Identification of a change in the thermal dynamics of a wall

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ABSTRACT

In this work the problem of the estimation of a change in the dynamics and time-varying thermal parameters for a wall from experimental data is solved in several ways. The problem is firstly solved by the application of a classical steady-state method. This method gives good results with time series long enough, but shows no predictive power. Secondly, linear time-invariant (LTI) dynamic statistical models in transfer function form are applied. They estimate the thermal parameters and also offer predictive power. Finally, linear and time-varying (LTV) continuous time stochastic modelling is applied. This last technique is able to estimate the thermal parameters, fits the time-varying dynamics and shows an accurate predictive power. The first two methods need some additional previous treatment of the time series by the analyst, while the last one doesn't.
The experimental data for the wall correspond to the first case proposed for resolution in the System Identification Competition III (SIC III, www.dynastee.info), organized by the DYNASTEE (DYNamic Analysis, Simulation and Testing applied to the Energy and Environmental performance of buildings) network. The implementation of the European Energy Performance of Building Directive requires adequate calculation and modelling tools which is the main reason why this third competition has been organised. In this context, SIC III has been organised with the objective of further develop knowledge of system identification applied to thermal performance assessment in the built environment.

1. INTRODUCTION

The considered case study, presented and made available in www.dynastee.info/events.php, is concerned with the monitoring of a wall in a house constructed in the 1990's to assess its thermal performance before and after the installation of cavity-fill insulation. The wall as-built consists in a lightweight concrete block and a cavity providing the insulation. Filling the cavity with insulating material should improve thermal performance, resulting in lower energy consumption to reach thermal comfort.
Monitoring was carried out during February and March 2007. A heat flux (q_i) meter was fixed to the internal surface of the wall using double sided tape. A temperature sensor was mounted within the room (T_{int}), a few centimetres away from the wall. An external air temperature sensor (T_{ext}) was mounted within a solar radiation shield fixed to the north facing exterior of the wall. Measurements were logged at 1 minute intervals and hourly averages recorded. 817 hours of data were collected. A section scheme of the wall and a plot of the collected data can be seen in figure 1.
It is asked for the day on which the insulation was added and the U-values before and after filling the wall cavity.
Several models can be considered to model the heat dynamics of building components (Jiménez

and Madsen, 2008). This work considers some of these approaches focusing in modelling time-varying thermal parameters of the studied wall.

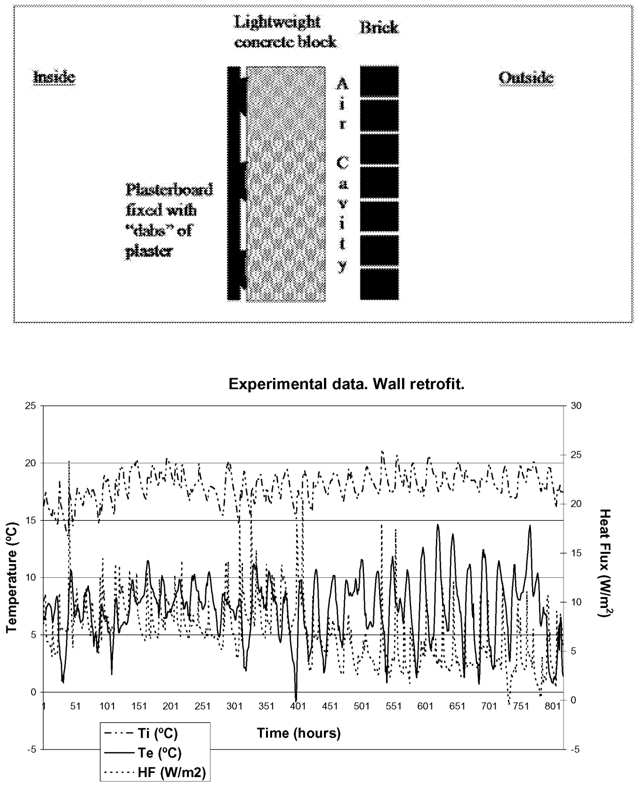

Figure 1. Section detail and plot of the experimental data for the retrofitted wall.

