A MODEL-BASED APPROACH TO THE COMMISSIONING OF HVAC SYSTEMS

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Abstract

The paper describes how first-principles models can be used to assist in the commissioning of HVAC systems. The techniques utilise models that are extended to treat different types of faults. A sequence of test signals is applied to the system under test and the measured sensor and control signals are used to estimate parameters of the models relating to certain faults. These parameter estimates are compared with values calculated from design information. Differences are taken to be evidence of faulty or unsuitable equipment, incorrect installation, or inadequate commissioning. Results are presented from tests carried out on an air handling unit test rig at Loughborough University. The work has been performed as part of a UK collaborative research project on the practical application of fault detection and diagnosis to HVAC systems and as part of IEA Annex 34.

1   Introduction

The performance of many HVAC systems is limited more by poor installation, commissioning and maintenance than by poor design [1]. In practice, systems are often poorly commissioned; possible reasons include:

- the time available for commissioning is frequently reduced by delays in construction;

- there is a shortage of skilled personnel to perform commissioning work;

- it is difficult to produce a well defined specification of certain aspects of performance, particularly dynamic performance;

- it is impossible to test fully the performance of the HVAC systems in an unoccupied building at any one time of year.

Computer-based control systems have the capability to collect and store sensor and control signals that could be analysed for diagnosing faults. In practice, only limited use is made of this information because manual analysis is time consuming and requires a certain level of expertise, whereas automatic methods have not yet been developed to the point where they can be implemented commercially.

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A considerable amount of research work has been carried out over the last five years on fault detection and diagnosis (FDD) in HVAC systems, much of it in IEA Annex 25 [2]. Some research on automated testing at the commissioning stage has also been performed [3, 4]. It is clear that the two topics are closely related; there is little point in setting up a performance monitoring system if the HVAC system is incapable of operating correctly because of incorrect installation and poor commissioning. In addition, most condition monitoring systems rely on measurements made on the correctly operating system to construct the reference against which future performance is compared. A collaborative research project to develop methods of integrating automated commissioning and condition monitoring and to study their performance when implemented in commercial office buildings is now under way in the UK. The project involves academic researchers, a consulting engineering practice, two controls companies and three building owners.

2   Model-based fault detection and diagnosis

Models can be used for both fault detection and fault diagnosis. Faults can be detected by using a model of the correctly operating process (a 'reference model'). Measured values of the control signals and some of the sensor signals are used as inputs to the reference model. The remaining sensor signals are then compared with the predictions of the reference model, as shown in Figure 1. Significant differences ('innovations') indicate the presence of a fault somewhere in the part of the system treated by the model.
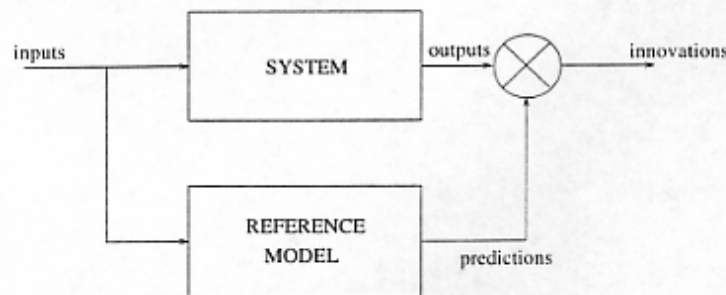

Figure 1: An innovations-based fault detection scheme

There are two basic types of reference model [5]:

Black box models: empirical models that embody no prior knowledge of the system (apart from that which is implicit in the choice of the inputs and outputs)

Physical models: models largely based on a first principles analysis of the system, which may include established empirical relationships (e.g. to predict heat transfer coefficients)

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The main advantage of black box models is that they can be chosen to be linear in the parameters, making the process of estimating the parameters of the model both less demanding computationally and more robust. One advantage of physical models is that the prior knowledge that they embody improves their ability to extrapolate to regions of the operating space for which no training data are available. They also require fewer parameters for a given degree of model accuracy. A further feature of physical models is that the parameters correspond to physically meaningful quantities, which has two advantages:

1. Estimates of the values of the parameters can be obtained from design information and manufacturers' data.

2. Abnormal values of particular parameters can be associated with the presence of particular faults.

The FDD methods presented here are based on physical models.

Fault diagnosis requires knowledge of how the system behaves when faults are present. There are two main approaches:

1. Analyse the way in which the innovations vary with operating point

2. Develop a model that treats faulty operation and estimate the values of the parameters that relate to particular faults

The first approach is usually implemented using a rule-based classifier. The rules, which may be Boolean or fuzzy, can either be based on expert knowledge or derived from simulations or experiments. The second approach, which is the one used here, involves estimating the parameters of the models from sensor and control signal measurements obtained from the system. In the approach adopted here, the model is extended to represent both faulty and correct operation; the estimated values for the parameters relating to the faulty behaviour are then used for fault diagnosis.

