On the Influence of the Reynolds-Stress Anisotropy Tensor on the Prediction of Wall-Affected Three-Dimensional Room Airflows

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ABSTRACT

It is well known that the turbulence anisotropy has a remarkable influence on the flow of three dimensional wall jets. So the accurate simulation of room airflows with air supplies mounted just below the ceiling requires a high-level turbulence closure.

Therefore in this paper the potential for the improvement of room airflow prediction by using different Reynolds stress models and a new nonlinear eddy viscosity turbulence model are discussed. For this purpose detailed three dimensional PIV measurement results of the velocity distribution and the Reynolds stress in a symmetrical model room are compared with the calculations using different turbulence models.

1. INTRODUCTION

In buildings often very exact thermal conditions are needed to ensure a high quality production process or comfortable indoor environment. To fulfil these demands the prediction of room airflows plays an important part in the development process of modern ventilation systems. For this reason an accurate prediction of the fluid motion in ventilated rooms is necessary. Computational Fluid Dynamics is a tool which can fulfil this requirement, but it strongly depends on the correct turbulence modelling. Most air flow simulations are based on the Reynolds Averaged Navier-Stokes equations (RANS). RANS allows to calculate the time-averaged flow by modelling the fluctuating flow components using turbulence models.

$$\frac{\partial(u_i u_j)}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu_0\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] - \frac{\partial\left(\overline{u_i' u_j'}\right)}{\partial x_j} \qquad (1)$$

First-order turbulence models use the Boussinesq hypothesis to determine the Reynolds stress tensor. Second-order turbulence models directly solve the transport equations for the Reynolds stresses.

The computational effort of the first-order models is clearly lower so that they are commonly used. Especially the $k-\varepsilon$ turbulence models are generally accepted as a good choice because they are numerical stable and often predict the mean flow variables in acceptable agreement with experiments (Nielsen, 1990; Nielsen, 1998; Voigt, 2000; Xu and Chen, 2000; Luo et al. 2004 etc.).

It is generally known that the Boussinesq assumption is not able to suitably reproduce the turbulent normal stresses (the Boussinesq hypotheses was original developed to reproduce only the turbulent shear stresses). For this reason the Boussinesq assumption fails e.g. in the prediction of the secondary motion in non-circular ducts or of the spreading behaviour of 3D wall jets. In both cases the flow pattern is strongly influenced by the redistribution of the turbulent normal stresses near the wall (wall reflection effect). This effect can be easily shown by the use of the axial vorticity transport equation for a fully developed flow in x-direction. For the axial vorticity

$$\Omega_1 = \frac{\partial u_2}{\partial x_3} - \frac{\partial u_3}{\partial x_2} \qquad (2)$$

the turbulence induced production and dissipation term in the transport equation of the axial vorticity can be summarized as follows:

$$S_{\Omega_1} = \frac{\partial^2\left(\overline{u_3'^2} - \overline{u_2'^2}\right)}{\partial x_2 \partial x_3} - \left(\frac{\partial^2 \overline{u_2' u_3'}}{\partial x_3^{\,2}} - \frac{\partial^2 \overline{u_2' u_3'}}{\partial x_2^{\,2}}\right) \qquad (3)$$

From Eq. (3) it can be derived that only local changes of the turbulent normal stress difference $\overline{u'_3} - \overline{u'_2}$ have an effect on the axial vorticity balance – but not the absolute values. Demuren and Rodi (1984) showed that the normal stresses excite the axial vorticity and the turbulent shear stresses $\overline{u'_2 u'_3}$ work against. Hence an accurate prediction of the turbulent secondary motion requires an exact balance of the turbulent stresses in the momentum equation (Bäumer 2000).

For a 3D wall jet Craft and Launder (2001) investigated the lateral movement of fluid away from the symmetry plane. They compared different Reynolds stress turbulence models with the experimental data of Abrahamsson (1997) and concluded that the prediction of the flow pattern is extremely sensitive to the used pressure strain model. The best results could be achieved with the two component pressure strain model (TCL).

Another investigation of the spreading mechanism of a 3D wall jet was done by Lübcke et al. (2003). In contrast to the investigations of Craft and Launder (2001) only low-Re $k-\varepsilon$ and algebraic Reynolds stress models were used. They showed that the modeling of 3D wall jets (a 3D wall jet with a round inlet area was considered) requires one quartic generator (defining equation with at least one fourth-order term). In contrast the secondary motion in a squared duct can be modeled with one quadratic generator.