2. CLASSICAL STEADY STATE METHOD

This method – also refered as the average method – provides quantitative and qualitative information about the measured data and allows to estimate, for experiments long enough, some of the parameters like the U-value of the wall (ISO 9869, 1994). The formula below gives the estimate of the U-value (in W/Km^2) of the average method:

$$U = \frac{\sum_{i=1}^{N} q_i}{\sum_{i=1}^{N} \left(T_{in,i} - T_{out,i} \right)}, \qquad (1)$$

where i stands for the index of an observation.
To obtain an accurate U-value with eq. (1) it is required at least 72 hourly-spatiated data. Other requirements for the accuracy of the average method according to (ISO 9869, 1994), such as "the estimate corresponding to the first 2/3 part of the test period should not deviate by more than 5% from the estimate corresponding to the last 2/3 part of that period" do not hold in this case, since there is an instant of time where thermal dynamics - and consecuently the parameters - change.
Anyway, the application of this method can give useful information: the estimate at the beginning of the time series must characterize the U-value of the wall before the insulation treatment and the estimate at the end of the period must characterize the U-value of the retrofitted wall. First, the average method has been applied to estimate the U-value with the first n hours of data, with n covering the whole data series (U-forwards). Secondly, the U-value has been estimated with the average method with the last n hours of data, with n covering the whole data series (U-backwards). The results of these calculations can be seen in figure 2.

It can be observed that in the sets of data with less than 80 points the U-value presents an oscillation. For the regime between 100 and 400 points the forwards estimation of the U-value is very stable and near 0.8 W/m^{2}K. The backwards estimation, seems more estable in the interval between 100 and 300, with an U-value near 0.4 W/m^{2}K. This estimation seems physically consistent, since the U-value after the retrofitting must be much more less than before.

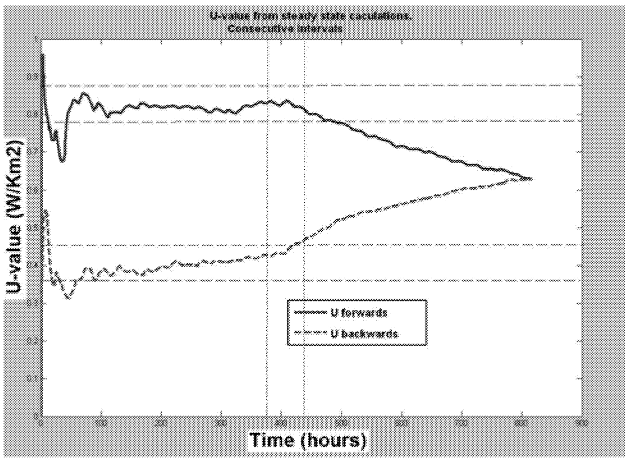

Figure 2. Estimation of the U-value "backwards" and "forwards" by the average method.

3. APPLICATION OF TRANSFER FUNCTION MODELS

An ARMAX model is considered (Box and Jenkins, 1976), to take into account the dynamics of the heat transfer through the wall (Norlén, 1993; Jiménez et al.; 2008):

$$A(q)Q(t) = B_1(q)T_{in}(t) + B_2(q)T_{ext}(t) + C(q)e(t), \quad (2)$$

where A, B_1 and B_2 are polynomials in the lag operator q, e(t) is white noise and the following steady state relation must be satisfied:

$$Q = U\left(T_{in} - T_{ext}\right) + e \quad (3)$$

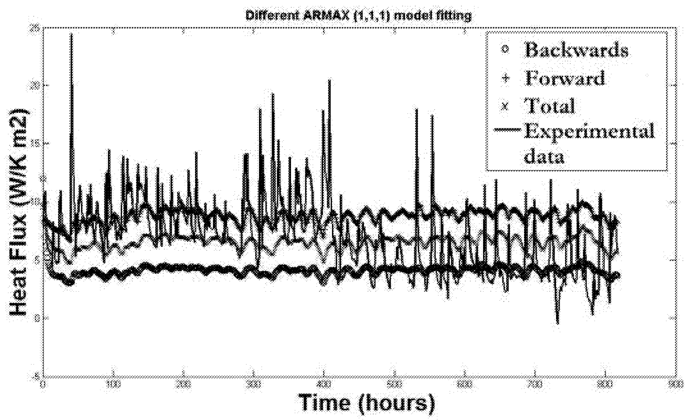

Figure 3. First order ARMAX model fitting with heat flux as output.

As a first approximation one could think of fitting one-order models to estimate the dynamics of the system. It is shown in figure 3 how heat flux dynamics is followed in the mean, while the peaks are far to be represented. The two different operation regimes are well represented in the backwards and forwards estimated models. The model obtained using all the data series exhibietes a good "mean performance", but fits worst in each regime than the estimated model using data only in that interval.

