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BUILDING THERMAL MODELLING : AN EXAMPLE OF RADIATIVE EXCHANGE BETWEEN SYSTEM AND ENVELOPPE TAKING ACCOUNT UNCERTAINTIES

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ABSTRACT:

The problem developed and analyzed in this paper is that of the estimation of the uncertainty associated with the results obtained by numerical simulation codes of physical systems induced from input data. Implicitly posed by the authors is the delicate question concerning the use of the output obtained by calculation codes used in prediction situations, and the reliability which can be attributed to such ouput. One example of thermal behaviour of simple physical systems is treated, serving as an illustration.

In a first introductory section, a general problem is developed allowing the more specific problem to be located and the theoritical context of the study to be determined. It is shown that the methods which allow determination of output uncertainty are linked to the nature of the models used and to their mathematical formalism, notably in the most commonly seen case when the formulation of explicit analytical solutions is absent. Two classic methods are then presented. One is a probability method, the Quasi Monte Carlo method; the other, a determinist method, a differential analysis with finite differences. These two methods are tested on the models represented by linear systems, quite frequent in the area concerned, with emphasis on the question of singularities, for small dimension matrixes.

One example of model of thermal behaviour of building premises is then treated. The radiative thermal exchanges are decoupled and convection is not treated, which leads to the resolution of a dimension 10 linear system. This model is extracted fractions of larger models of thermal behaviour of buildings, allowing a simplified presentation of the methods proposed, the objective of which is its sequential application in large calculation codes.

The comparison of the two methods leads to conclusions in favour of differencial analysis, which is clearly more economical in calculation time and which makes it possible to identify sensitive data with significant bearing on output uncertainty. Nevertheless, it is emphasized that for this method it is essential to enter into the calculation code formalism in order to express the partial derivatives of the transfer function. Globally, the authors conclude that a relative superiority of the differential analysis exists, particularly in the case of large codes where Monte Carlo use would be prohibitive in calculation time.

KEYWORDS :
Uncertainties, Reliability, Thermal Building Behaviour, Singularity, Linear model, Monte-Carlo, Finite Differences Differential Analysis.

1. INTRODUCTION : DESCRIPTION OF THE PROBLEM

Let us consider a building model of which all the output can be grouped in the vector S with his components as walls surfaces temperatures, air rooms temperatures, air velocities in rooms, air humidities, ... This vector can be expressed in the form of a linear or non linear relation □ of the model's input data and the control parameters according to the very general expression presented by relation (1). Thus :

$$S = □ (E , C) \qquad (1)$$

where $S = \{s_1, s_2, ..., s_q\}$ is the output vector with dimension q.
$E = \{e_1, e_2, ..., e_n\}$ is the input data vector with dimension n. E contains the whole thermal and geometric building data. For example : south area windows, east area windows, ..., insulation thermal conduction coefficients, concrete thermal conduction coefficients, materials thickness....
$C = \{c_1, c_2, ..., c_m\}$ is the control parameters vector, with dimension m. For example: reference temperature in rooms, maximum heating power, ...
E and C constitute the data. The function □ is a function representative of the model, known analytically, numerically or even experimentally.

The problem of estimation of the reliability of results, i.e. error analysis, consists therefore in evaluating the repercussions on the components of the output S of disturbances generated on the elements of E and C. In this perspective, the fluctuations affecting the data must be distinguished. The most direct method is to associate a variation interval with each component e_i and c_i :

$$e_i \in [e_i - \Delta e_i , e_i + \Delta e_i]$$
$$c_i \in [c_i - \Delta c_i , c_i + \Delta c_i] \quad , \text{where } i = 1, ..., n \text{ or } m.$$

Likewise, resolving the problem of error analysis will consist to reach a fluctuation interval on the output s_j :

$$s_j \in [s_j - \Delta s_j, s_j + \Delta s_j] \quad , \text{where } j = 1, ..., q.$$

The variable of output s_j may be considered as a function F_j (j ith component of □). Taking an elementary case of 2 dimensions for the sake of convenience, we have a surface. The uncertainty analysis will proceed in function of the domain defined by the different variation limits. It is therefore expected that the optimal combination which produces the maximal amplitude variation of the response S be located in this zone.

2. DETERMINATION OF OUTPUT UNCERTAINTY INTERVAL

2.1 Reference method : Monte Carlo method (MC)

In the Monte Carlo method, a probability density is assigned to all of the input model data which may be affected with uncertainties. For each simulation carried out, a value is randomly selected for all uncertain data according to their respective probability density, and all the uncertain parameters are simultaneaously disturbed. The Monte Carlo method thus makes it possible to take full account of the diverse interactions taking place among all input data of the model. The simulation product is

then saved, and the process is reinitialised, using a unique and different set of input data for each operation.