2.1 Commissioning

The commissioning of an HVAC system involves the balancing of the water and air circuits, performance validation, and the identification and elimination of any design or installation faults. The techniques described in this paper are intended to assist in the performance validation and fault diagnosis aspects of commissioning. There are two important respects in which fault diagnosis at commissioning time differs from on-going fault diagnosis:

1. The reference model for correct operation is derived from design information and manufacturers' data, rather than from measured data taken from the target system.

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2. The tests are performed when the building is unoccupied, allowing the HVAC system to be exercised over its operating range in a systematic way.

Note that 'commissioning' may take place at any time during the operating life, not just before hand-over. Refurbishment of the building, replacement of part of the HVAC system or an increased concern about performance may result in a recommissioning of the HVAC system. The methods described here are equally applicable to all these situations.

This paper considers an approach that uses a physical model of the target system. The HVAC system is exercised in a systematic way over its operating range and the resulting set of control signals and sensor signals is used to estimate the values of the parameters of a physical model that treats important faults as well as correct operation. These parameter values are then compared with the corresponding values obtained directly from design information and manufacturers' data. Significant differences in the parameter values can then be attributed to incorrect installation or equipment malfunction. The approach has the following requirements, which may prove costly in the short term:

1. It requires the use of design information and manufacturers' data for most items of equipment.

2. It requires the use of non-linear parameter estimation methods, which are computationally expensive.

In the longer term, the use of a common database throughout the building life-cycle, in particular, the design, construction and commissioning phases, should address the first problem and the oft-cited continuing increase in affordable computing power should address the second problem.

For the purpose of the work described here, the commissioning phase can be considered complete once any significant differences between the parameter values estimated at commissioning time and the values derived from design information have been resolved.

3 Trial on real AHU

3.1 Description of system

The system used to test the FDD procedures consists of an air handling unit serving four rooms in a laboratory, each of which has its own VAV terminal box. The air-handling unit, which is depicted in Figure 2, consists of a mixing box, a cooling coil, a heating coil, a steam humidifier and supply and return fans connected to variable speed drives.
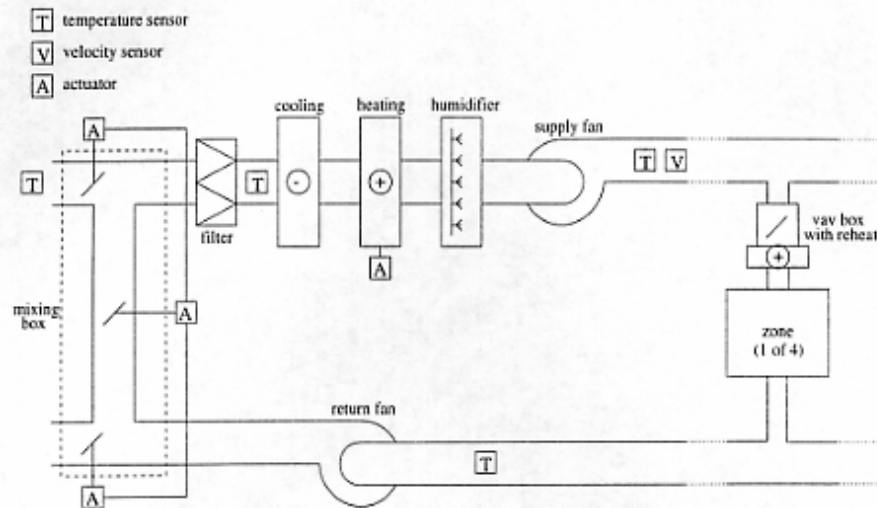
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Figure 2: The test system (only the sensors and actuators used in the study are shown)

3.2 Modelling

Models for the mixing box and heating coil subsystems of the air handling unit were produced based on first principles analyses of the components, supplemented by empirical relationships.

3.2.1 Actuator

The actuators connected to the final control elements (i.e. the valve and the dampers) are motor-driven. Assuming that the size of the dead-band in the positioner is small, the main departures from ideal behaviour in steady state are then likely to be due to hysteresis and to mismatch between the range of movement of the actuator and the corresponding final control elements. The hysteresis associated with the heating coil and mixing box studied here are believed to be small and have not been treated. The actuator model does, however, allow account to be taken of dead bands that may exist at either end of the travel. The position of the final control element, s, is given by:

$$s = \frac{u - u_l}{u_h - u_l}, \text{ where } 0 \leq s \leq 100\,\%$$

(1)

where u is the signal issued by the controller. If u_l and u_h are in the range $0 - 100\,\%$, they are the lowest and highest control signal values that produce movement of the final control element. If $u_l < 0$ or $u_h > 100\,\%$, the actuator is unable to fully close, or fully open,

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the final control element. However, in the work reported here, u_l and u_h were constrained to the range $0 - 100\%$ since it is difficult to distinguish the behaviour of the system when the range of movement of the actuator is too small from the behaviour of the system when there is leakage of the final control element or there is some form of under-capacity (e.g. coil fouling).

