

# **THERMOECONOMICS, A DESIGN TOOL FOR THE NEXT CENTURY**

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Thermoeconomics is the combination of thermodynamics using exergy with economics. The objective of thermoeconomics is to minimise a total cost function, which includes capital, maintenance and running costs. This will establish the most cost effective, or optimal, design parameters. This paper shows the application of thermoeconomic analysis to a building services system, consisting of a constant air volume (CAV) air conditioning system. The specific cost of indoor environment cooling is optimised in terms of system variables, such as water and air temperatures and mass flow rates. The method is firstly to determine the most significant variables from the detailed exergy analysis. Secondly, the process may be optimised by implementing the most appropriate design modifications. From the work presented, significant and unexpected improvements to both thermal and economic design are made.

## **INTRODUCTION**

This paper describes the use of Thermoeconomics (Lozano & Valero 1993, Lozano et al 1994) in an application to Building Services Engineering design by means of an example. A similar analysis has been previously carried out on a displacement ventilation system (Tozer & Missenden 1996). The analysis involves the use of simultaneous exergetic life-cycle cost equations, in order to determine the optimum system design and operating conditions for the installation (Tozer et al 1995b). The innovative element in exergy analysis is that it quantifies the usefulness of energy rather than just its quantity. In thermodynamic terms exergy is similar to the Gibbs Function, or “free energy”, but related to a “useful” datum. The concept of exergy is described mathematically in Appendix A and further references are available (Tozer 1995, Tozer & James 1995) for the interested reader. In contrast to previous analysis, exergy has a positive value for cooling energy, which would have a negative value for energy. Also, energy is always transformed and never destroyed but exergy is always destroyed in real systems. The quantity of exergy lost in a process provides a measure of its thermodynamic irreversibility, i.e. the extent to which it has deviated from ideal conditions.

Exergy analysis of systems has previously been impracticable both because of the large number of simultaneous equations and of their frequent implicit form. However, a high capacity equation solving computer software package (EES 1994) has recently become available, working within PC based Windows. It can solve a large number of simultaneous algebraic equations, limited only by hardware memory. To solve or to minimise / maximise a set of equations an iterative procedure is used, and therefore appropriate initial values of all variables have to be provided together with their upper and lower limits. The software also conveniently includes a wide range of mathematical and thermodynamic library functions.

The method was applied to a constant volume air conditioning system, consisting of a water chiller, a chilled water system, an air handling unit (AHU) and ductwork. A mathematical model was developed, each component being described in terms of its physical and economic parameters. The effects of alterations to the system were analysed and optimisation was performed for several cases and the results presented and discussed.

### CONSTANT AIR VOLUME AIR CONDITIONING SYSTEM

The system investigated consisted of an air handling unit (AHU) that was supplied with chilled water by a water chiller having a reciprocating compressor and an air cooled condenser. Initially, this was considered to operate with chilled water flow and return temperatures having standard design values. The AHU leaving air was then distributed through a ductwork system. The air exhausted from the room returned through a ceiling plenum to the extract side of the AHU, where some recirculation took place.