To better represent the dynamics of the wall, higher order ARMAX models must be fitted to the experimental data. Once this has been done, one can check the following features:

1. The orders of the fitted models increase to (10,10,8) and (10,10,9) in the forward case and (10,10,1), (10,10,6) and (10,10,1), depending on the exact interval selected for the estimation.
2. There is a final interval (near the half of the time series) where the forward predicted heat flux diverges from the experimental data (provided the interval used for the estimation doesn't contain the former).
3. There is a time interval at the beginning of the data series (corresponding to the complementary of the previous one) in which the backwards predicted heat flux diverges from the experimental data (provided the interval used for the estimation doesn't contain the former).
4. In the part of the series with the best aproximation, the peaks of the heat flux are better approximated than the first order case, but still away from a reproduction.

In Figure 4 it is shown the estimation performed forward with the first 380 points (order 10,10,8) and backwards with data from the point 450 and on (order 10,10,10). As in the first order case, the transition near point 400 can be noticed.

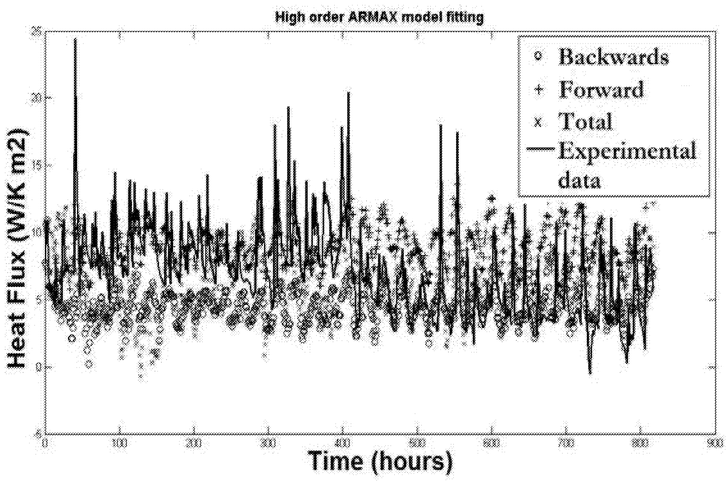

Figure 4. Estimation of the higher ARMAX models considered.

Features 1-4 mentioned before are also noticed in figure 4. Numerical results for different intervals are presented in table 1. As in the application of the steady state method, the results show physical consistency, giving a lesser value for the U-value after retrofittting the wall. It must be highlighted the low relative value of the associated error of the parameter identified.

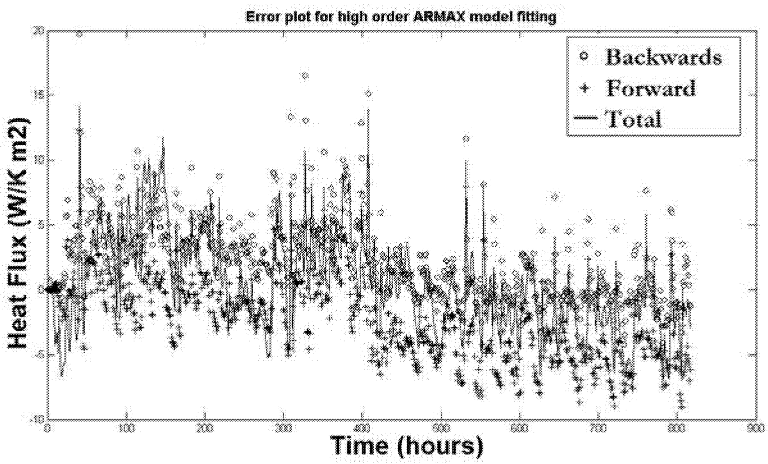

Figure 5. Residual for the high order ARMAX models.

The residuals of the time series are mostly comprised between -5 and +5 W/m^2K, which represents a high relative percentage. Figure 5 shows the residuals for the same models as estimated in figure 4.

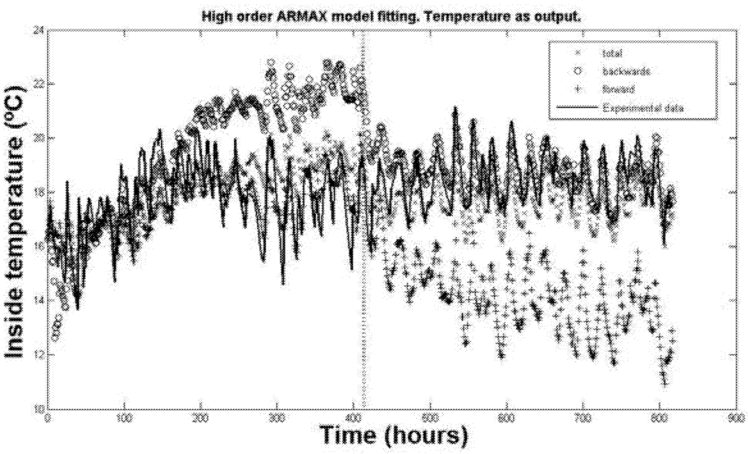

Figure 6. High order ARMAX model fitting with inside temperature as output

As reported in previous works other assignment of inputs and output can be considered (Jiménez et al., 2008). In this case it could be considered the inside temperature of the wall (T_{int}) as an output and the outside temperature and the heat flux through the wall as inputs to the model and following an analog procedure.