3.2.2 Mixing box

The thermal performance of a mixing box depends on the relative flow rates of the outside and recirculated air. These flow rates depend not only on the characteristics of the dampers and the resistances of the adjacent ductwork and fittings but also on the characteristics of the rest of the duct system and on the fan control strategy. It is, therefore, difficult to produce a simple first principles model of a mixing box and so an empirical model has been developed. The fraction of outside air, g, is given by:

$$g = l_r + (1 - l_o - l_r) \left[\frac{b^n + (u - b)^n}{b^n + (1 - b)^n} \right], \text{ for } u > b, \tag{2}$$

$$= l_r + (1 - l_o - l_r) \left[\frac{b^n - (b - u)^n}{b^n + (1 - b)^n} \right], \text{ for } u \leq b, \tag{3}$$

$$n = e^a \tag{4}$$

where u is the control signal to the mixing box, with $u = 1$ corresponding to a demand for full outside air. l_r is the fractional flow due to leakage in the recirculation damper and l_o is the fractional flow due to leakage in the outside air damper. The shape of the characteristic within the active range is determined by a and b. a defines the degree of non-linearity, with $a > 0$ corresponding to a rapid opening characteristic at each end of the range and $a < 0$ corresponding to a slowly opening characteristic. b defines the degree of asymmetry of the characteristic, being the value of the input, u, at which the point of inflection occurs.

The mixed air temperature, t_m, is given by:

$$t_m = g\, t_o + (1 - g)\, t_r \tag{5}$$

where t_o is the temperature of the outside air and t_r is the temperature of the recirculated air.

3.2.3 Control valve

The valve model treats the behaviour of a three-port valve with either a linear characteristic or an exponential (equal percentage) characteristic and also treats leakage. The model of the inherent characteristic is:

$$f(s) = l_v + (1 - l_v)\frac{(e^{\beta s} - 1)}{(e^{\beta} - 1)} \qquad \text{(exponential)} \qquad (6)$$
$$= l_v + (1 - l_v)s \qquad \text{(linear)}$$

where $f(s)$ is the fractional flow through the valve at the stem position s. l_v is the fractional leakage, and β is the curvature parameter. The form of the function has been chosen to approximate the high rangability of modern valves, whilst maintaining the continuity of its first derivative, which is important for certain parameter estimation techniques.

When a valve is installed in a circuit, the actual (installed) characteristic of the valve differs from the inherent characteristic. The installed characteristic is given by:

$$f'(s) = \frac{1}{\sqrt{1 + A(f^{-2}(s) - 1)}} \qquad (7)$$

where the function $f(.)$ is defined by Equation 6, and A is the authority of the control port of the valve.

3.2.4 Heating coil

The overall conductance, UA, of an air to water heat exchanger is determined by three resistances in series: the convective resistances on the air and water sides and the resistance of the tubes. If both the air and the water flow are assumed to be turbulent, the overall conductance is given by:

$$\frac{1}{UA} = \gamma \frac{r_a v_a^{-0.8} + r_m + r_w v_w^{-0.8}}{A} \qquad (8)$$

where v_a and v_w are the air and water velocities and A is the face area of the coil. Typical values of r_a, r_m and r_w for different types of coil are given in [7]. The value of γ is chosen so that the calculated value of UA equals the design value, UA_D, when v_a and v_w have their design values, $v_{a,D}$ and $v_{w,D}$.

The heat transferred between the fluids is calculated using the NTU-effectiveness method. Heating coils in HVAC systems have a cross-flow configuration in which the water flow is mixed and the air flow is unmixed. The effectiveness for this configuration depends on which fluid has the minimum heat capacity rate, i.e.

$$\varepsilon = \left(\tfrac{1}{C_r}\right)\left(1 - exp\left\{-C_r[1 - exp(-NTU)]\right\}\right) \qquad \text{for } C_{min} = C_a \qquad (9)$$
$$= 1 - exp\left(-C_r^{-1}\left\{1 - exp[-C_r(NTU)]\right\}\right) \qquad \text{for } C_{min} = C_w \qquad (10)$$

where:

$$NTU = \frac{UA}{C_{min}} \qquad (11)$$

$$C_r = \frac{C_{min}}{C_{max}} \qquad (12)$$

where C_a and C_w are the air and water capacity rates, and C_{min} and C_{max} are the minimum and maximum of the air and water capacity rates. (The capacity rate is the product of the mass flow rate and the specific heat capacity.)

The air outlet temperature, t_{ao}, is then:

$$t_{ao} = t_{ai} + \varepsilon \frac{C_{min}}{C_a}(t_{wi} - t_{ai}) \qquad (13)$$

where t_{ai} and t_{wi} are the air and water inlet temperatures, respectively.