A schematic diagram of the system is shown in Figure 1

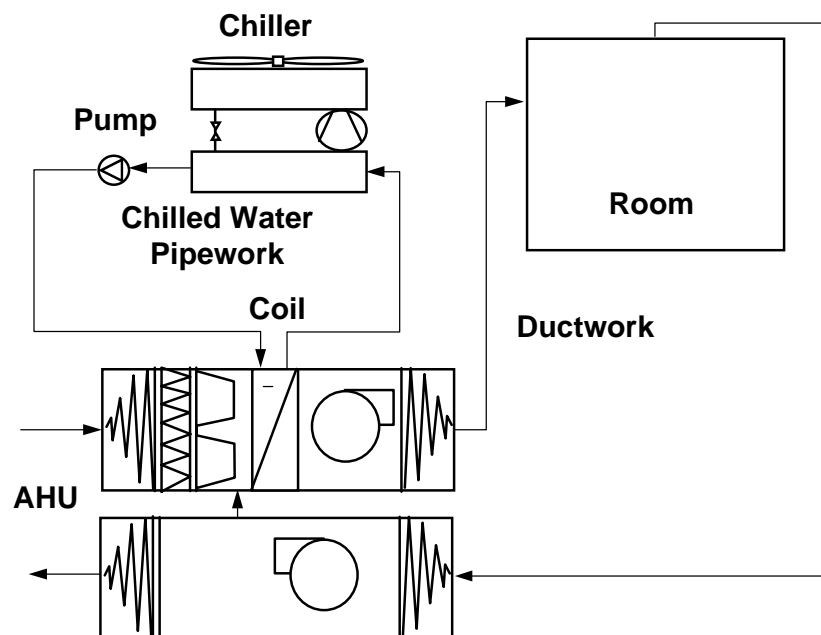


Figure 1: Constant Air Volume System

## DESIGN CONSIDERATIONS

Standard design criteria for such a system would typically assume about 7°C and 12°C for respective chilled water flow and return temperatures. The indoor conditions would be typically 22°C dbt and 55% rh. A sensible heat ratio of 0.95 determines the supply air temperature of 15.2°C. The base study considers 500 hr (equivalent full load) of cooling, (i.e. chiller operation) with 3000 hr of system use (fan operation) annually. Other situations are later considered including the minimisation separately of life cycle costs and of initial costs. An outdoor design condition was assumed of 28°C dbt, 50% rh.

## AIR CONDITIONING THERMOECONOMIC MODEL

A mathematical model should predict the same behaviour as a purpose built physical model would demonstrate experimentally, by solving simultaneous algebraic equations. These should be validated with experimental data, i.e. manufacturer's information and test data, and adjustments made to calibrate the equations for real situations. A large number of equations may provide more accuracy but at the expense of more complexity. As an example, a simple mathematical model of a fan characteristic would consist of a quadratic or polynomial equation, to fit the pressure-flow curve. If used in conjunction with the system characteristic, the operating point is obtained by solving two equations simultaneously, for the two variables of pressure and air flow.

The thermodynamic analysis uses the Second Law and Availability (Moran 1989), or Exergy (Wepfer 1979, Haywood 1991, Horlock 1992, Kotas 1995). Minimising the objective function will establish the most cost effective design parameters for the specific design configuration analysed. The components modelled for this exercise were the chiller, chilled water pipework and pump, dehumidifying cooling coil, air handling unit, ductwork and extract fan. The mathematical model comprises firstly a thermodynamic model which determines the functions regarding system variables and secondly a thermoeconomic model which evaluates exergy and cost flows.

The number of simultaneous equations, which equals the number of solvable variables is (rounded to the nearest 10):

- Thermodynamic model:		220 equations
- Chiller:	100 equations	
- Chilled water pipework, lagging / pump:	20 equations	
- Cooling coil	70 equations	
- AHU, ductwork, lagging and extract fan	30 equations	
- Thermoeconomic model:		60 equations
- Annual running costs:	10 equations	
- Capital costs:	10 equations	
- Life cycle costs:	10 equations	
- Exergy flows:	10 equations	
- Cost flows (exergy & capital costs):	20 equations	
- Total:		280 equations

The number of equations and variables is therefore high, and for clarity the chiller equations only will be briefly described. A simplified refrigeration cycle can be represented by four conditions on the pressure - enthalpy diagram, but if all the components are considered together with their pressure losses and temperature changes the cycle is represented by nine conditions. Each of these requires up to five parameters including temperature, pressure, enthalpy, entropy, specific volume and quality. To model the chiller it is also necessary to consider all of the heat exchangers and their subsections, i.e. the condenser has superheating, condensing and subcooling areas. Log mean temperature differences, heat exchange areas and heat transfer coefficients in terms of variable convection factors have also been considered. Compressor volumetric and isentropic efficiencies are functions of further variables. The remaining systems were modelled with similar levels of complexity.

