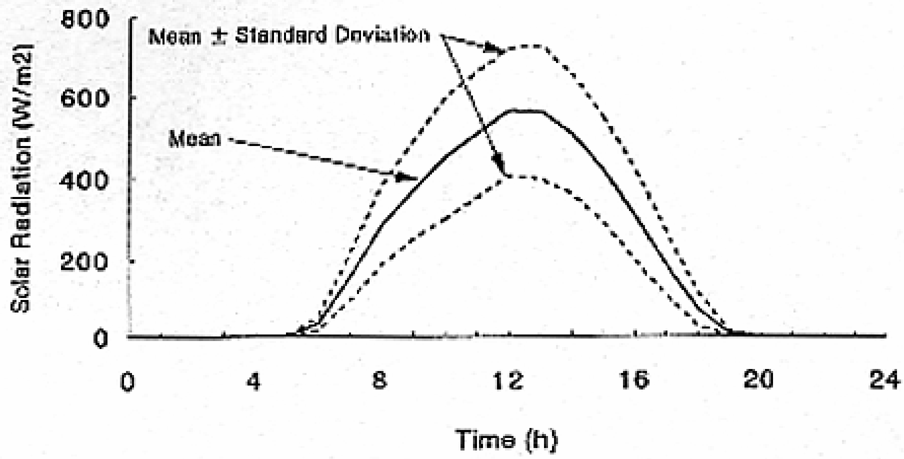

Figure 1 Mean and standard deviation of solar radiation

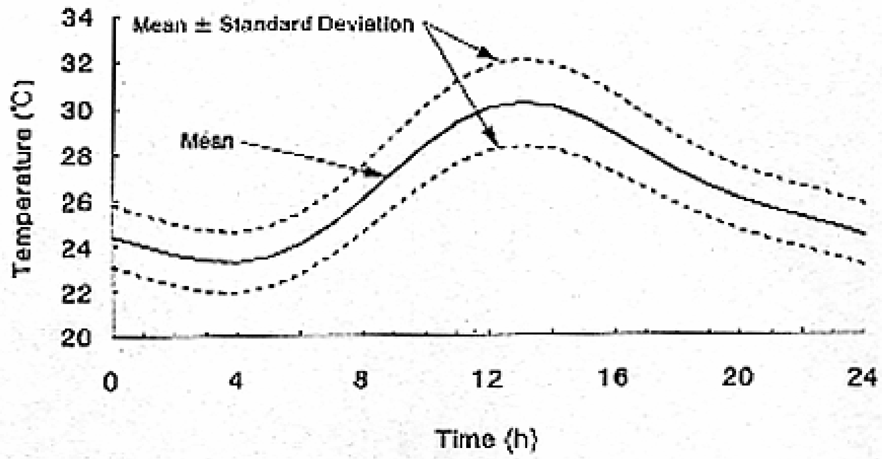

Figure 2 Mean and standard deviation of outdoor temperature

Figure 1 Mean and standard deviation of solar radiation
Figure 2 Mean and standard deviation of outdoor temperature