The total uncertainty of simulation results can be expressed by the standard deviation :

$$s_D(p_j) = [\frac{1}{(N-1)}(\sum P_{jk}^{\ 2} - N.(m(P_{jk}))^2)]^{\frac{1}{2}}$$

(2)

where N is the number of simulations and m the average of the output values, P_{jk} the probability value of p_j parameter.

An estimation of s_D can be deduced after each simulation and the precision of this estimation can be determined by using a distribution of ☞" to calculate an interval of reliability for s_D [1] . The precision of s_D only depends on the number of simulations carried out, as shown by relation (2). The main inconvenience is that the sensitivity of the predictions related to the individual variations of each parameter is not accessible, since these input data all vary simultaneaously. The algorithm used is that of the IMSL Library [2]. Its consistency with an exact Gaussian is very good.

Another inconvenience is the large time computation when using MC method for complex models as building thermal behaviour simulation or fluid mechanic simulation. Figure 1 shows such variance variation with number of computations $s_D^2 = f(N)$ in case of thermal system model. We observe number of computations must be greater than about 100 for a reliability interval less than 15% of final s_D value (i.e. for a very great number of computations). Applications studied in §3 involved about 800 computations to reach 5% of final s_D value.

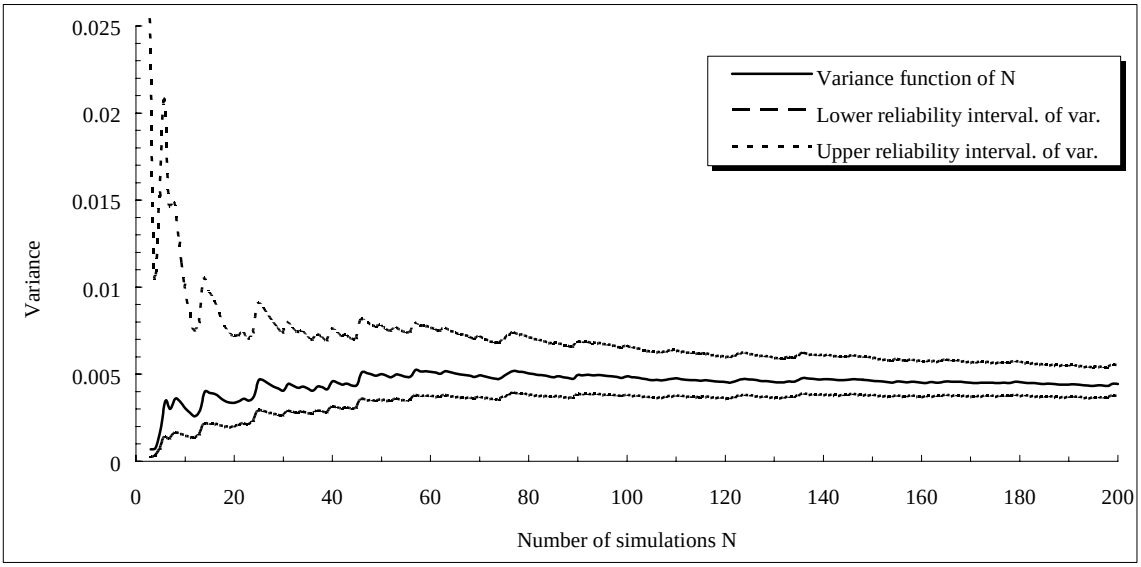

Figure 1 - Variance variation with number of computations $s_D^2 = f(N)$ with Monte Carlo method.

2.2 Finite Differences Differential Analysis method (FDDA)

Two possible cases can then be distinguished :

- The function □ has no singularity on the domain of fluctuation of the disturbed elements of the model. An analysis is done in first order of each output s_k. If the function □ is differentiable to the point considered, we have :

$$\Delta s_k = \sum_{j=1}^{m} \left| \frac{\partial F_k}{\partial e_j} \right| \Delta e_j + \sum_{j=1}^{n} \left| \frac{\partial F_k}{\partial c_j} \right| \Delta c_j \qquad (3)$$

where m, n are the numbers of disturbed input data and parameters. F_k correspond to the expression of □ relative to the output s_k under considered.

- If the function □ exhibits one or more singularities, it is necessary to proceed to a case by case study.