The chiller was modelled against published manufacturers' data (York 1994) and the design parameters were confirmed by the Company. The software package includes the thermodynamic properties of most refrigerants and the model was based on HFC 134a. For pipework and ductwork modelling, standard literature (CIBSE 1986) was referred to for Moody diagram equations. Cooling coils were modelled using the standard methods described in the literature (ASHRAE 1993).

The model philosophy considered the following:

- Electric energy was derived from the public supply and maximum demand charges were disregarded as only the cooling season was considered where these were normally not charged.
- The chiller was considered to remain under constant load and therefore the output varied according to the overall efficiency of the system.
- The design parameters considered as variables were; both entering and leaving chilled water temperatures, pipework friction loss, number of coil rows and ductwork velocity.

- The chilled water pipework and air ductwork configuration were considered as constant and the pump and fan sizes varied in terms of their respective mass flow rates and pressure drops.
- The size of the AHU varied in terms of the air volume

### COST ALLOCATION USING EXERGY

Exergy flows are determined throughout the system, starting from the exergy provided by the electricity supply. Its magnitude decreases through the system until the air is discharged to the outdoors (exergy reference state) where the exergy value is zero. The costs are evaluated on a yearly basis and each plant item satisfies a cost equation based on exergy costs, i.e.: the costs entering a plant equals the costs leaving the plant.

$$\text{EXergy flow cost in} + \text{Capital cost in} + \text{ENergy cost in} = \text{EXergy flow cost out} \quad [1]$$

In the case of the AHU the exergy flow cost in is that of the chilled water, the energy cost is the fan energy cost, the capital cost corresponds to the cost of the AHU and the exergy flow cost out is that of the cold air produced. All these costs have to be related to the same time interval, i.e. one year.

$$c_i^b b_i \text{eflh}_i + c_i^w w_i \text{eflh}_i = c_{i+1}^b b_{i+1} \text{eflh}_{i+1} \quad [2]$$

This equation only considers the exergy and energy flows and provides useful information on the relative efficiency of each plant comprising the overall system. The suffix i indicates entering the plant and i+1 indicates leaving the plant. The following equation provides useful information as it includes all costs involved (exergy, energy and capital costs).

$$c_i^{bz} b_i \text{eflh}_i + z_i \text{crf} \cdot f_{ma} + c_i^w w_i \text{eflh}_i = c_{i+1}^{bz} b_{i+1} \text{eflh}_{i+1} \quad [3]$$

### COST CALCULATIONS

The specific exergy costing throughout the process from the chiller plant to the final product of building cooling, of the base system as described previously is indicated in Figure 2. Two figures are given for each stage numbered 1 to 9, the cbz figures indicate the cost taking into account the exergy and plant costs, whereas the cb figures indicate these costs in terms of the exergy costs alone. The term "z" refers to capital costs, "w" to electric power and plant identities are given by suffixes.

