A DYNAMIC NONLINEAR MODEL OF A THERMOSTATIC VALVE

Lars Henrik Hansen *†
Henrik Madsen †
Jan Holst ‡
Niels Bidstrup *
Pierré Vadstrup *

* Grundfos A/S, Denmark.
† Dept. of Math. Modelling, DTU, Denmark.
‡ Dept. of Math. Statistics, LTH, Sweden.

Abstract

An experimental setup of a water based central heating system has been used to measure the thermo dynamical behavior of a thermostatic valve in order to identify an adequate mathematical model.

The identification of the thermostatic valve is based on nonlinear Grey-Box modelling. Grey-Box modelling is characterized by the fact that, in the modelling procedure, partial known information from physics about a system is combined with information from data. The parameters of the model are estimated by a Maximum Likelihood Method. In the paper an overview of this modelling procedure is given. The estimated model of the applied thermostatic valve is a first order differential equation with a static non-linear output description.

The presented results constitute a part of a research project, where the primary goal is to establish a collection of dynamic models for the components of water based central heating systems in single family houses.

Keywords: Identification; thermodynamics; nonlinear modelling; stochastic differential equations; constrained parameter estimation.

1 Introduction

During the last couple of decades the energy consumption of buildings has been reduced, mainly motivated by increasing energy costs. The reduction has been accomplished through improved building constructions, more insulation and introduction of local room temperature controllers. In the case of water based heating system, the introduction of thermostatic valves and pumps has increased the thermal comfort and significantly reduced the energy symposium.

Research in building materials and building constructions, e.g. low energy buildings and zero energy buildings, continues to yield a decreasing energy consumption. This development increases the demands on the control equipment of the heating system, as the smaller energy demand implies a more sensitive heat balance.

Today the design of a central heating installation is mainly based on static considerations. However, in the future this aspect might not be sufficient. The dynamic properties of the central heating system and its control equipment have to be taken into account in order to handle the increasing sensitivity of the heat balance in buildings.

This study focus on the thermodynamic properties of a thermostatic valve. A mathematic model of the thermostatic valve is identified based on the Grey-Box modelling technique, where physical knowledge is combined with information from measurements. In this paper a gas based (saturated vapor) thermostatic valve is considered, due to its relative small time constant and its small hysteresis band. These two properties are very important for obtaining an efficient control in low energy buildings.

2 Thermostatic Valve Model

Today the thermostatic valve is a standard component in a water based heating system. Different thermostatic valve designs exist. In the following the gas based (saturated vapor) thermostatic valve (see e.g. (*Danfoss, 1988)*) is considered. This type of thermostatic valve consists of a thermostatic head (sensor part) and a valve body, as shown in Figure 1. The displacement of the valve spindle (valve stem) is denoted *x,* while *s* is related to the displacement of the spring and the set point temperature. When the thermostatic head is turned, the spring displacement is altered and thus also the set point, T_{set}.

The thermostatic sensor operates as a P-controller, as sketched in Figure 2, where the valve spindle position *x* is determined by the surrounding air temperature. According to the specifications for the thermostatic valve the P-band of the thermostatic sensor has a width of 6 K. At point A the valve is fully open and at S point it is closed. A definition of the temperature point S is given in the European standard EN 215-1 (see e.g. *(Dansk Standard, 1988)*). Point C in Figure 2 is equivalent to the set point temperature T_{set}.

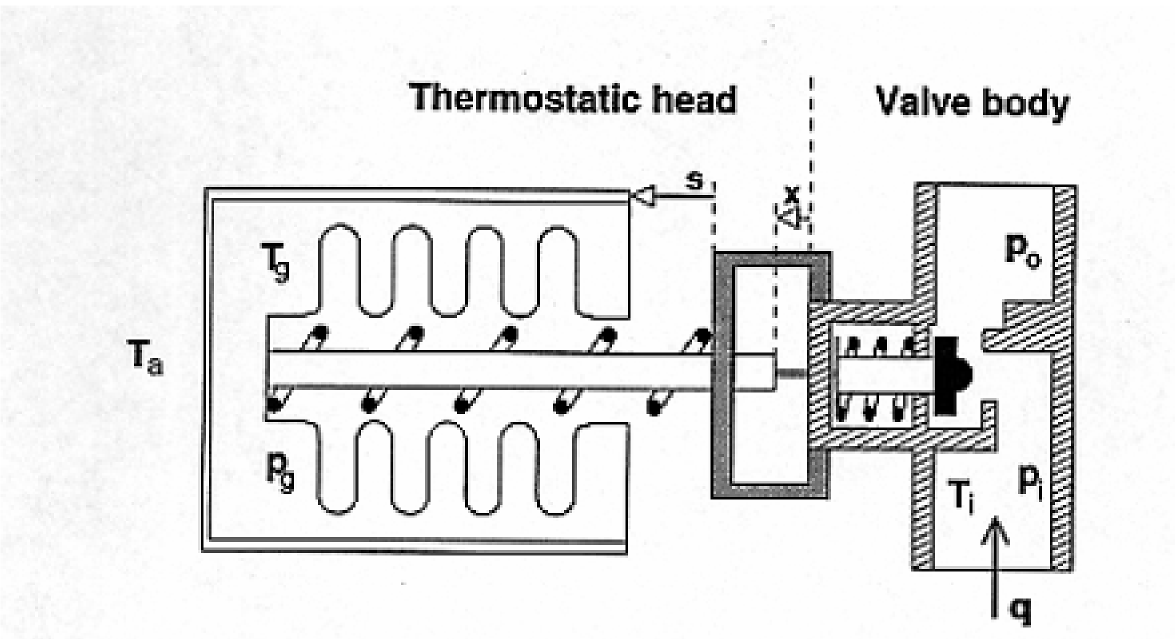
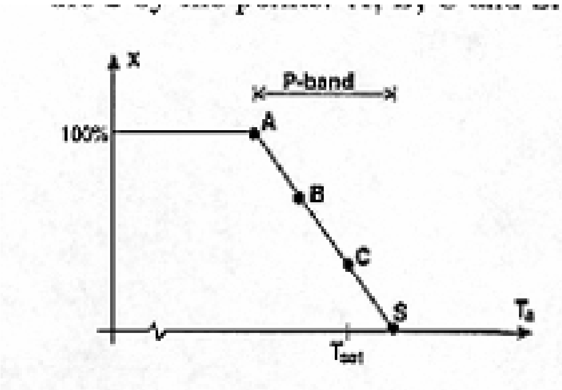