2.1.1 Evaluation of partial derivatives

The calculation of partial derivatives can be developed according to various approaches in function of the complexity of the model studied. The ideal case lies in the possibility of analytically establishing these primary derivatives. The direct differential analysis then proceeds to the derivation of equations of the model. Thus, designating by
S =$\{s_1,s_2,...,s_q\}$ the output vector of a system of equations (which can be non-linear) :

$$f(s^n, \ldots, s^1, \dot{s}, \alpha_1, \ldots, \alpha_m) = 0 \qquad (4)$$

with $\dot{s} = \frac{\partial s}{\partial t}$, χ_i grouping together the data (control parameters and input data of the system considered). The s^n, ... , s^j, ... s^1 denote derivatives of space of the order *n,..., j,...1*. Introducing as a new solution to the problem the function of sensitivity of output $\chi = \left. \frac{\partial s}{\partial \alpha_i} \right|_{\alpha_0}$, its differentiation gives the following system [3] :

$$\left. \frac{\partial f}{\partial s^n} \right|_{\alpha_0} \chi^n + \ldots + \left. \frac{\partial f}{\partial s} \right|_{\alpha_0} \chi = - \left. \frac{\partial f}{\partial \alpha} \right|_{\alpha_0} \qquad (5)$$

These equations of output sensitivity produce the sensitivities of all output on the parameter or input data observed. This approach may be adopted whenever the elements of the above equation (5) are easily accessible. However this opportunity is not frequent due either to the high degree of complexity of the equations, or to the fact that the model is not explicit (numerical or experimental model). In addition, the use of this approach automatically makes numerous modifications of the theoretical model necessary to be able to recuperate the derivatives. This continues to be difficult to carry out on complex models.

The possibility that remains is to resort to approximate calculations using the method of finite differences. Each partial derivative will be evaluated using the following relation :

$$\frac{\partial F_k}{\partial e_j} \approx \frac{F_k(E+\delta E, C) - F_k(E, C)}{\delta e_j}$$

(6)

with,

$$\delta E = [0, \ldots, 0, \delta e_j, 0, \ldots 0]^T$$

where δE is equal to a weak disturbance (compared 1) to the j ith position, and zero everywhere else. For a first order calculation , δe_j is included between 10^{-3} and 10^{-6} .

Therefore m + n + 1 evaluations of □ are necessary to calculate the ◑s_k.

Other authors have described Finite Difference method in sensitivity analysis context. It is important to note Finite Difference method (marked FD) [4] is different of Finite Differences Differential Analysis method (marked FDDA). Indeed, FD method gives an approximate value for derivative $\frac{\partial F_k}{\partial e_j}$ as shown in figure 2, for a central value $e_{j,o}$, between $e_{j,o}$ - Δe_j and $e_{j,o}$ + Δe_j , where [e_j - Δe_j , e_j + Δe_j] is the data variation interval defined in §1. Our calculation reach a numerical estimation derivative value with a very small displacement δe_j around $e_{j,o}$ value (6). An approximative derivatives calculation is nevertheless correct for linear function F_k with smooth variations, but is not efficient for non-linear cases with rapid variations which are not studied here.

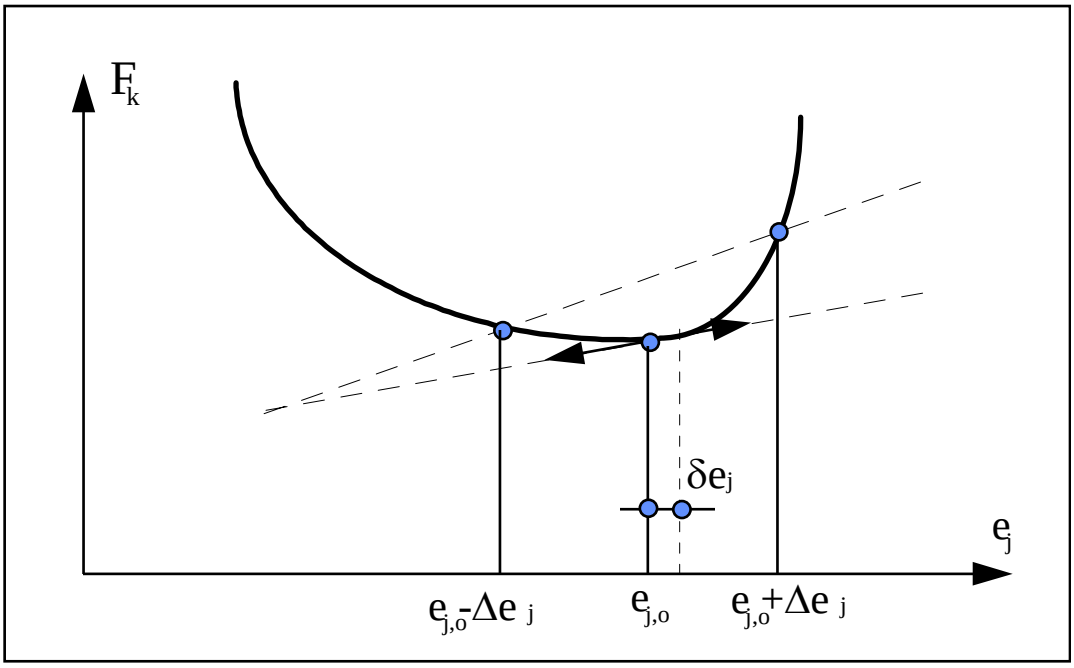

Figure 2 - The two possible derivative calculations.