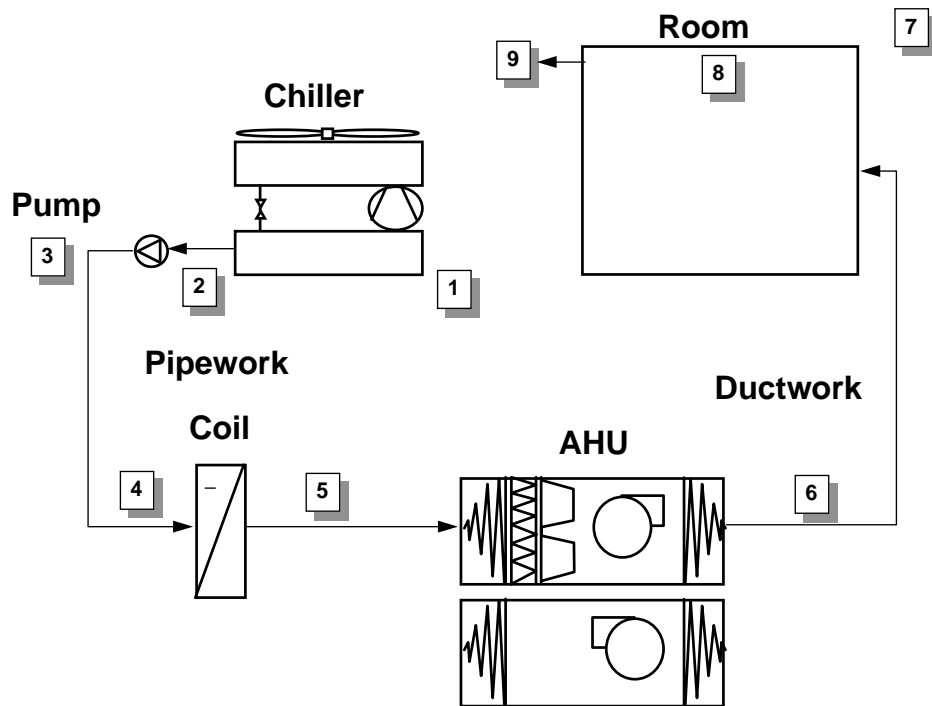


Figure 2: Main cost flows of system

### ANALYSIS OF COSTS

The method of approach used in this section was to determine the relative cost increase, between the exergy flow into and out of each plant, resulting from exergy loss. An appraisal of these costs indicates that the cost of chilled water, from point 1 to 2, has the largest increase with respect to its incoming value. The next highest ratio is for the air handling unit. In order to improve the overall system efficiency an appropriate assessment of real achievable economic and exergy factors may then be considered. The relevant parameters are indicated in Table 1.

Flow	Exergy kW (b)	Exergy Cost £/kWh (c <sup>b</sup> )	Exergy and Capital Cost £/kWh (c <sup>bz</sup> )
1	59.51	0.05	0.05

2	8.99	0.34	0.95
3	8.99	0.35	1.20
4	8.93	0.35	1.57
5	6.46	0.49	2.26
6	5.42	0.85	4.42
7	5.42	0.85	6.39
8	3.19	1.51	11.01
9	0	na	na

Table 1: Summary of Exergy and Flow Costs

The above table shows that the contribution to costs is most significant in the chiller as observed in points 1,2. In comparison, the AHU costs appear relatively small. This suggests that optimisation should include the effects of both chilled water and supply air temperatures. It also suggests that the air temperatures could be allowed to fall somewhat with little cost penalty. Also, once the chiller is optimised, particularly in regard to its range, little alteration can be expected.

Taking the above changes into account the system costs were optimised in terms of the variables described. The equation solver minimised the value of lcc in terms of the same independent variables. It also minimised the capital costs in terms of the same variables.

### OPTIMISATION OF THE OBJECTIVE FUNCTION

Thermoeconomic optimisation consists of minimising an objective function, which is usually a total cost relating both capital and running costs. This objective function will have a minimum, corresponding to an optimum, or minimum waste or cost. Independent variables were selected as degrees of freedom, each one of which could be allowed to vary and hence be optimised. The independent variables selected were the leaving chilled water temperature, chilled water temperature difference, supply air temperature, pipework friction loss, number of coil rows and ductwork air velocity. In this case the chiller selected is constant and the output room cooling to the building is variable. Therefore the objective function was established as the annual life cycle cost per unit of cooling capacity of air to the building:

$$lcc = \frac{crf \cdot f_{ma} z + c^w (eflh_c w_c + poh_c (w - w_c))}{Q_r} n \quad [4]$$

The first term on the RHS of the equation represents the contribution of capital and maintenance cost whereas the second term represents the element of operating costs. A distinction has been made between the "equivalent full load hours" of the chiller compressor and the "plant on hours" of the rest of the plant including condenser fans, chilled water pump, air handling unit and extract fans.