Figure 1: Schematic drawing of a thermostatic valve (based on saturated vapor).

The so called P-band temperature X_p (a relative temperature) is defined as:

$$X_p = T_{set} + 2 - T_a \qquad (1)$$

where T_a denotes the ambient temperature. As sketched by the curves A, B, C and S in figure 3, the relation between the pressure drop over the valve $\Delta p = p_i - p_o$ and the corresponding flow is influenced by the valve openings indicated in Figure 2 by the points: A, B, C, and S.



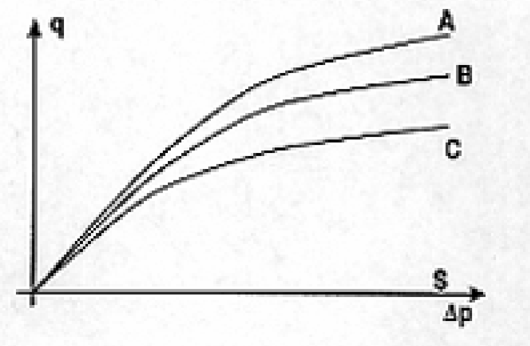


Figure 2: *P-band of the thermostatic sensor. Valve opening versus ambient temperature.*

Figure 3: *Sketch of valve characteristics at different valve openings.*

The flow q is proportional to the square root of the pressure drop Δp over the valve (*Fox and McDonald, 1985)*, i.e. in the case of a thermostatic valve:

$$q = K_v(X_p)\sqrt{\Delta p} \qquad (2)$$

where $K_v(X_p)$ denotes the valve constant, which is controlled by the thermostatic sensor.

The purpose of a thermostatic valve is to control the water flow through a radiator in order to maintain a certain room temperature, as sketched in Figure 4. The control properties of the closed loop system in Figure 4 is normally assessed by the *P-band* of the thermostatic valve. When the heat loss of a room is found, a suitable radiator can be selected and the nominal water flow can be calculated. Based on the desired pressure drop over the valve and on the nominal flow, a thermostatic valve can be selected, i.e. a thermostatic valve, which has approximately the desired pressure drop and flow at X_p = 2K. A P-band of 2 K is often used as an appropriate compromise between thermal comfort and stability in the hydraulic system.

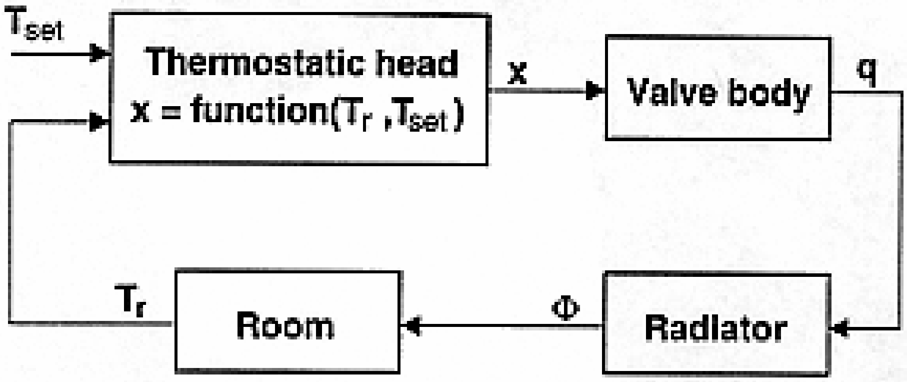


Figure 4: *Block diagram of the thermostatic valve control loop.*

As indicated in Figure 4, the flow is controlled by the thermostatic sensor (and the valve pressure - not sketched). The valve pressure Δp depends on the pressure generated by the pump and on the actual distribution of hydraulic resistances in the heat network (pipes, radiators, valves, etc.). When the thermostatic valve is in its P-band, the hydraulic resistance of the valve is dominating. Hence in this case both q and Δp depend on the thermostatic sensor.