3.3 Results of pre-commissioning tests

A set of tests was first performed to check the operation of the sensors and actuators.

3.3.1 Sensor calibration

The relative calibration of the outside, mixed, supply and return air temperature sensors was checked by moving them to the same location. The differences between the readings were less than 0.1 K. Since the sensors are nominally identical, it is reasonable to assume that the relative errors in the measurements of the local temperature do not greatly exceed 0.1 K across the range of temperatures encountered in the test reported below (i.e. the offset error is taken to be the dominant error). More significant sources of error arise from poor mixing and poor placement of point sensors, as discussed below.

3.3.2 Mixing box actuators

The outside air damper is operated by a motor-driven actuator that includes a positioner. The recirculation and exhaust air dampers are operated by actuators that have no positioner but are slaved off the master actuator that operates the outside air damper.

Figure 3 is a plot of the positions of the damper spindles, as indicated by the dials on the actuators, vs. control signal. It is immediately apparent that the actuators do not respond to control signals between 0 and 20 %. Inspection showed that the control strategy had been configured for actuators with an active input range of 0-10 V whereas the actual actuators (which were replacements) have an active input range of 2-10 V.
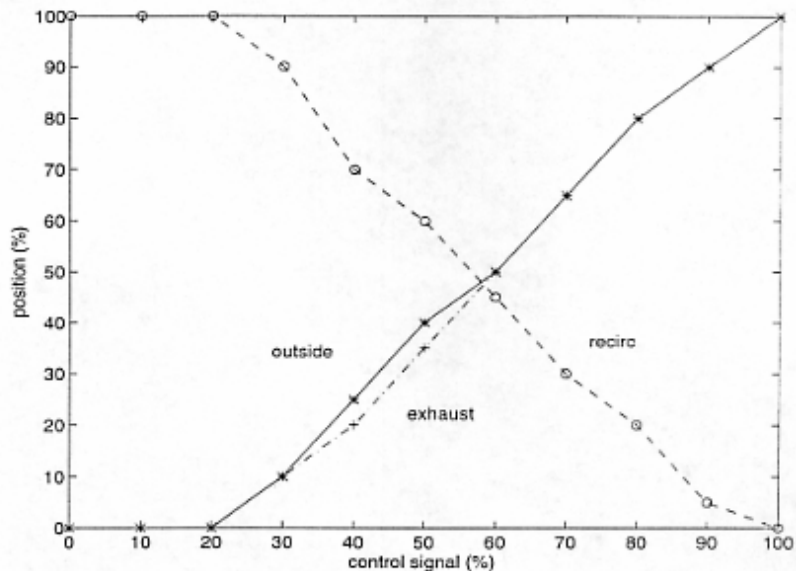

Figure 3: The positions of the mixing box damper actuators vs. control signal

3.3.3 Heating coil actuator and valve

The three port control valve for the heating coil has a nominally linear characteristic but the actuator has a jumper that can be set to give either an exponential (equal percentage) or a linear characteristic. The result of selecting the exponential characteristic is shown in Figure 4, which is a plot of actuator position, as indicated by the angular position of the screw thread that produces the translational motion of the valve stem, vs. control signal. The valve reaches the fully open position at ~90 %. Figure 5 shows the installed characteristic of the valve, obtained by measuring the relationship between the water mass flow rate through the heating coil and the valve control signal. Measurements were made with the valve both opening and closing and no measurable hysteresis (< 1 %) was detected. The authority of the valve was measured to be 0.49.
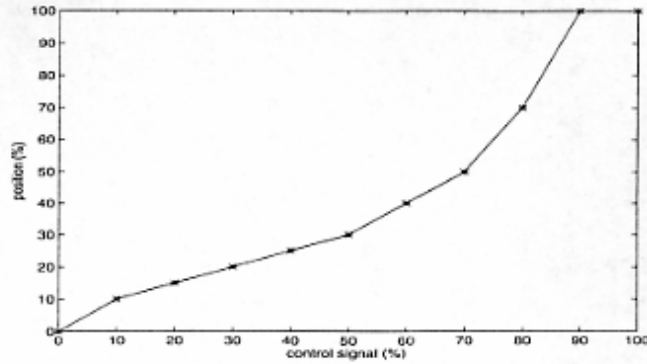
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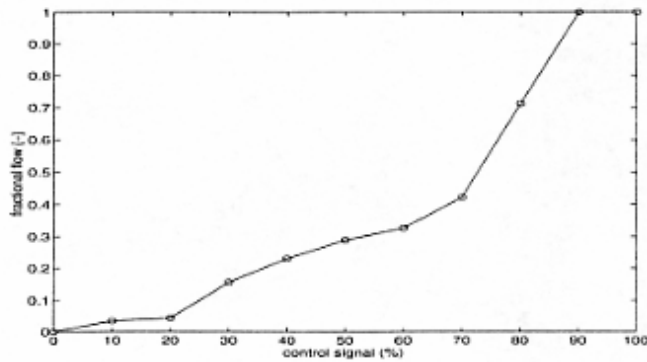Figure 4: The position of the heating coil actuator *vs.* control signal