Results are presented in table 2 and in figure 6 (for [1,200] forward and [700,817] backwards, respectively) and are consistent with all previous considerations. The divergence in temperature is more noticeable than in the heat flux, becoming a more illustrative situation of the change of the dynamics. . These models can also be applied to evalute the effect produced in the indoors temperature by the insulation added.

4. DEVELOPMENT AND APPLICATION OF A LINEAR TIME-VARIANT (LTV) MODEL

The dynamics of a wall can also be modelled through an electrical circuit analogy. One of the simplest circuits to identify the thermal dynamics of a wall is shown in figure 7, representing the wall as a two resistances and one capacitor.

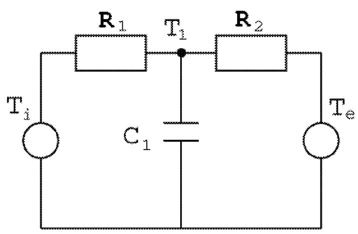

Fig. 7. Thermal circuit for the dynamics of a wall. R_1, R_2 and C_1 are functions of time.

The wall with a change in the dynamics at the instant t_0, can be modelled with the same kind of circuit, but component characteristics time-varying. The adition of insulation at t_0 can be modelled as two resistances and one capacitance, each of these parameters in the form of a heaviside function of time with the discontinuity at the same position for all these parameters.

Table 1. Numerical results for different ARMAX fitted models considering the heat flux density as output.

Time interval	U	Error	Model	Time interval	U	Error	Model
[1,150]	0.9039	0.002	10,10,8	[650,817]	0.3428	1.54 E-4	10,10,10
[1,200]	0.7851	3.6 E-4	10,10,8	[600,817]	0.3303	6.1475 E-5	10,10,10
[1,250]	0.7880	1.58 E-4	10,10,8	[550,817]	0.3542	8.0164 E-5	10,10,10
[1,300]	0.8261	1.078 E-4	10,10,8	[500,817]	0.3478	5.8 E-5	10,10,10
[1,350]	0.8451	1.3 E-4	10,10,9	[450,817]	0.3216	5.8 E-5	10,10,10
[1,375]	0.7819	2.4 E-4	10,10,8	[425,817]	0.3420	6.48 E-5	10,10,10
[1,400]	0.7732	2.394 E-4	10,10,8	[400,817]	0.3549	7.59 E-5	10,10,10
[1,425]	0.6736	2.4 E-4	10,10,8	[375,817]	0.3205	8.38 E-5	10,10,1
[1,450]	0.6141	4.437 E-4	10,10,8	[300,817]	0.1757	0.006	10,10,1
[1,500]	0.6572	0.0014	10,10,8	[200,817]	0.2543	0.003	10,10,1

Table 2. Numerical results for different ARMAX fitted models considering the indoor temperature as output.

Time interval	U	Error	Model	Time interval	U	Error	Model
[1,200]	0.7358	0.0927	10,10,1	[600,817]	0.4309	0.0065	10,10,1
[1,250]	0.7988	0.0109	10,10,1	[550,817]	0.4222	0.0011	10,10,1
[1,300]	0.7998	0.018	10,10,1	[500,817]	0.425	0.002	10,10,1
[1,330]	0.8070	0.0104	10,10,1	[575,817]	0.4247	0.0018	10,10,1
[1,350]	0.8289	0.016	10,10,1	[650,817]	0.4148	0.0012	10,10,1
[1,375]	0.8259	0.0083	10,10,1	[450,817]	0.4332	0.03	10,10,1
[1,400]	0.8342	0.0045	10,10,1	[400,817]	0.4434	0.0048	10,10,1
[1,425]	0.8137	0.0133	10,10,1	[350,817]	0.4946	0.0189	10,10,1