In (*Svensson, 1978)* the following 4 properties of the thermostatic valve are reported: a time delay, a time constant, hysteresis and a dependency of the supply water temperature. Meanwhile, *(Dansk Standard, 1988)* describes the following 4 properties: time constant, hysteresis, dependency of the supply water temperature and elasticity with respect to the set point temperature (due to the pressure drop over the valve). In *(Gammelby, 1974)*, 4 thermostatic valve types are presented and the above mentioned properties are discussed. It is further concluded that, the gas based thermostatic valve is the most advantageous, due to its relatively small time constant, the very small hysteresis band and its small dependency of the supply water temperature.

Based on the considerations above, the following assumptions about the gas based thermostatic valve will be applied:

1. No hysteresis.

2. No supply water temperature dependency.

3. No elasticity.

Under these assumptions the temperature of the gas T_g in the thermostatic head is described by :

$$\frac{dT_g}{dt} = \frac{1}{\tau}(T_a - T_g) \qquad (3)$$

where τ denotes the time constant of the gas temperature, when T_g is exposed to the ambient temperature T_a (see Figure 1).

The stationary relation between the P-band temperature X_p and the valve flow q is usually listed in the data sheet of a thermostatic valve. Data for the considered thermostatic valve are presented in Table 1.

X_p	0.5	1.0	1.5	2.0	6.0	K
q	0.22	0.40	0.55	0.65	0.85	m^3/h
K_v	0.22	0.40	0.55	0.65	0.85	$m^3/h/Bar^{0.5}$

Table 1: *RA-FN 15 data, at Δp = 1 Bar (Danfoss, 1988).*

As defined by the standard (EN 215-1), the flow q must be zero, when the relative temperature X_p is zero. The K_v values in Table 1 are calculated using (2). Based on the data in Table 1 a relation between the P-band temperature X_p and the valve constant K_v can be found. An adequate description is :

$$K_v(X_p) = a \tanh (b\, X_p) \tag{4}$$

The parameters of this function have been found by non-linear least squares estimation and the estimated parameters are shown in Table 2. The estimated standard deviation of the residuals is denoted σ_F. An analysis of (4) yields:

$$\begin{aligned} X_p \rightarrow 0 \ \text{implies that}\ K_v \rightarrow 0 \ \text{and}\ \frac{d K_v}{d x_p} \rightarrow a\, b \\ X_p \rightarrow \infty \ \text{implies that}\ K_v \rightarrow a \ \text{and}\ \frac{d K_v}{d x_p} \rightarrow 0 \end{aligned} \tag{5}$$

Table 2: *Estimated parameters.*

Parameter	a	b	σ_F
Estimate	0.85	0.51	$4.23 \cdot 10^{-3}$
Unit	$m^3/h/Bar^{0.5}$	1/K	$m^3/h/Bar^{0.5}$

A dynamic thermostatic valve model can now be established. In the dynamic case, the ambient temperature T_a in (1) is replaced by the sensor gas temperature T_g. Hence, the dynamic model of a (gas based) thermostatic valve is:

$$\frac{dT_g}{dt} = \frac{1}{\tau}(T_a - T_g) \tag{6}$$

$$K_v(T_g) = \begin{cases} a \tanh(b(T_{set} + 2 - Tsubg)) & \text{, for } T_{set} + 2 > T_g \\ 0, & \text{for } T_{set} + 2 \leq T_g \end{cases} \tag{7}$$

where the input is denoted by the ambient temperature T_a (air temperature). The output of the thermostatic valve model is the valve constant K_v, which is generated as a non-linear function of the gas sensor temperature T_g. It is noted that K_v is indirectly measured by the fraction $K_v = q/\sqrt{\Delta p}$.

3 Experiments

The thermostatic valve experiments have been conducted in connection with a heating experiment *(Hansen, 1996)* in a test house (*Rasmussen and Saxhof, 1982*) at the Department for Buildings and Energy (IBE) at DTU. A test setup water based central heating system designed/build by Grundfos A/S was used as heating system. For further information see *(Hansen, 1996)*.

The measured quantities (defined in Figure 5) are:

- T_a, the air temperature next to the thermostatic head.
- p_i, p_o, the inlet and outlet pressure, respectively.
- q, the valve flow.

The basic idea was to manipulate the ambient temperature around the thermostatic head. For this purpose a modified heat blower was used. The blower part was blowing continuously, while the heat element was on/off controlled by the data acquisition system, as sketched in Figure 5.