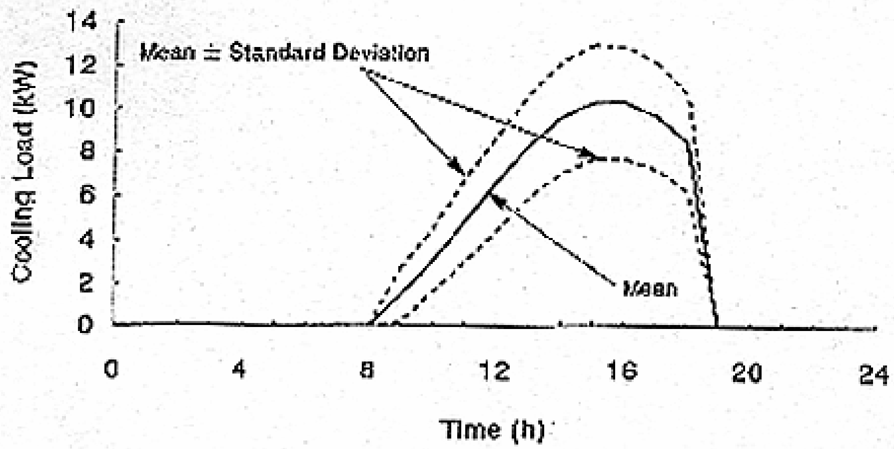

Figure 3 Mean and standard deviation of space cooling load

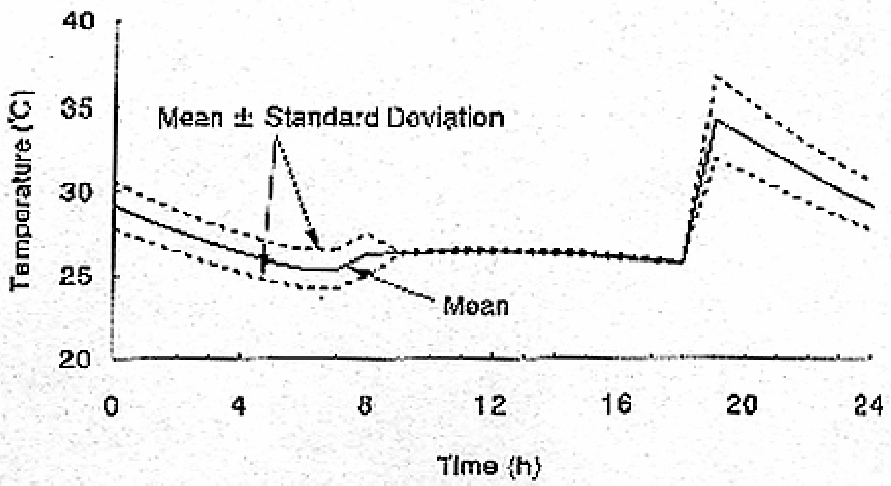

Figure 4 Mean and standard deviation of room air temperature

Figure 3 Mean and standard deviation of space cooling load
Figure 4 Mean and standard deviation of room air temperature

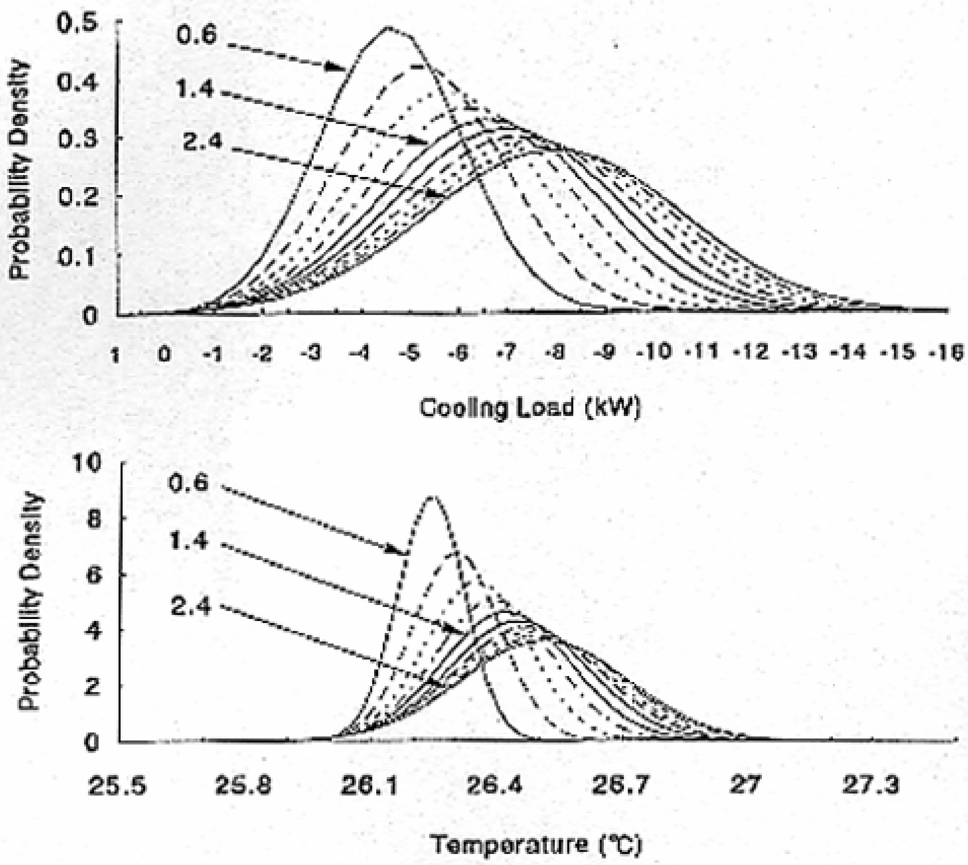

Figure 5 Relationship between thermal conductivity and probability density functions of cooling load and room air temperature