## OPTIMISATION OF DESIGN PARAMETERS

Using the EES equation solver software package previously referred to, the set of 280 equations in 280 unknowns was solved. The software additionally enables functions to be minimised in terms of successive independent variables. Two broad cases of design optimisation are presented; the first is for minimum life cycle costs (lcc), and the second is for minimum capital costs. As a first stage, the optimisations were carried out for a group of different air supply temperatures and their associated room relative humidities. As a second stage, the yearly specific life cycle cost (lcc) and capital costs were minimised in terms of the variables described, thus producing Tables 2 and 3.

Parameter (500 eflh)	Base Design	Design @ 50%rh	Design @ 45%rh	Design @ 40%rh	Design @ 35%rh	Min. LCC Design
Room Relative Humidity %	55	50	45	40	35	32
Supply Air Temperature °C	15.4	13	11.5	9.9	8.2	6.7
Off Coil Temperature °C	13.6	10.8	9.1	7.3	5.1	3.5
No of coil rows	3	6	7	7	8	9
Leaving Chilled Water Temperature °C	7	7.6	6.4	4.4	2.6	1.4
Entering Chilled Water Temperature °C	12	15.6	14.4	12.8	11.4	10.4
Chilled Water Range K	5	8	8	8.2	8.8	9
Duct Air Velocity m/s	8	8.3	8.6	9.2	9.7	10
Duct Friction Loss Pa/m	0.34	0.44	0.54	0.76	1.01	1.20
CHW pwk velocity m/s	1.3	1.5	1.5	1.5	1.5	1.5
CHW Friction Loss Pa/m	200	320	330	360	390	410
Fresh Air %	16	22	25	29	33	37
Cooling Capacity kW	120	125	117	106	96	90
<b>OUTPUT % of Cooling</b>	<b>100</b>	<b>104</b>	<b>97</b>	<b>88</b>	<b>80</b>	<b>75</b>
LIFE CYCLE COST £ / kW life	3312	2703	2571	2473	2416	2404
<b>LIFE CYCLE COST %</b>	<b>100</b>	<b>82</b>	<b>78</b>	<b>75</b>	<b>73</b>	<b>72.6</b>

Table 2: Thermodynamic parameters of optimised designs on life cycle costs



The optimum operating conditions were constrained by the chiller manufacturer who allowed extension of the temperature differential of 7.5K to a maximum of 12K, providing minimum flow rates were satisfied to guarantee heat transfer and that the compressor motor was not overloaded.

The chilled water supply temperature had to be above the minimum permissible for water systems, otherwise a glycol or brine comparison would have been required.

The room air relative humidity needed to be within an acceptable comfort range.

The supply air temperature needed to be combined with an acceptable room air mixing system.

Parameter (0 eflh)	Base Design	Design @ 50%rh	Design @ 45%rh	Design @ 40%rh	Minimum Capital Cost Design
Room Relative Humidity %	55	50	45	40	35
Supply Air Temperature °C	15.4	13.5	12	10.4	8.8
Off Coil Temperature °C	13.6	10.8	9.1	7.3	5
No of coil rows	3	8	8	8	12
Leaving Chilled Water Temperature °C	7	7.8	6.4	3.7	2.8
Entering Chilled Water Temperature °C	12	17.8	15.6	15.1	14.6
Chilled Water Range K	5	10	9.2	11.4	11.8
Duct Air Velocity m/s	8	11.9	12.1	12.6	14.3
Duct Friction Loss Pa/m	0.34	1.04	1.28	1.98	2.6
CHW pwk velocity m/s	1.3	1.5	1.5	1.5	1.5
CHW Friction Loss Pa/m	200	380	380	460	480
Fresh Air %	16	20	24	28	31
Cooling Capacity kW	120	123	114	103	95
<b>OUTPUT % of Cooling</b>	<b>100</b>	<b>103</b>	<b>95</b>	<b>86</b>	<b>79</b>
LLC @ 500 EFLH £/kW life	3312	2827	2647	2531	2549
% of LLC @ 500 EFLH %	100	85	80	76	77
CAPITAL COST £ / kW	2484	1937	1840	1771	1729
<b>CAPITAL COST %</b>	<b>100</b>	<b>78</b>	<b>74</b>	<b>71</b>	<b>70</b>

Table 3: Thermodynamic parameters of optimised designs on minimum capital costs

If the life cycle cost at 500 EFLH of Table 3 is compared to that of Table 2, it can be seen that by optimising a design on capital costs 80% to 93% of life cycle costs are saved.