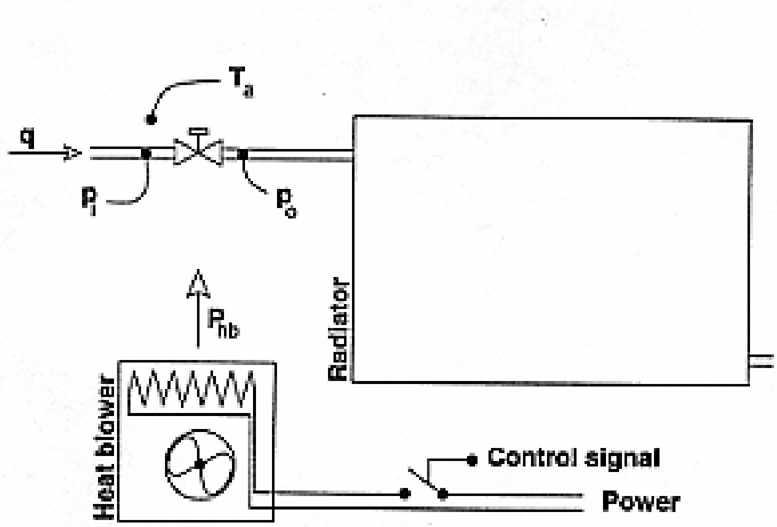


Figure 5: *Sketch of the experimental thermostatic valve setup.*

Figure 5: *Sketch of the experimental thermostatic valve setup.*

A predefined PRBS-sequence (Pseudo Random Binary Signal, see *(Godfrey, 1980)*) was used to control the heat element. In order to justify the assumption about no dependency on temperature of the supply water, the supply temperature was set to 30°C (i.e. close to the air temperature). The pump pressure was approximately constant. Due to the heat blower arrangement and the low supply temperature, the system in Figure 4 is operating in open loop during the experiment. The time constant of the thermostatic valve was expected to be in the interval 1-30 minutes. Thus, the data were sampled with a sample time of 10 seconds. Step "4" on the thermostatic head was selected to be the set point (~23°C).

In order to be able to perform a cross validation of the experimental results, two data sets were collected. The two data sets, sequences I and II are shown in Figure 6 and Figure 7, respectively. Each plot contains T_a [°C] in the first subplot, q [l/h] in the second, Δp [mWc] (1.0 meter-Water-column is equivalent to 0.1 Bar) and the pump pressure [mWc] (the dotted line) in the third subplot and a pseudo measurement of the valve constant K_v [m^3/h/$Bar^{0.5}$] (i.e. calculated as $q/\sqrt{\Delta p}$) in the last subplot.

A PRBS-signal can be characterized by two parameters *(Godfrey, 1980)*: 1) the number of the shift register stages n and 2) the smallest period time where the signal is constant, T_{PRBS}. The following PRBS-signals were used:

1. PRBS with $n = 3$ and $T_{PRBS} = 50$ seconds.

2. PRBS with $n = 5$ and $T_{PRBS} = 20$ seconds.

Two remarks on the data:

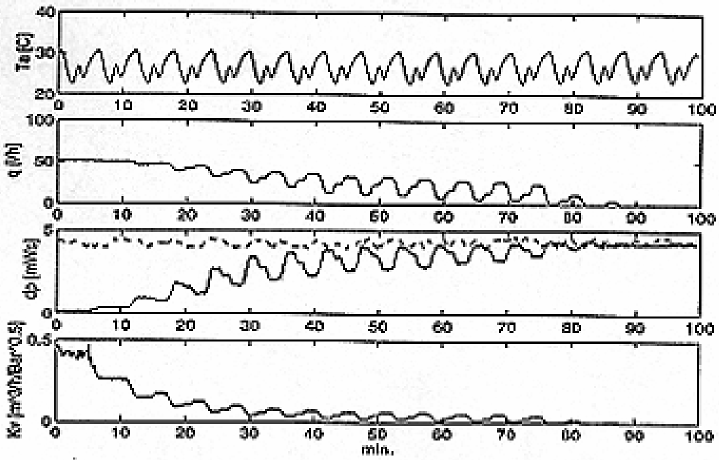


Figure 6: *Measured data: Sequences I. The first subplot contains T_a [°C], the second q [l/h], Δp [mWc] and the pump pressure [mWc] (the dotted line) in the third subplot and a pseudo measurement of the valve constant K_v [$m^3/h/Bar^{0.5}$] in the last subplot.*

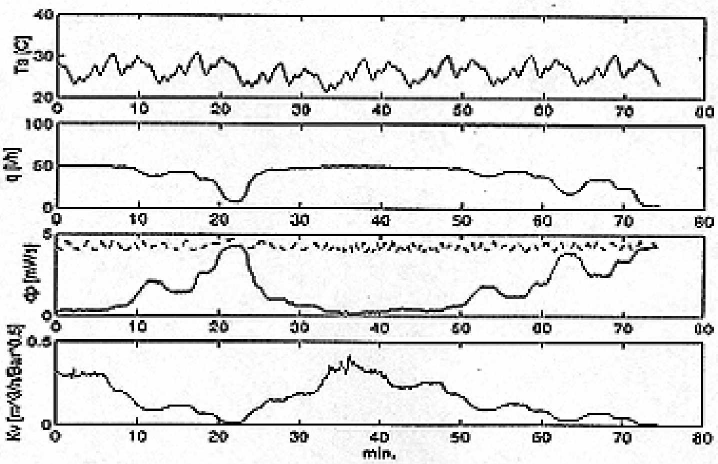


Figure 7: *Measured data: Sequences II (contents as in Figure 6).*

Figure 6: *Measured data: Sequences I. The first subplot contains T_a [℃], the second q [l/h],*

Δp [mWc] and the pump pressure [mWc] (the dotted line) in the third subplot and a pseudo measurement of the valve constant K_v [$m^3/h/Bar^{0.5}$] in the last subplot.
Figure 7: *Measured data: Sequences II (contents as in Figure 6).*

1. The measured ambient temperature T_a is - due to the experimental setup - a combination of a slightly low-passed version of the control signal to the heat element and a slowly varying room temperature.

2. When the valve is almost full open (X_p large), the calculated valve constant is noisy, see e.g. the first 5 minutes in the last subplot of Figure 6. This is mainly due to insufficient measuring accuracy.