In the case of approximative calculation, derivative $\partial F_k / \partial e_j$ is obtained with $e_{i,o}$ - Δe_i and $e_{i,o}$ + Δe_i interval. Calculation with a weak displacement δe_j gives a better accuracy.

2.2.2 The particulary case of linear systems

Physical systems described by linear systems noted in the matrix form A X = B are considered, where A is a square matrix in *n x n* dimensions, and the vectors X and B are in *n* dimensions. The linear system's solution is written $X = A^{-1} B$. For line *k*, we obtain :

$$X_k = \sum_{j=1}^{n} \alpha_{kj} \; b_j \qquad (7)$$

where α_{kj} is the *kj* term of the inverse matrix A^{-1}.

The disturbance of a term of the system is denoted by the symbol δ. Disturbances on A and B provide disturbances on X :

$$(A + \delta A) (X + \delta X) = B + \delta B \qquad (8)$$

Our analysis is limited to the first order, which means that the second order term $\delta A . \delta X$ of relation (8) is neglected, so :

$$A X + A \delta X + \delta A X \approx B + \delta B \qquad (9)$$

We try to construct a framing for the output vector X. If we only consider disturbances on B vector, differentiating (7) :

$$dX_k = \sum_{j=1}^{n} \alpha_{kj} \, db_j \qquad (10)$$

and consequently,

$$\Delta X_k \approx \sum_{j=1}^{n} \left|\alpha_{kj}\right| \; \Delta b_j \qquad (11)$$

Given the linearity of the system, the framing of the fluctuations is correct.
Following the same reasoning as previously, and differentiating with disturbances only affecting matrix A terms, we obtain :

$$dX_k = \sum_{l=1}^{n} \left(\sum_{j=1}^{n} b_j \frac{\partial \alpha_{kj}}{\partial a_{kl}} \right) da_{kl} \qquad (12)$$

and finally,

$$\Delta X_k \approx \sum_{l=1}^{n} \left| \left(\sum_{j=1}^{n} b_j \frac{\partial \alpha_{kj}}{\partial a_{kl}} \right) \right| \Delta a_{kl} \qquad (13)$$

Yet, explicit form of α_{kj} is rarely avalaible and derivative analytic calculation of $\frac{\partial \alpha_{kj}}{\partial a_{kl}}$ terms is not generally possible. So, the problem is solved by using general formulation (3) where □ is represented by $A^{-1}B$ operator, and where $s_k = X_k$.

3. APPLICATION TO A RADIATIVE EXCHANGE THERMAL MODEL

3.1 Model of radiative exchange between an emitter and its enclosure.

The aim here is to determine the emission of a radiant panel towards the front, that is towards the room in which it is placed, generally against an inner wall. This heat flux is obtained indirectly by the intermediary measure of surface temperatures of the enclosure of which the radiosities are deduced. The determination of the heat flux towards the back is known and will not be treated here.

This method of studying emitters, used by INARD and MOLLE [5], concerns only plane radiant strips (of negligible thickness) and leads to the evaluation of the radiative flux P . As the emitter can always be considered near the YZ plane, it can be written (figure 3) :