Figure 5: The water mass flow rate through the heating coil *vs.* control signal

3.4   Results of commissioning tests

3.4.1   Mixing box

The mixing box was tested by applying the sequence of step changes in the demanded position of the mixing box actuators shown in Figure 6. The test was performed at the design air flow rate. Figure 7 shows the measured temperatures. As indicated in Figure 2, the return air temperature is measured upstream of the return fan and the temperature of the recirculated air can be estimated by adding 1.0 K, the measured temperature rise across the return fan. The outside air temperature sensor is located in the inlet duct. The mixed temperature sensor is a single point sensor immediately down stream of the filter, which is located immediately down stream of the mixing plenum. The supply air temperature sensor, which is situated downstream of the supply fan, can be used to

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provide an alternative estimate of the mixed air temperature by subtracting 1.75 K, the measured temperature rise across the supply fan. The boiler and chiller were switched off to avoid any heating or cooling due to leakage in the heating or cooling coil valves.
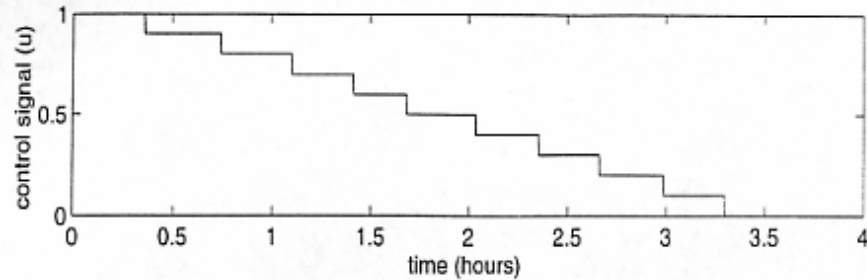

Figure 6: The demanded position of the outside air dampers used in the mixing box test
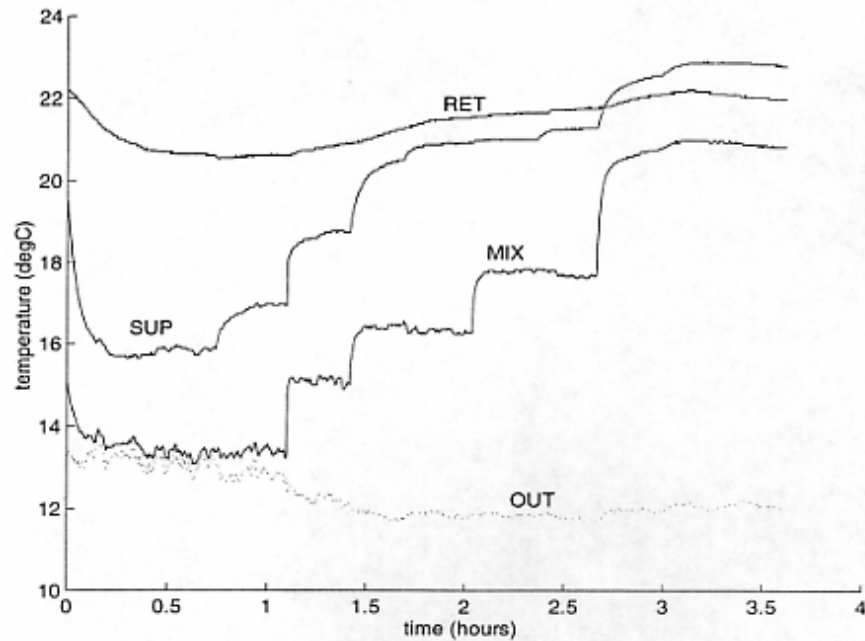

Figure 7: The temperature sensor readings recorded in the mixing box test. 'OUT' indicates the inlet duct sensor, 'RET' the return duct sensor, 'MIX' the sensor downstream of the mixing plenum and 'SUP' the supply air sensor.