4 The Grey-Box Modelling Method

In this section the method used to estimate the parameters of the model (6)-(7) of the thermostatic valve is briefly described. The method is a maximum likelihood method for estimating parameters in stochastic differential equations based on discrete time measurement data. For a more detailed description of the method we refer to *(Madsen and Melgaard, 1991*) or *(Melgaard and Madsen, 1993)*. A description of a linear case is given in *(Madsen and Holst, 1995)*.

The observations are given in discrete time, and, in order to simplify the notation, we shall assume that the time index k belongs to the set {0, 1, 2, ..., N}, where N is the number of observations. Introducing:

$$\mathcal{I}(k) = [\boldsymbol{Y}(k), \boldsymbol{Y}(k-1), \ldots, \boldsymbol{Y}(1), \boldsymbol{Y}(0)]^T \tag{8}$$

i.e. $\mathcal{I}(k)$ is a vector containing all the observations up to and including sample number k. In the case of equidistant sampling with sampling time T, the time t and the sample number k are related as $t = kT$.

Using matrix notation the stochastic differential equation describing the dynamics of the thermostatic valve can be written as an Itô Equation (see e.g. *(Øksendal, 1995)*):

$$d\boldsymbol{X}(t) = f(\boldsymbol{x}, \boldsymbol{U}, t)dt + \boldsymbol{G}(\boldsymbol{U}, t)d\boldsymbol{w}(t) \tag{9}$$

where $\boldsymbol{X}$ is the state vector, $\boldsymbol{U}$ an input (e.g. control) vector, $\boldsymbol{w}$ a vector Wiener process (or Brownian motion) (see e.g. *(Kloeden and Platen, 1995)*), and $\boldsymbol{G}$ is a matrix describing any input or time dependent variation related to how the variation generated by the Wiener process enters system.

For the observations we assume the discrete time relation:

$$\boldsymbol{Y}(k) = h(\boldsymbol{X}, \boldsymbol{U}, k) + \boldsymbol{e}(k) \tag{10}$$

where $\boldsymbol{e}(k)$ is assumed to be a Gaussian white noise sequence independent of $\boldsymbol{w}$. All the unknown parameters, denoted by the vector $\boldsymbol{\theta}$, are embedded in the continuous time state space model (9) and (10).

The likelihood function is the joint probability density of all the observations assuming that the parameters are known, i.e.

$$L'(\theta;Y(N)) = p(Y(N)\,|\,\theta)$$
$$= p(Y(N)\,|\,Y(N-1),\theta)\,p(Y(N-1)\,|\,\theta) \tag{11}$$
$$= \left(\prod_{k=1}^{N} p(Y(k)\,|\,Y(k-1),\theta)\right)p(Y(0)\,|\,\theta)$$

where successive applications of the rule $P(A \cap B) = P(A|\,B)P(B)$ is used to express the likelihood function as a product of conditional densities.

In order to evaluate the likelihood function it is assumed that all the conditional densities are Gaussian. In the case of a linear state space model it is easily shown that the conditional densities actually are Gaussian (*Madsen and Melgaard, 1991*). In the more general non-linear case the Gaussian assumption is an approximation.

The Gaussian distribution is completely characterized by its mean and covariance. Hence, in order to parameterize the conditional distribution, we introduce the conditional mean and covariance as:

$$\boldsymbol{ì}(k|k-1) = E\,[\mathbf{Y}(k)|\boldsymbol{l}(k-1),\theta] \tag{12}$$

$$\mathbf{R}(k|k-1) = V\,[\mathbf{Y}(k)|\boldsymbol{l}(k-1),\theta] \tag{13}$$

respectively. It may be noticed that these correspond to the one-step prediction and the associated covariance, respectively. Furthermore, it is convenient to introduce the one-step prediction error (or innovation):

$$\boldsymbol{\varepsilon}(k) : \boldsymbol{Y}(k) - \boldsymbol{ì}(k|k-1) \tag{14}$$

When calculating the one-step prediction and its variance, an iterated extended Kalman filter is used. The extended Kalman filter is simply based on a linearization of the system equation (9) around the current estimate of the state (see *(Gelb, 1974)*). The iterated extended Kalman filter is obtained by local iterations of the linearization over a single sample period.

Using (11) - (14) the conditional likelihood function (conditioned on $\boldsymbol{Y}(0)$) becomes:

$$L(\theta;Y(N)) = \prod_{k=1}^{N}((2\pi)^{-m/2}\det R(k/k-1)^{-1/2}$$
$$\exp(-\frac{1}{2}\varepsilon(k)^{T} R(k/k-1)^{-1}\varepsilon(k))) \qquad (15)$$

where m is the dimension of the $\boldsymbol{Y}$ vector (number of outputs). Consider the logarithm of the conditional likelihood function:

$$\log L(\theta;Y(N)) = -\frac{1}{2}\sum_{k=1}^{N}(\log(\det R(k/k-1)) + \varepsilon(k)^{T} R(k/k-1)^{-1}\varepsilon(k))$$
$$-N\frac{m}{2}\log(2\pi) \qquad (16)$$

The maximum likelihood estimate (ML-estimate) is the parameter vector ^θ, which maximizes the likelihood function. Since it is not possible, in general, to perform the optimization analytically, a numerical method has to be used. A reasonable method is the quasi-Newton method.