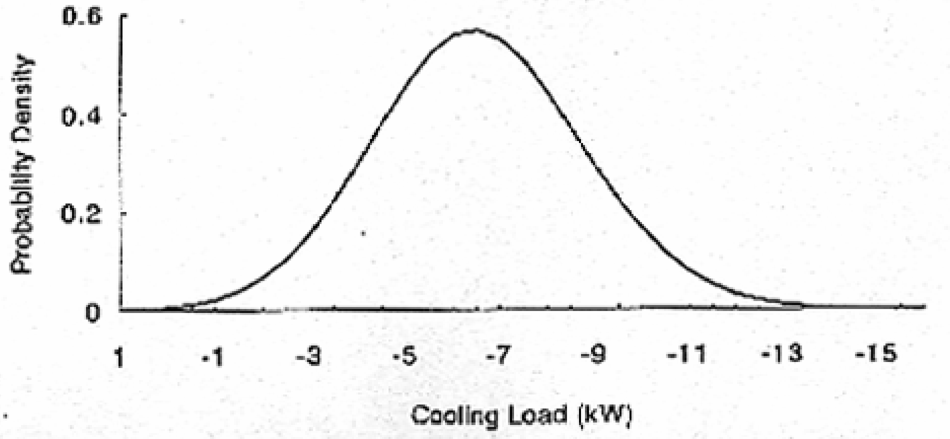

Figure 6 Probability density function of cooling load under random thermal conductivity

Figure 5 Relationship between thermal conductivity and probability density functions of cooling load and room air temperature.

Figure 6 Probability density function of cooling load under random thermal conductivity

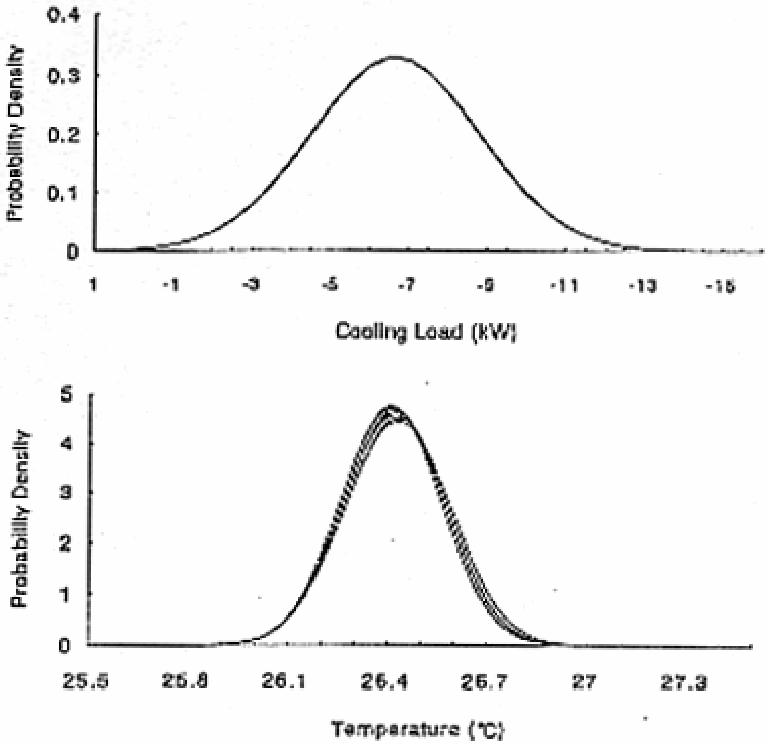

Figure 7 Relationship between heat capacity and probability density functions of cooling load and room air temperature

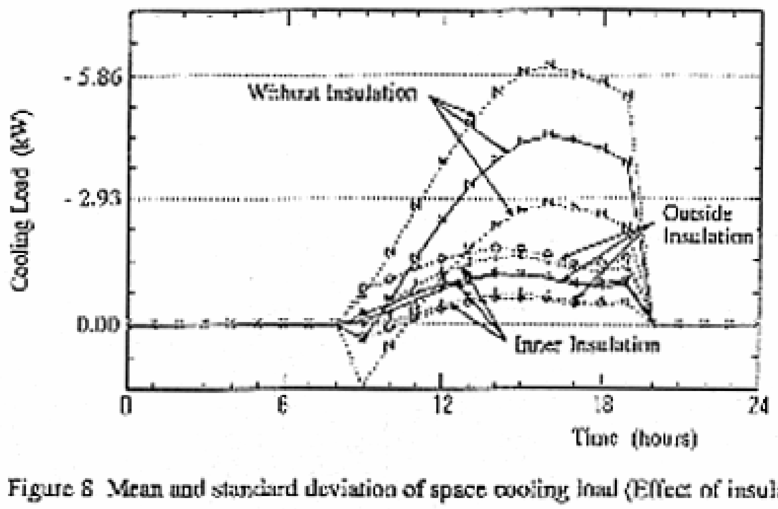

Figure 8 Mean and standard deviation of space cooling load (Effect of insulation)