Significant differences between the behaviour of the mixed air sensor and that of the supply air sensor are apparent. These differences are repeatable, and are probably due
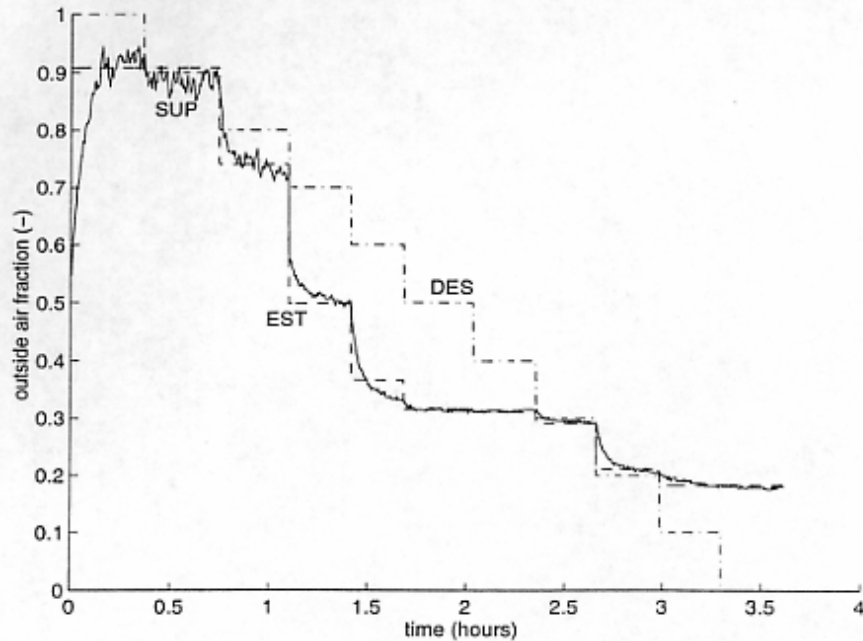
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Figure 8: The outside air fractions inferred from the measurements and predicted by the model. 'SUP' indicates the fraction inferred from the mixed air and supply air sensors. 'DES' and 'EST' indicate the fractions predicted by the model using the design and estimated parameter values.

to imperfect mixing of the two air streams in the vicinity of the single point mixed air sensor. In particular, near the beginning of the test, when the demanded outside air fraction is unity, the supply air temperature sensor reading exceeds the outside air temperature sensor reading by more than the temperature rise across the supply fan, indicating significant leakage through the recirculation damper. The agreement between the outside air temperature sensor reading and the mixed air temperature sensor reading indicates that the air leaking through the recirculation damper largely by-passes the mixed air temperature sensor.

Figure 8 shows the outside air fraction as predicted by the model and as inferred from the reading of the supply air sensor. The return and supply air temperature measurements were corrected for the temperature rises across the fans, as indicated above. The model predictions were generated using two sets of parameters, given in Table 1. The design values are ideal values, corresponding to a mixing box with a linear response, no leakage and actuators whose active ranges correspond to the required movement of the dampers. The estimated values were obtained by fitting the combination of the actuator model and

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Table 1: Mixing box model parameters

Parameter	Symbol	Design Value	Estimated Value
lower limit of active range (%)	u_l	0.0	17.9±0.4
upper limit of active range (%)	u_h	100	84.9±0.5
leakage at $u = 0$ (%)	l_o	0.0	18.3±0.2
leakage at $u = 100\%$ (%)	l_r	0.0	9.1±0.4
degree of curvature (-)	a	0.0	1.00±0.03
position of midpoint (%)	b	50	36.1±0.5

the mixing box model to the mixed air temperature inferred from the supply air sensor reading using a non-linear optimisation method [8]. Data collected when one or more of the measurements was changing significantly were not used in the estimation. The technique used to filter the data is described in [9]. The small values of the uncertainties in the estimated values is a consequence of the use of an empirical model with enough degrees of freedom to produce a good fit to the measured data. The r.m.s. difference between the predictions of the model and the measurements is 0.11 K.

No significant change is observed in the measured outside air fraction as the control signal falls below 20 %. This can be attributed to the mismatch between the controller output and the actuator input range that was observed during the pre-commissioning tests. There appears to be significant leakage through the outside air damper and/or the exhaust air damper and somewhat less leakage through the recirculation air damper. These faults are also apparent in the values of the model parameters estimated from the measurements, which are given in Table 1. The response of the mixing box is significantly non-linear: the estimated value of a (1.0) corresponds to an approximately cubic variation of the outside air fraction about the point of inflection. The point of inflection, which occurs at $u=36\%$, is significantly displaced from the mid point of the control signal range and even more significantly displaced from the centre of the active range of the actuators ($20< u <100\%$). This asymmetry may be attributed to the considerable difference between the sizes of the outside and recirculation dampers. The recirculation damper is significantly larger, so that, once it is partially open, it has minimal effect on the flow rates.