An estimate of the uncertainty of the parameters is obtained by the fact that the ML-estimator is asymptotically Gaussian distributed with mean θ and covariance :

$$\boldsymbol{D} = \boldsymbol{H}^{-1} \qquad (17)$$

where the matrix $\boldsymbol{H}$ is given by :

$$h_{ij} = -E\left[\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log L(\theta;Y(N))\right] \qquad (18)$$

An estimate of $\boldsymbol{D}$ is obtained by equating the observed value with its expectation and applying:

$$h_{ij} \approx -\left(\frac{\partial^2}{\partial\theta_i\partial\theta_j}\log L(\theta;Y(N))\right)_{/\theta=\hat{\theta}} \qquad (19)$$

The above equation can be used for estimating the variance of the parameter estimates. The variances serve as a basis for calculating t-test values for tests under the hypothesis that the parameter is equal to zero. Finally, the correlations between the parameter estimates are readily found based on the covariance matrix $\boldsymbol{D}$.

5 Estimation and Results

The first task is to reformulate the physical model of the thermostatic valve (equations (6) and (7)) as a stochastic differential equation :

$$DT_g(t) = -\frac{1}{T}(T_g(t) + T_a(t))dt + dw(t) \tag{20}$$

with the observation equation:

$$y(k) = a \tanh(b(T_{set} + 2 - T_g(k))) + e(k) \tag{21}$$

The output of the model *y(k)* denotes the calculated valve constant K_v at sample time $t = kT$. The system noise process *w(t)* is assumed to be a Wiener process and the measurements noise *e(k)* is assumed to be sequence of Gaussian white noise. It is further assumed that *w(t)* and *e(k)* are mutually uncorrelated.

The next step is to determinate whether all parameters are identifiable. The parameters of the model are: τ, $T_g(0)$, T_{set}, *a, b,* and the two variances σ_w^2 and σ_e^2, i.e. in total 7 parameters. In an equivalent linear model (single input, single output) it is only possible to identify 5 parameters (of which 2 are noise parameters (*Melgaard, 1994)* and one the initial value). The considerations below (2) combined with the analysis in (5) (and the second comment in Section 3) indicate that the information in data about the valve constant will be small - in the case when X_p is large. Hence, it will not be possible to estimate parameter *a* with *simple* experimental setup. Due to this fact, *a* is set to the estimated value given in Table 2.

In order to further decrease the number of parameters to be estimated, the following constraint is applied:

$$T_g(0) = T_{set} + 2 - \frac{1}{b} \operatorname{atanh}\left(\frac{K_{v0}}{a}\right) \tag{22}$$

where K_{v0} denotes the valve constant (known from data) at time 0. $T_g(0)$ is equivalent to $p(\mathbf{Y}(0)/\theta)$ in (11). Thus, the number of parameters to be estimated is reduced to 5.

The estimation results from the two sets of data are shown in Table 3. Both the estimated parameters and their standard deviations are shown. The presented estimates are found to be consistent in the way that the estimation procedure converges to the same set of estimates independent of the initial settings. Furthermore, t-test values and other statistical tests have not caused any doubt about the estimates.

Table 3 : *Parameter estimates.*

Seq.	Par.	τ	T_{set}	b	σ^2_w	σ^2_e
I	Est. Std.	769 149	24.97 0.208	0.126 $26.0\cdot10^{-3}$	$2.44\cdot10^{-4}$ $1.06\cdot10^{-4}$	$9.99.10^{-6}$ $4.68\cdot10^{-8}$
II	Est. Std.	756 174	25.03 0.326	0.159 $39.0\cdot10^{-3}$	$2.80\cdot10^{-4}$ $1.44\cdot10^{-4}$	$9.99.\cdot10^{-6}$ $5.10\cdot10^{-8}$
	Unit	sec.	°C	1/°C	$(°C/sec.)^2$	m^6/l^2/Bar

The estimate values of τ are around 13 minutes, which is quite reasonable. The estimated values of T_{set} are around 25°C, which is somewhat larger than expected, as step "4" should be equivalent to 23°C according to the specifications. It should be noted, however, that under normal conditions the airflow around the thermostatic valve is small (compared with the airflow in the experiments). Furthermore, the temperature of the water was only 30°C. A higher supply temperature would probably lead to a smaller set point temperature. The estimated values of b are both smaller than the estimated parameter value given in Table 2. Based on the analysis in (5) it can be concluded that the derivative of the valve constant with respect to the P-band temperature is smaller than expected. In other words, the hydraulic valve resistance is increasing faster, when the P-band temperature is decreasing. This fact could be explained by a smaller P-band of the system.

The number N of samples used in the estimation, the variance of the one-step predictor error Var (Pe) and the negative log-likelihood function *Neg.L* - the cost function (e.g. (16)) evaluated in the found optimum - are shown in Table 4, together with *Neg.L/N* in the last column.