Figure 7 Relationship between heat capacity and probability density functions of cooling load

and room air temperature
Figure 8 Mean and standard deviation of space cooling load (Effect of insulation)

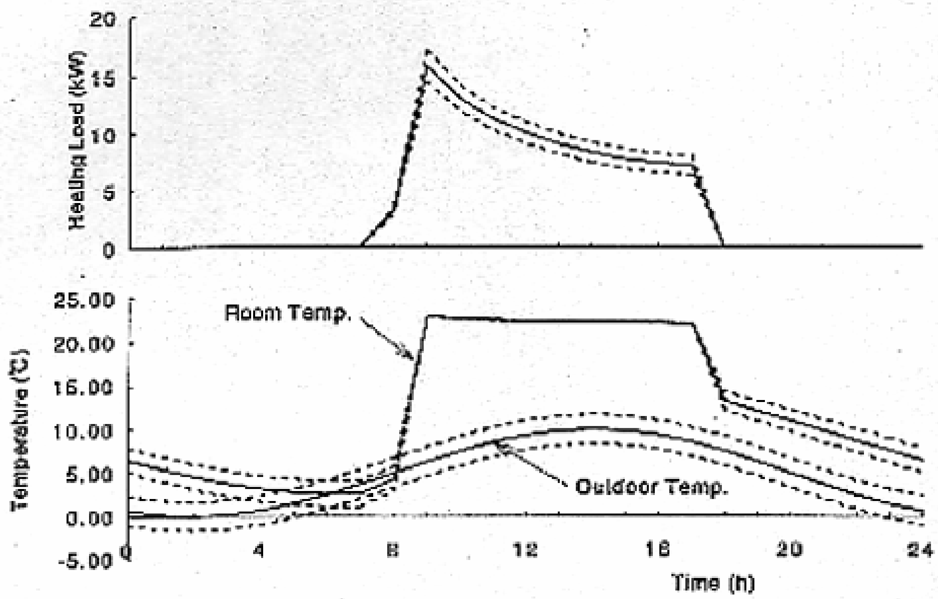

Figure 9 Means and standard deviations of space heating load, outdoor and room air temperatures under deterministic infiltration rate

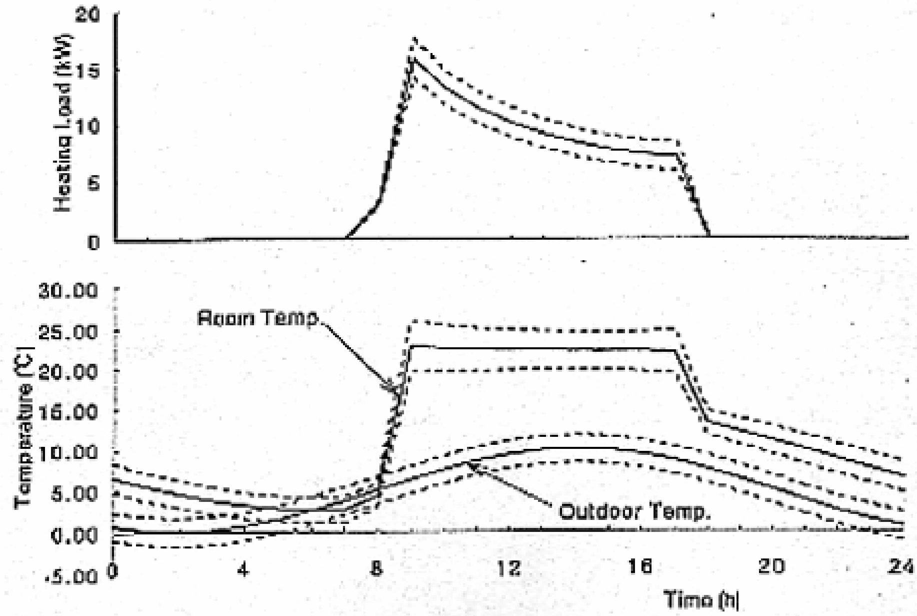

Figure 10 Means and standard deviations of space heating load, outdoor and room air temperatures under random infiltration rate

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Figure 9 Means and standard deviations of space heating load, outdoor and room air temperatures under deterministic infiltration rate

Figure 10 Means and standard deviations of space heating load, outdoor and room air temperatures under random infiltration rate