$$P = P_f + P_b = (\text{Radiat. Flux Forward}) + (\text{Radiat. Flux Back})$$

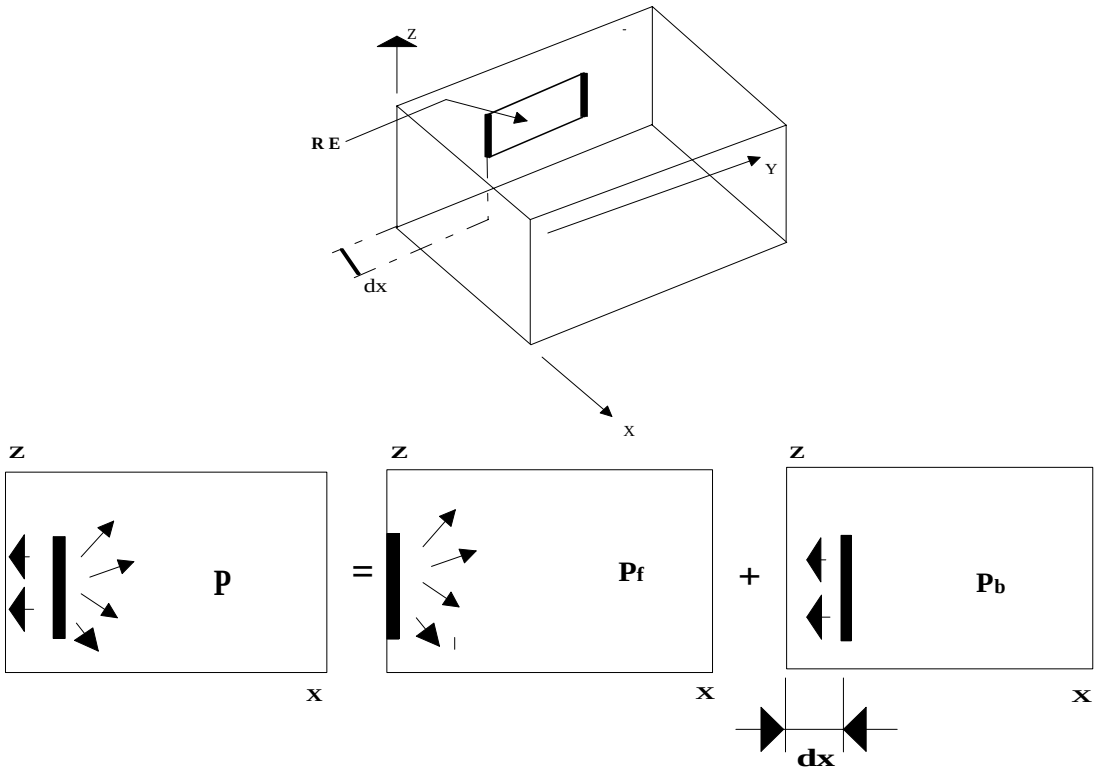

Figure 3 - Sketch of the experimental apparatus used for the evaluation of the radiant emitter RE heat flux.

P_f is the net flux evaluated through the general relation :

$$\Phi_{i,net} = \frac{\varepsilon_i S_i}{1-\varepsilon_i}(\sigma_0 T_i^4 - J_i)$$

established for a grey, diffusing surface i the medium separating this surface from the other surfaces of the enclosure is assumed perfectly transparent. If RE designates the radiant emitter, the situation is :

$$P_f = \frac{\varepsilon_{RE} S_{RE}}{1-\varepsilon_{RE}}(\sigma_0 T_{RE}^4 - J_{RE})$$

where $\sigma_0 = 5{,}68 \times 10^{-8} W/m^2.K^4$, the Stefan - Boltzmann constant, T_{RE} the absolute surface temperature (K); J, the radiosity of the surface (W/m²), S_{RE} the area of the surface (m²) and ε the global and hemispheric emissivity of the surface. The radiosity J_{RE} can only be determined by the complete evaluation of all radiative exchanges in the enclosures. J_{RE} is therefore one of the components of the radiosity vector J:

$$J = \left[J_1, J_2, ... J_j ... J_n\right]^T$$

solution of the linear system A J = B :

i = 1 $$\sum_{j=1}^{j=n}\left[\delta_{1j} - (1-\varepsilon_1) F_{1j}\right] J_j = \varepsilon_1 \sigma_0 T_1^4$$

.....

□i, $$\sum_{j=1}^{j=n}\left[\delta_{ij} - (1-\varepsilon_i) F_{ij}\right] J_j = \varepsilon_i \sigma_0 T_i^4$$

n is the number of surfaces making up the enclosure (*n* = 10 in this case). F_{ij} the view factor of S_i towards S_j . The matrix system has a dimension of 10x10.

$$a_{ij} = \delta_{ij} - (1-\varepsilon_i) F_{ij} \quad \text{and} \quad b_i = \varepsilon_i \sigma_0 T_i^4$$

3.2 Uncertainties calculation.

As we show in §2.2.2, we can calculate total fluctuation ΔJ_k of the J_k component of *J* radiosity vector, resulting from the disturbances of A matrix and B vector.

$$\Delta J_k = \left(\Delta J_k\right)_{\Delta A} + \left(\Delta J_k\right)_{\Delta B}$$

In this application, input data vector **E** is :

$$E = [T_1, T_1, ..., T_{10}, \varepsilon_1, \varepsilon_1, ..., \varepsilon_{10}, F_{11}, F_{12}, ...F_{10,1}, ...F_{10,10}, \sigma_o]^T$$

with Δ**E** which is reduced to :

$$\Delta E = [\Delta T_1, \Delta T_1, ..., \Delta T_{10}, \Delta\varepsilon_1, \Delta\varepsilon_1, ..., \Delta\varepsilon_{10}]^T$$

with 10 temperatures and 10 emissivities uncertainty intervals, because we consider there is not or negligible uncertainties on F_{ij} form factors and σ_o Stefan - Boltzmann constant.
Relation □ is represenred by $A^{-1}B$ operator, and where $s_k = J_k$. Relation (3) becomes :

$$\Delta J_k = \sum_{j=1}^{n} \left| \frac{\partial J_k}{\partial T_j} \right| \Delta T_j + \sum_{j=1}^{n} \left| \frac{\partial J_k}{\partial \varepsilon_j} \right| \Delta \varepsilon_j \qquad (14)$$