Table 4 : *Performance characteristics*

Seq.	N	Var(PE)	*Neg.L*	*Neg.L/N*
I	441	$4.50\cdot10^{-5}$	-1469	-3.3
II	446	$6.88\cdot10^{-5}$	-1442	-3.2

The prediction and simulation performances of the dynamic estimated models are shown in Figures 8 and 9, respectively. The first of the subplots contains the measured valve constant, the one-step predicted valve constant and a simulation of valve constant. The prediction error is shown in the second subplot. It can be observed that the prediction error is big, when the valve constant is big due to the limited information in data (see second remark in Section 3). In the first subplot, the dash-dot-dash line indicates the simulated valve constant based on the achieved parameter estimates and the measured ambient temperature T_a as

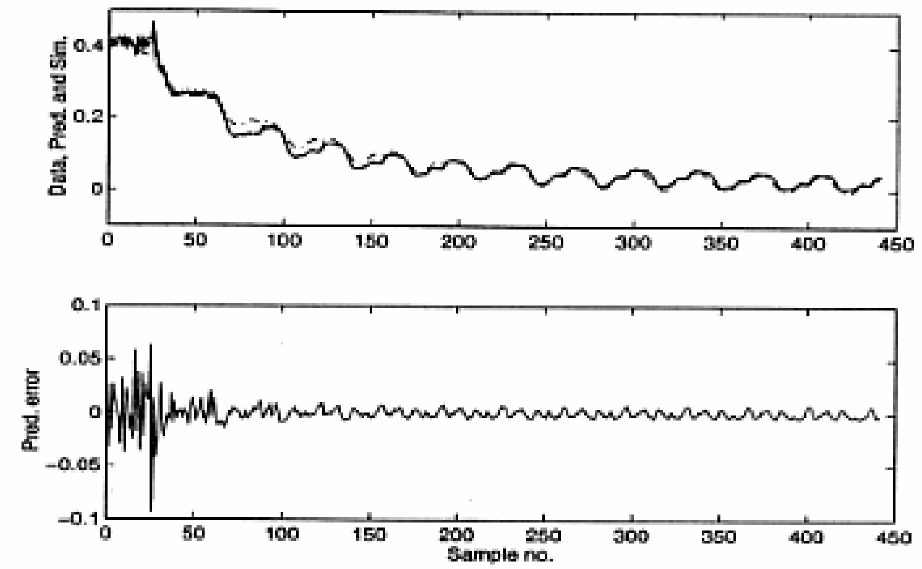


Figure 8: *Data, prediction, simulation (in the first subplot) and prediction error (in the second subplot) of sequences I. The dash-dot-dash line in the first subplot indicates the simulated valve constant.*

Figure 8: *Data, prediction, simulation (in the first subplot) and prediction error (in the second subplot) of sequences I. The dash-dot-dash line in the first subplot indicates the simulated valve constant.*

input in the simulation. Also the stimulated valve constant is quite close to the real valve constant.

One way to evaluate the achieved estimates is to perform a cross validation, i.e. to use the estimated parameters from one estimation in a simulation with the other set of data. Thus, defining the following cases:

1. Using the estimated parameters from sequence II in a simulation with the data from sequence I.

2. Using the estimated parameters from sequence I in a simulation with the data from sequence II.

a comparison can be performed.

The results of the cross validation are shown in Table 5. The cost function values in Table 5 are of course larger than the cost function values in Table 5 (i.e. which are based on the optimal parameters). The variance of the one-step prediction error in the cross validation case is virtually of the same size as in the optimal case.

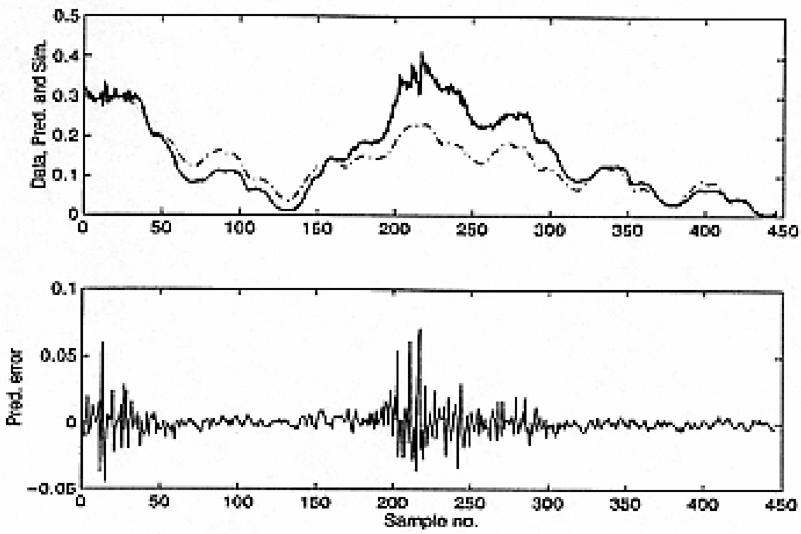


Figure 9: *Data, prediction, simulation and prediction error of sequences II (as in Figure 8).*

Figure 9: *Data, prediction, simulation and prediction error of sequences II (as in Figure 8).*

Table 5: *Cross validation characteristics.*

Case	Var(PE)	*Neg.L*
1	$6.85 \cdot 10^{-5}$	-1451
2	$4.51 \cdot 10^{-5}$	-1421

The achieved estimates for the two data sequences can be compared, as shown in Figure 10, based on the intervals of the estimates. A visual test can easily be performed by looking at the estimates, in order to decide whether or not the individual parameter estimate from each sequence are alike. It is clearly seen that the two set of parameters do not significantly differ. The two set of parameter estimates are alike.