In order to appreciate the effect of different plant running hours, a series of design optimisations were carried out for different Equivalent Full Load Hours (EFLH). A constant design was assumed at 45% room relative humidity and the results are indicated in Table 4.

Table 4 clearly indicates the little influence on running hours on the design optimisation, once these are above 500 EFLH. Zero EFLH equates to optimising the design based on minimising the capital costs only. With the exception of zero EFLH the selection of chilled water temperatures remain the same for the rest of the EFLH. Only minor differences are noted in the design parameters of ductwork and pipework.

Parameter (45% relative humidity)	Optimise d Design	Optimised Design	Optimised Design	Optimised Design
EFLH	0	500	1000	2000
Supply Air Temperature °C	12	11.5	11.5	11.5
Off Coil Temperature °C	9.1	9.1	9.1	9.1
No of coil rows	8	7	7	7
Leaving Chilled Water Temperature °C	6.4	6.4	6.4	6.4
Entering Chilled Water Temperature °C	15.6	14.4	14.4	14.4
Chilled Water Range K	9.2	8	8	8
Duct Air Velocity m/s	12.1	8.6	8.5	8.3
Duct Friction Loss Pa/m	1.28	0.54	0.53	0.50
CHW pwk velocity m/s	1.5	1.5	1.4	1.4
CHW Friction Loss Pa/m	380	330	315	280
Cooling Capacity kW	113	117	117	117

Table 4: Thermodynamic parameters of optimised designs in terms of EFLH

## CONCLUSIONS

A thermoeconomic analysis has been carried out on an air conditioning system which comprises of an air cooler water chiller and a supply/extract AHU. Components have been modelled within the system both in respect of their physical, thermodynamic and economic performance.

The concept of exergy and its associated value, termed thermoeconomics, has been applied to determine the internal costs of each process within the system. Using these, the internal costs have been used to evaluate design modifications or improvements as these costs relate to both the plant capital cost and maintenance and to the fuel or energy costs. It has indicated correctly that the critical design parameters are the chilled water temperatures.

It was shown to be possible to optimise designs for various room relative humidities and that the point of inflection occurred at 32% rh, and a supply air temperature of 6.7°C, achieving a 28% saving in life costs. However, at 45 % rh and a supply temperature of 11.5°C, a saving of 22% in life costs was still achieved with the more practicably acceptable conditions stated.

A similar analysis carried out with respect to minimising capital costs indicated that the point of inflection occurred at 35% rh, and a supply air temperature of 8.8°C, achieving a 30% saving in capital costs. However, at 45 % rh and a supply temperature of 11.5°C, a saving of 26% in capital costs was still achieved. If designs are optimised on capital costs, savings on optimisations based on life cycle costs of 80% to 93% are achieved.

The practical application of this technique to building service is feasible. It indicates the need to overcome limitations historically indicated by manufacturers in their catalogues for practical reasons. Also, it has the potential to model beyond the designer's original best judgement, providing a fundamental method for innovative and cost effective design, thus providing a powerful design tool for the next century.