3.5 Heating coil subsystem

The heating coil was tested at 100 % and 50 % of the design air flow rate by applying the sequence of step changes in the demanded position of the control valve shown in Figure 9. Figure 10 shows the measured temperatures. The value of mixed air temperature predicted by the mixing box model is shown rather than the measured value, since the mixing box test showed that the measured value is not a good indication of the true value. Figure 11 shows the measured and predicted values of the air side approach, α, defined by:

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$$\alpha = \frac{t_{ao} - t_{ai}}{t_{wi} - t_{ai}},\qquad(14)$$

The tests were performed with the mixing box set to provide full recirculation. The off-coil air temperature, t_{ao}, was calculated from the supply air temperature sensor measurement by subtracting the estimated temperature rise across the supply fan (1.75 K at 100 % fan speed, 1.04 K at 52 % fan speed). The on-coil air temperature, t_{ai}, was taken to be the value of the mixed air temperature predicted by the mixing box model using the estimated parameter values given in Table 1. The oscillation in the inlet water temperature is due to cycling of the boiler. The decrease in the water inlet temperature observed when the control signal falls below 30 % occurs because, as shown in Figure 5, the water flow rate through the coil is very small and the heat loss through the imperfectly insulated pipework becomes significant.

The model predictions were generated using the values of the design and estimated parameters given in Table 2. The design values are ideal values, corresponding to a valve with no leakage, a typical exponential characteristic and an authority of 0.5, an actuator whose active range corresponds to the required movement of the valve and a coil whose UA value is defined by the design operating conditions. The estimated values were obtained by fitting the combination of the actuator model, the valve model and the heating coil model to the off-coil air temperature inferred from the supply air sensor reading. The r.m.s. difference between the predictions of the model and the measurements is 0.9 K.

The authority of the control valve was measured to be 0.49. No value for the curvature parameter, β, was given by the manufacturer and so an effective value was estimated. The difference between the estimated value of 1.39 and the value of 3.0 that is typical of valves found on larger air handling units is due, in part, to differences between the actual and predicted heating coil behaviour as well as to differences in valve behaviour. A more normal situation would be for the value of β to be known and the actual authority not to be known. It would then be appropriate to fix β and estimate the value of the authority.
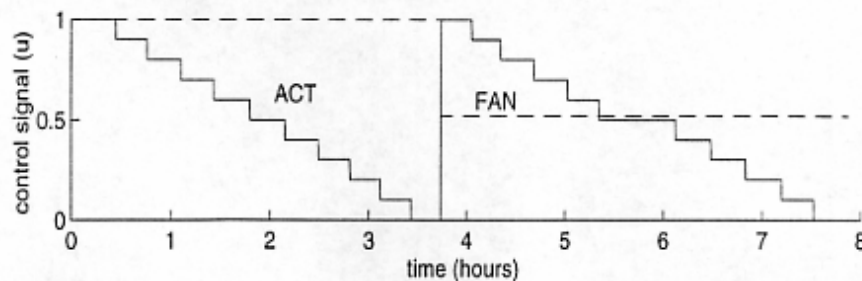

Figure 9: The demanded position of the actuator, 'ACT', and the fan speed, 'FAN', used in the heating coil test
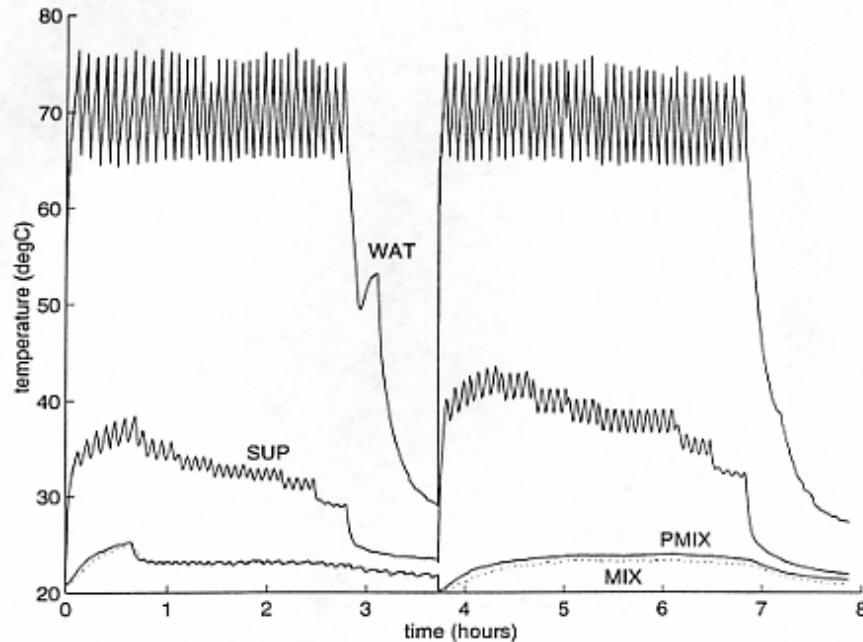
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Figure 10: The temperature sensor readings recorded in the heating coil test. 'RET' indicates the return duct sensor, 'SUP' the supply air sensor, 'MIX' the mixed air sensor and 'WAT' the inlet water sensor. 'PMIX' indicates the value of the mixed air temperature predicted from the mixing box model.