We have to calculate derivatives :

$$\frac{\partial J_k}{\partial T_j} \approx \frac{J_k(T_j + \delta T_j) - J_k(T_j)}{\delta T_j} \quad \text{and} \quad \frac{\partial J_k}{\partial \varepsilon_j} \approx \frac{J_k(\varepsilon_j + \delta \varepsilon_j) - J_k(\varepsilon_j)}{\delta \varepsilon_j}$$

In a first time, we have :

$$J = A^{-1}B$$

which provides *central* values $J_k(T_j)$ *and* $J_k(\varepsilon_j)$. Note that $J_k(T_j) = J_k(\varepsilon_j) = J_k$, same value.

$$J = \begin{bmatrix} J_1 \\ J_2 \\ \cdots\cdots \\ J_k \\ \cdots\cdots \\ J_{10} \end{bmatrix} \qquad \textbf{\textit{requiring 1 computation}}.$$

Then, we have to calculate :

$J(T_1+\delta T_1)$, $J(T_2+\delta T_2)$, ..., $J(T_{10}+\delta T_{10})$,
$J(\varepsilon_1+\delta\varepsilon_1)$, $J(\varepsilon_2+\delta\varepsilon_2)$, ..., $J(\varepsilon_{10}+\delta\varepsilon_{10})$, ***requiring 20 computations.***
with :

$$J(T_1 + \delta T_1) = \begin{bmatrix} J_1 + \delta J_1\big|_{\delta T_1} \\ J_2 + \delta J_2\big|_{\delta T_1} \\ \cdots\cdots\cdots\cdots \\ J_k + \delta J_k\big|_{\delta T_1} \\ \cdots\cdots\cdots\cdots \\ J_{10} + \delta J_{10}\big|_{\delta T_1} \end{bmatrix} \quad \cdots\cdots\cdots\cdots \quad J(T_{10} + \delta T_{10}) = \begin{bmatrix} J_1 + \delta J_1\big|_{\delta T_{10}} \\ J_2 + \delta J_2\big|_{\delta T_{10}} \\ \cdots\cdots\cdots\cdots \\ J_k + \delta J_k\big|_{\delta T_{10}} \\ \cdots\cdots\cdots\cdots \\ J_{10} + \delta J_{10}\big|_{\delta T_{10}} \end{bmatrix}$$

and

$$J(\varepsilon_1 + \delta \varepsilon_1) = \begin{bmatrix} J_1 + \delta J_1\big|_{\delta \varepsilon_1} \\ J_2 + \delta J_2\big|_{\delta \varepsilon_1} \\ \cdots\cdots\cdots\cdots \\ J_k + \delta J_k\big|_{\delta \varepsilon_1} \\ \cdots\cdots\cdots\cdots \\ J_{10} + \delta J_{10}\big|_{\delta \varepsilon_1} \end{bmatrix} \quad \cdots\cdots\cdots\cdots \quad J(\varepsilon_{10} + \delta \varepsilon_{10}) = \begin{bmatrix} J_1 + \delta J_1\big|_{\delta \varepsilon_{10}} \\ J_2 + \delta J_2\big|_{\delta \varepsilon_{10}} \\ \cdots\cdots\cdots\cdots \\ J_k + \delta J_k\big|_{\delta \varepsilon_{10}} \\ \cdots\cdots\cdots\cdots \\ J_{10} + \delta J_{10}\big|_{\delta \varepsilon_{10}} \end{bmatrix}$$

Derivatives are calculated with components of these 21 vectors :

$$\frac{\partial J_k}{\partial T_1} = \frac{\left(J_k + \delta J_k\big|_{\delta T_1}\right) - J_k}{\delta T_1} \qquad \frac{\partial J_k}{\partial \varepsilon_1} = \frac{\left(J_k + \delta J_k\big|_{\delta \varepsilon_1}\right) - J_k}{\delta \varepsilon_1}$$

$$\frac{\partial J_k}{\partial T_2} = \frac{\left(J_k + \delta J_k\big|_{\delta T_2}\right) - J_k}{\delta T_2} \quad \text{and} \quad \frac{\partial J_k}{\partial \varepsilon_2} = \frac{\left(J_k + \delta J_k\big|_{\delta \varepsilon_2}\right) - J_k}{\delta \varepsilon_2}$$

$$\frac{\partial J_k}{\partial T_{10}} = \frac{\left(J_k + \delta J_k\big|_{\delta T_{10}}\right) - J_k}{\delta T_{10}} \qquad \frac{\partial J_k}{\partial \varepsilon_{10}} = \frac{\left(J_k + \delta J_k\big|_{\delta \varepsilon_{10}}\right) - J_k}{\delta \varepsilon_{10}}$$

Finally, we obtain uncertainty of J_k with relation (14).
Numerical data values of our example are $\varepsilon_i = 0.9$ for i = 1,10
and $i \neq 4$, $\varepsilon_4 = \varepsilon = 0.93$.
T = 273[**1**]+[22.1,22.1,22.1,81.4,22.1,18.8,18.2,18.8,18.6,19.8] where [**1**] is the vector unity.