6 Concluding Remarks

A dynamic nonlinear model of a thermostatic valve has been established and validated using two sequences of measurements. A set of parameters is estimated for each of the sequences. The dynamic model with the estimated parameters gives a good description of the measurements.

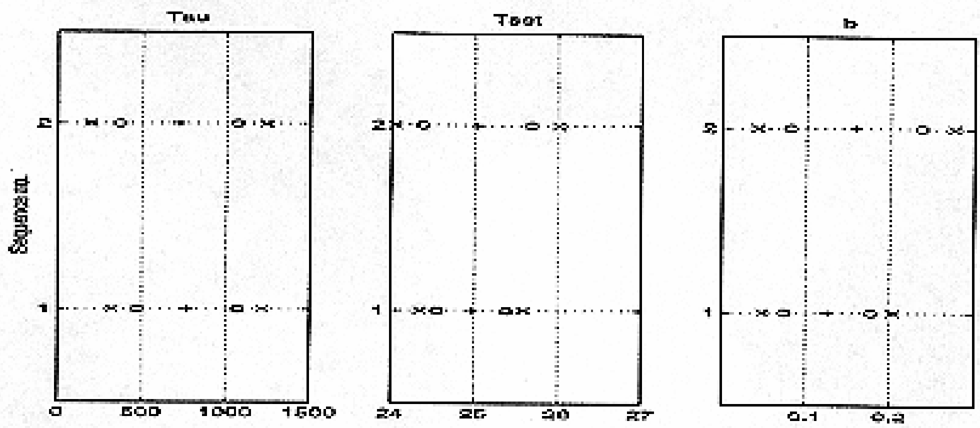


Figure 10: *Confidence intervals of the estimated parameters: τ, T_{set} and b. Used symbols: estimate(+), 95%(o) and 99%(x).*

Figure 10: *Confidence intervals of the estimated parameters: τ, T_{set} and b. Used symbols: estimate (+), 95%(o) and 99%(x).*

The validation of the results includes a cross validation, where the two sets of parameter estimates have been compared using their confidence intervals. Also the cross validation indicates that very reasonable results are found.

Hence, the real time constant is approximately 13 minutes. The set point temperature turned out to be about 2°C larger than the value given in the specification. This could be explained by the low supply temperature.

The described dynamic model of thermostatic valve is implementable as a component in our library of dynamic models. The model will be extended with a dynamic model for the dependency of the water supply temperature. Furthermore, the influence of hysteresis is to be investigated.

Acknowledgment

This work has been financial supported by the Danish Energy Agency, the Danish Academy of Technical Sciences and Grundfos A/S.

The authors are also grateful to Bjarne Saxhof and Jørgen M. Schultz, the Department for Buildings and Energy at DTU, for their support in connection with the experiments in the test house.

References

Danfoss (1988). *Radiatortermostat/Ventilhuse, RA 2000/RA-FN og RA-G.* Danfoss A/S, VD.51.A5.01 edition.

Dansk Standard (1988). Thermostatic Radiator Valves. Part 1: Requirements and Test

Methods. (Danish title: Termostatiske radiator ventiler. Del 1: Krav og prøvningsmetoder). Technical report, Dansk Standard. DS/EN 215-1.

Fox, R.W. and McDonald, A. T. (1985). *Introduction to Fluid Mechanics.* John Wiley & Sons.

Gammelby, J. (1974). Heizkörperthermostate. Grundsätze und Eigenschaften. *The Danfoss Journal*, 3.

Gelb, A. (1974). *Applied Optimal Estimation.* MIT Press, New York.

Godfrey, K. R. (1980). Correlations Methods. *Automatica*, 16(2):527-534.

Hansen, L.H. (1996). Dynamic Analysis of a low Energy Test House and a Central Heating System. Technical Report No. 20, Department of Mathematical Modelling, Technical University of Denmark.

Kloeden, P. an Platen, E. (1995). *Numerical Solutions of Stochastic Differential Equations*. Springer-Verlag, 2 edition.

Madsen, H. and Holst, J. (1995). Estimation of Continuous-time Models for the Heat Dynamics of a Building. *Energy and Buildings*, 22:67-79.

Madsen, H. and Melgaard, H. (1991). The Mathematical and Numerical Methods used in CTLSM. Technical Report No. 7, IMSOR, DTH.

Melgaard, H. (1994). *Identification of Physical Models.* PhD thesis, IMM, DTU.

Melgaard, H. and Madsen, H. (1993). CTLSM, Continuous Time Linear Stochastic Modelling. In Bloem, J.J., editor, *Workshop on Application of System Identification in Energy Savings in Buildings*, pages 41-60. Commission of the European Communities, DG XII.

Øksendal, B. (1995). *Stochastic Differential Equations.* Springer-Verlag, 4 edition.

Rasmussen, N. H. and Saxhof, B. (1982). Simultaneous Testing of Heating Systems. Technical Report No. 128, Thermal Insulation Laboratory, Technical University of Denmark.

Svensson, A. (1978). Functionality of Thermostatic Radiator Valves. (Swedish title: Radiatortermostatventilors funktion). Technical report, The national Swedish Institute for Building Research. Lägesrapport M78:4.