The abrupt decline in the approach when the control signal falls below 30 % can be explained by the installed characteristic of the actuator/valve combination, as shown in Figure 5, possibly exacerbated by a transition to laminar flow in the tubes of the coil. The failure of the coil model used here to account for this behaviour is compensated for by a significantly non-zero estimate of the value of the lower limit of the active range (17 %), which appears from Figure 5 to be non-physical.

The small value of the estimated leakage parameter and the good agreement between the design and estimated UA values indicate that the heating coil subsystem is free from major defects. It is unclear from the open loop tests reported here quite how difficult the coil will be to control, but it would appear that careful tuning of the supply air temperature controller will be required to avoid instability at low duty.
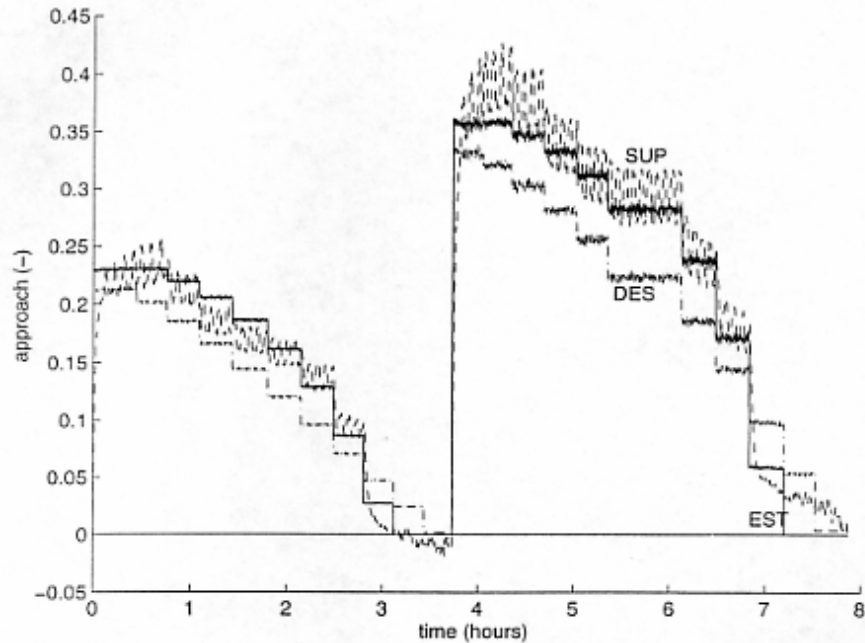
Figure 11: The values of airside approach inferred from the measurements 'MEAS' and predicted by the model. 'DES' and 'EST' indicate the fractions predicted by the model using the design and estimated parameter values.

4 Discussion and conclusions

The purpose of performing the detailed tests reported here was to investigate the adequacy of the models used; several improvements to the models were made during the course of the work. The time required to perform such detailed tests is excessive for practical performance validation. Further analysis of the results reported here, together with the results of tests on other systems, is required to determine how the testing period might be reduced whilst still obtaining the information required to detect and to diagnose faults. It would appear that it is most important to characterise the behaviour near the ends of the operating range, since this is where most faults manifest themselves, e.g. leakage, actuator range mismatch and under-capacity. The question of appropriate threshold levels for the different fault parameters remains to be addressed.

Another practical issue is the amount of design data required to configure the models. The only design parameters used here are the coil UA and maximum air and water velocities, although the availability of a value for the parameter that defines the valve characteristic would allow a meaningful value of the authority to be estimated. The UA

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Table 2: Heating coil model parameters

Parameter	Symbol	Design Value	Estimated Value
lower limit of active range (%)	u_l	0.0	16.9±1
upper limit of active range (%)	u_h	1.0	90±5
curvature (-)	β	3.0	1.39±0.21
inherrent leakage (%)	l_v	0.0	0.0±0.4
authority (-)	A	0.5	n/a
maximum water velocity (m.s^{-1})	$v_{w,D}$	0.276	n/a
maximum air velocity (m.s^{-1})	$v_{w,D}$	1.74	n/a
face area (m^2)	A	0.48	n/a
UA (kW.K^{-1})	UA	0.252	0.264±0.005

and maximum air velocity are easily obtained, but the water velocity may be more difficult to determine since it depends on the number of circuits as well as the internal diameter of the tubes. A study of the range of coil designs encountered in air handling units is required to determine if the coil model can be recast in terms of fractional flow rates rather than absolute velocities, whilst still retaining approximately constant coefficients. This would allow the part load behaviour of (dry) coils to be treated by a model with just one parameter (the UA).

To summarise, the extension of a model-based condition monitoring method to fault detection and diagnosis at commissioning time has been described. Trials on an air handling unit have been performed to demonstrate the operation of the method and a number of faults identified in the mixing box. The work has been performed as part of a UK collaborative research project on the practical application of fault detection and diagnosis to HVAC systems and as part of the UK contribution IEA Annex 34.

Acknowledgements

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