Putting all these hypotheses together we can write a model for the wall:

$$dT_1 = \left[-\left(\frac{1}{R_1} + \frac{1}{R_2} \right) \frac{1}{C_1} T_1 + \left(\frac{1}{R_2 C_1} \quad \frac{1}{R_1 C_1} \right) \begin{pmatrix} T_{ext} \\ T_{\text{int}} \end{pmatrix} \right] dt + \sigma_{11} dw \tag{8}$$

$$Q = -\frac{1}{R_1} T_1 + \left(0 \quad \frac{1}{R_1} \right) \begin{pmatrix} T_e \\ Ti \end{pmatrix}, \tag{9}$$

$$R_i = \frac{R_i^b}{1 + e^{-\gamma (t_0 - t)}} + \frac{R_i^a}{1 + e^{\gamma (t_0 - t)}}, \tag{10}$$

Where t_0 is the instant when the insulation was added and i = 1,2,3 and R3 = C1. CTSM (Kristensen et al., 2003) has been used as software tool to estimate the parameters of the considered model. As recommended in CTSM user manual (Kristensen et al., 2003), the heaviside function has been changed for the eq. 10 with a value of $\gamma=10^7$ to allow this, to estimate the parameters correctly, since the heaviside function is discontinuous. The supercripts "b" and "a" stand for before and after the retrofitting, respectively.

Estimating the parameters of this model with the whole set of experimental data gives results summarised in table 3.

These results are consistent with all previous results presented in sections 2 and 3, and also provide an estimation of the time of the change of the dynamics of the wall without need of further pre-processing the input data.

Table 3. Parameter estimation using a simple LTV model.

U-value before	0.8078
U-value after	0.4272
Time of change	390

It can be seen in figure 8 that this model can reproduce the experimental behaviour both backwards and forwards. This model also shows a good fit to the heat flux peaks before and after the installation of the insulation.

If identifications by separation of intervals were performed, one could have noticed that even in the worse region it presents less error in the peak tracking that the transfer function models considered. At this point, it should be remarked that the same linear and time-variant model can be used for estimation in intervals of data without change in the dynamics. A value of the time of change close to zero or greater than the leght of the interval considered will take into account of this phenomenon, without the need of reformulating the model and the analysis algorithms.

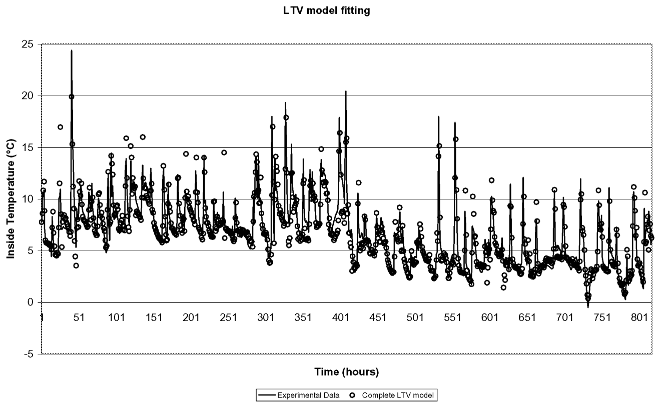

Figure 8. Time-varying dynamic model estimated with the whole series without previous treatment.

For example, in the backwards estimation, the probabilities of being negligible the resistances and capacity of the "previous insolation" part are near one (0.9992 for R_{1a}, R_{2a} and C_{2a}). For the estimated time-of-change, a negligible probability is obtanied for this parameter of 0.9998.

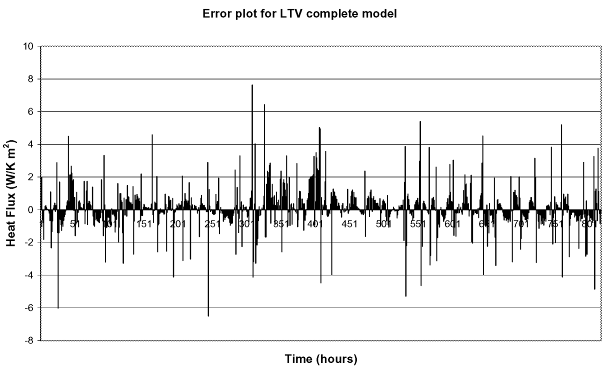

Figure 9. Residuals for the complete LTV estimation

In the figure 9 a plot of the error is presented. It can be seen that the error plot is nearly homogeneous as before as after the global identification, with values between -2 and 2 W/m^2K.

5. CONCLUSION

Several methods have been applied to identify a change in the dynamics os a wall. The steady-state method solves the problem in a simple way but give no information about the dynamics of the system.

Transfer function form models considered represent the dynamic better than the average, but in principle is difficult to select the right interval to estimate.

A LTV model has been developed and applied, showing a good predictive capability, capturing the dynamics of the model.

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