Magnitudes of interval uncertainties are :

$$\varepsilon_i = \varepsilon_i)_{exp} \square\ 4.5 \times 10^{-2}$$
$$T_i = T_i)_{exp} \square\ 0.2\ K$$

3.3 Results.

Table 1 sums up the results obtained respectively by the MC method and by FDDA method. It presents only a few examples. For brevity, all of the results obtained for each component have not been included. The superposition of the framing by FDDA and of the MC ellipsoid is good (figure 4).

This original approach, initiated in this work, clearly offers quite a satisfactory estimation of the uncertainty on the output of the model. Consistency with the Monte Carlo simulations is excellent, and our approach, though limited to the first order, deserves complete confidence as to the boundaries it produces. Table 1 sums up the different uncertainties calculated by the two methods. Note that the Monte Carlo method and its estimation of the reliability ellipsoid have not, on certain components of J, included all the fluctuating values. This problem did not occur with FDDA.

As for time computation performance, time rate is about 1 to 40, for FDDA and MC method.

J components	Non-disturbed Values	Uncertainties estimated by FDDA (21 simulations)	FDDA Uncertainties in %	Uncertainties estimated by MC (800 simulations)*	MC Uncertainties in %
J 1	429.0	□ 43.4	□ 10.1	□ 39.0	□ 9.1
J 2	428.8	□ 43.2	□ 10.0	□ 39.3	□ 9.2
J 3	428.9	□ 43.8	□ 10.2	□ 38.8	□ 9.0
J 4	862.1	□ 62.1	□ 7.2	□ 71.4	□ 8.3
J 5	429.0	□ 43.6	□ 10.1	□ 39.7	□ 9.2
J 6	412.7	□ 48.3	□ 11.7	□ 45.5	□ 11.0
J 7	410.4	□ 43.0	□ 10.5	□ 36.7	□ 8.9
J 8	412.8	□ 48.1	□ 11.7	□ 46.1	□ 11.2
J 9	411.7	□ 42.8	□ 10.4	□ 36.6	□ 8.9
J 10	417.5	□ 43.2	□ 10.3	□ 37.7	□ 9.0

Table 1 - Uncertainties associated with each component of J calculated by FiniteDifferences Differential Analysis (FDDA) and Monte Carlo (MC)

* number of computation must be greater than about 100 for a reliability interval less than 15% of final standard deviation s_D value. This case involved about 800 computations to reach 5% of final s_D value.

DEPECKER & *al.* [6] had examined this model of radiosities using the conditioning number. The authors had thus calculated :

$$\text{cond(A)} = 1.169 \quad \text{et} \quad \|\delta A\|_2 = 0.683, \|\delta A\|_2 / \|A\|_2 = 0.657, \|\delta B\| / \|B\|_2 = 0.053$$

from which

$$\|\delta J\|_2 / \|J\|_2 = 3.586$$

was obtained using the expression of FRANKLIN [7]. From the results just presented, we find by FDDA, for norm of the vector of radiosities :

$$\|\delta J\|_2 / \|J\|_2 = 0.1926$$

Consequently it can be simultaneously concluded that the model of radiosities, such as has been considered here, is stable in the face of the disturbances undergone, and that the uncertainties taken into account on the temperatures and the emissivities impose variations of about 12% maximum on each component of J. In other words, the preceding work [6] based on the notion of majorant tended to aggrandise the uncertainty affecting the resolution results. We go then from an (absolute) majorant of relative uncertainty of 358% to an uncertainty estimation (non-absolute, but representative) of 19%.

Let us note that it is the definition itself of the majorant which led to uncertainties of very high amplitudes. The subsequent use of these models, in engineering practice, is unquestionably enriched further by this second result than by the first. While the conditioning number makes it possible to base decisions on the global stability of a model in the face of uncertainties on certain of its parameters, we are now in a position to offer a satisfactory estimation of variation limits for each component of the output vector, with a calculation cost that remains acceptable.

4. CONCLUSION

We have compared, through the use of the example or radiative thermal exchange in building, two uncertainty evaluation methods. These two methods involve different approaches. The first is a probabilistic type (Monte Carlo, MC), the second a deterministic type (Finite Differences Differential Analysis, FDDA).

The FDDA method has to be particularly dependable and effective. As regards dependability, we have observed that the uncertainty interval described by this method almost systematically includes the values computed by the MC method. In addition, this framing of the MC results by first order approximation of FDDA does not lead to an excessive extension of the area of uncertainty of the results, but on the contrary, narrows the extrema of the cloud.

The interval which the FDDA method leads to is therefore always more pessimistic than that obtained by the MC method, but the difference does not exceed 2%. The FDDA first order approximation therefore proves to be satisfactory.