### SYMBOLS USED

b	Thermal exergy (kW)	c	Cost (£/kWh)
crf	Capital recovery factor	dbt	Dry bulb temperature (°C)
eflh	Equivalent full load hours (hours)	f	Factor
h	Enthalpy (kJ/kg)	HFC	Hydro-Flour-Carbon
llc	Life cycle cost per output (£/life kg/s)	m	Mass flow rate (kg/s)
n	Number of years (years)	poh	Plant on hours (hours)
Q	Thermal load (kW)	rh	Relative humidity (%)
RHS	Right hand side	s	Entropy (kJ/kgK)
T	Absolute temperature (K)	w	Electric power (kW)
W	Work (kW)	z	Plant cost (£)
φ	Destroyed exergy (kW)		

### Suffixes

a	air	afan	ahu fan
afanm	ahu fan motor	ahu	air handling unit
ahuf	ahu fan	c	cooling / compressor
ch	chiller	chw	chilled water
cond	condenser	dwk	ductwork
efan	extract fan	efanm	extract fan motor
ma	maintenance	p	pump
pwk	pipework	s	entropy
0	reference state		

### Superscripts

b	exergy	z	capital cost
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## APPENDIX A: SHORT REVIEW OF EXERGY THEORY

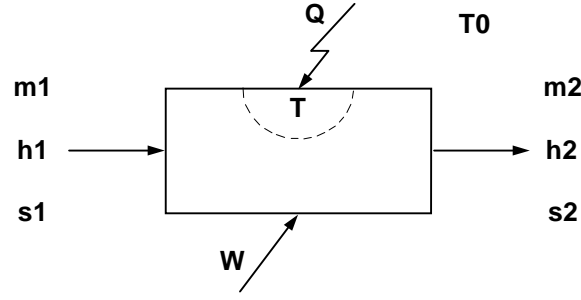


Figure A1: Thermodynamic system in stationary state

Consider a system that works in a stationary state (Lozano & Valero 1993). The balances of mass, energy and entropy are:

$$m_1 = m_2 = m \quad [A1]$$

$$W = m(h_2 - h_1) - Q \quad [A2]$$

$$\phi_s = m(s_2 - s_1) - \frac{Q}{T} \geq 0 \quad [A3]$$

where  $\phi_s$  is the generated entropy due to internal irreversibilities.

Note:

$\phi_s = 0$  implies a reversible process

$\phi_s > 0$  implies a irreversible process

Given  $T_0$  as the ambient temperature, and combining the energy equation [2] with the entropy equation [3], and operating as [2]- $T_0$ [3]:

$$W = m[(h_2 - T_0 s_2) - (h_1 - T_0 s_1)] - Q\left(1 - \frac{T_0}{T}\right) + T_0 \phi_s \quad [A4]$$

This equation provides the exergy balance of the system and all the terms in the equation have exergy units. The following terms can be identified, where the final term is zero for a reversible process

$m(h - T_0 s)$  exergy of flow

$Q\left(1 - \frac{T_0}{T}\right)$  exergy of heat

$T_0 \phi_s$  exergy destruction

To produce a change from state 1 to 2 on the mass flow in a system that only exchanges heat with the ambient ( $T = T_0$ ), a minimum amount of work will be required. It will be equal to the difference of exergy of flow between states 2 and 1 which is equal to

$m[(h_2 - T_0 s_2) - (h_1 - T_0 s_1)]$ , when the process is internally reversible ( $T_0 \phi_s = 0$ ).

Another point of interest in refrigeration is to consider  $Q$  as the cooling capacity of a room at temperature  $T$ . The heat dissipated to the outdoor environment at  $T_0$ , is  $Q_0$ . The refrigeration plant works a cycle in a closed system. Applying the exergy equation [4] to this system:

$$W = Q_0 \left( 1 - \frac{T_0}{T_0} \right) - Q \left( 1 - \frac{T_0}{T} \right) + T_0 \phi_s$$

[A5]

$$W = -Q \left( 1 - \frac{T_0}{T} \right) + T_0 \phi_s \quad [A6]$$

From equation [6] it can be seen that the minimum amount of work required for refrigeration is  $Q(T_0 - T)/T$  which corresponds to the Carnot reversible cycle ( $T_0 \phi_s$ ), where more work will be required for colder rooms.