As regards effectiveness, the calculation times necessary to obtain the uncertainty interval are far below those of the MC method. As for performance, calculating time rate is 1 to 40, and for other examples not presented in this paper 1 to 100, showing that FDDA method is more economic than the probabilist reference MC method. The only precaution necessitated by the FDDA method is within the determination of the calculation step of partial derivatives. This is a particularity of the FDDA method which must be enters in the calculation code, in the different stages of resolution. But it is likely that in deterministic type approaches, a sequential analysis of the numerical and mathematical treatment of the model is essential. Nonetheless, when this is done, and the partial derivatives are coded, the uncertainty of the output vector is easily calculated.

Finally we note that this method imposes few *a priori* restrictions concerning the nature and the amplitude of the uncertainties associated with data.

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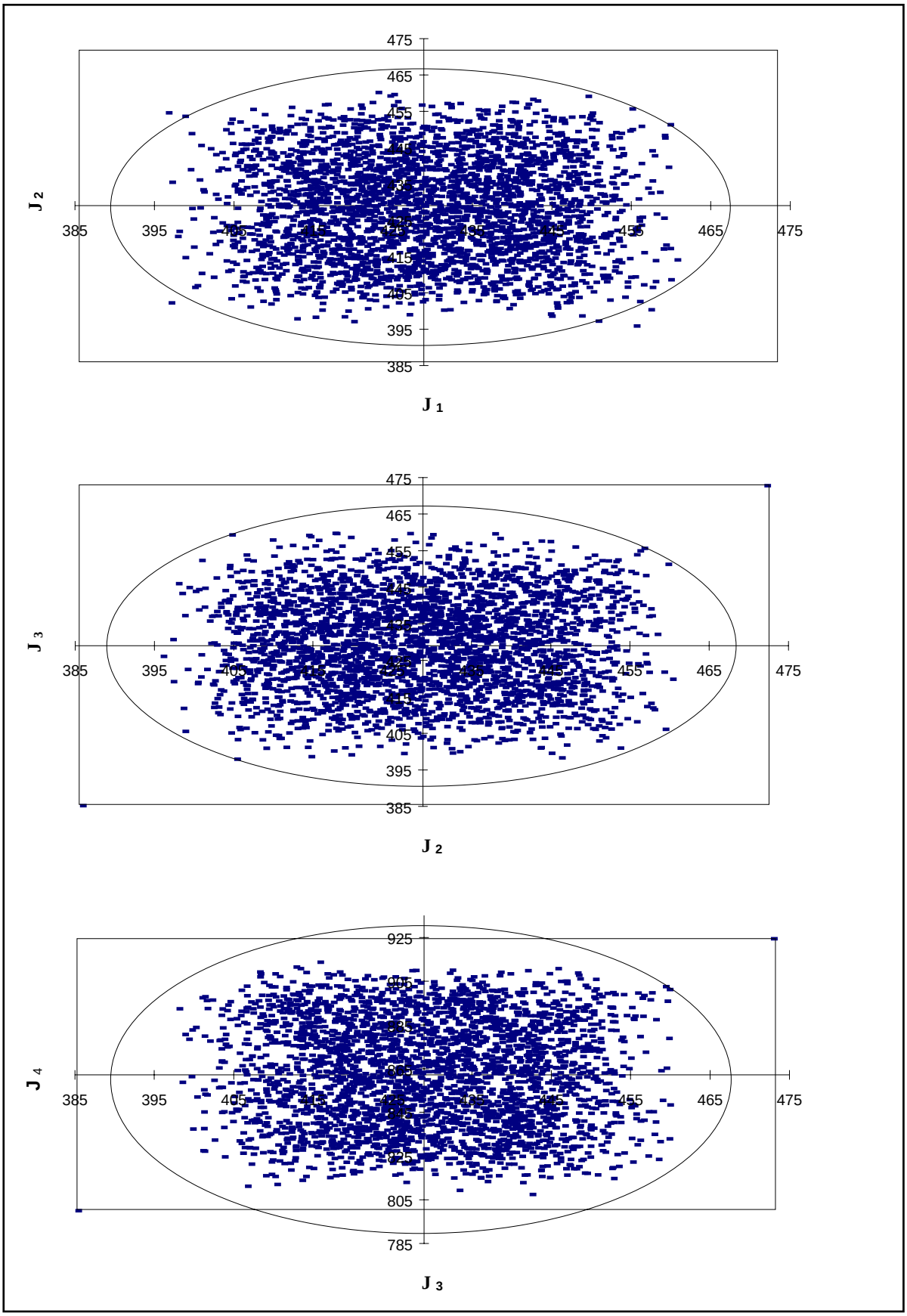

Figure 4 - Superposition of Monte Carlo values on Finite Differences Differential Analysis intervals. Presented here are three projections of Monte Carlo sets of points on planes (J_1, J_2), (J_2, J_3) and (J_3, J_4). It can be observed that the domain of the Finite Differences Differential Analysis indeed includes all of the MC points