The impact of the turbulent normal stresses on the flow pattern of three dimensional wall jets indicates that higher order turbulence models can better predict room airflows. Therefore in this paper the potential of differential (RSM) and algebraic (ARSM) Reynolds stress turbulence models are discussed for an isothermal 3D room airflow. Besides the familiar standard $k-\varepsilon$ and RSM models with linear pressure strain terms a new ARSM $k-\varepsilon$ turbulence model is presented.

2. TURBULENCE MODELING

In this paper only isothermal incompressible flows are investigated. Source terms which take into account system rotation or buoyancy effects are not considered. Since the paper is focused on turbulence modelling model equations and constants not commonly known are briefly summarized.

2.1. Differential Reynolds stress turbulence models

The RSM uses for every turbulent stress a transport equation of the following form:

$$\frac{D\left(\rho \overline{u'_i u'_j}\right)}{Dt} = D^T_{ij} + D^L_{ij} + P_{ij} + \phi_{ij} - \varepsilon_{ij} \qquad (4)$$

Where D^T_{ij}, D^L_{ij}, P_{ij}, ϕ_{ij} are turbulent diffusion, molecular diffusion, stress production and pressure strain terms and ε_{ij} is the dissipation tensor. To close the equations the terms D^T_{ij}, ϕ_{ij} and ε_{ij} need to be modelled. Besides, the pressure strain term has a significant influence on the anisotropy tensor (b_{ij}). Therefore the right choice of the pressure strain term is a prerequisite for an accurate prediction of room airflows. The basic concept of the pressure strain term modelling divides it into a slow and a rapid part. The slow term describes the interaction between the pressure and velocity fluctuation and the rapid term the interaction between the pressure fluctuation and the time-averaged velocity field. To ensure a stable solution generally linear slow and rapid terms are used with the following components:

$$\phi_{ij} = \phi_{ij,1} + \phi_{ij,2} + \phi_{ij,w} \qquad (5)$$

$$\phi_{ij,1} + \phi_{ij,2} = -C_1 \rho \frac{\varepsilon}{k}\left[\overline{u'_i u'_j} - \frac{2}{3}\delta_{ij} k\right] - C_2\left[P_{ij} - \frac{2}{3}\delta_{ij} P\right]$$

$$\phi_{ij,w} = C'_1 \frac{k}{\varepsilon}\left(\overline{u'_k u'_m}\, n_k n_m \delta_{ij} - \frac{3}{2}\overline{u'_i u'_k}\, n_j n_k - \frac{3}{2}\overline{u'_j u'_k}\, n_i n_k\right)\frac{k^{3/2}}{C_l\, \varepsilon\, x_n} + C'_2\left(\phi_{km,2}\, n_k n_m \delta_{ij} - \frac{3}{2}\phi_{ik,2}\, n_j n_k - \frac{3}{2}\phi_{jk,2}\, n_i n_k\right)\frac{k^{3/2}}{C_l\, \varepsilon\, x_n}$$

Because the linear terms are not able to consider the redistribution of the turbulent normal stresses near the wall a wall reflection term must be added. Different choices of the model constants C_1 and C_2 allow to modify the anisotropy tensor b_{ij}. For a homogeneous turbulent shear flow it can be shown that the anisotropy tensor components b_{12}, b_{11}, b_{22} and b_{33} are related to each other. For this reason an individual adjustment of the model constants to ensure a correct modelling of both - the normal and the shear stresses - is not possible. So an individual weighting of the model constants in order to improve the prediction of the normal stresses or the shear stresses is necessary. In this paper two different approaches for the model constants are used. The first approach was proposed by Gibson and Launder (1978) and corresponds to the well-known standard values for the isotropisation of the production model. In this paper the model is designated as the RSM-IP-GL model (IP stand for isotropisation by production). The second approach was proposed by Gibson and Younis (1986) and is denoted in this paper as the RSM-IP-GY model. The proposed model constants are summarized in Table 1.

Table 1. Model constants for the pressure strain model

	C_1	C_2	C_1'	C_2'
model	-	-	-	-
RSM-IP-GL	1.80	0.60	0.50	0.30
RSM-IP-GY	3.00	0.30	0.75	0.50

2.2. Algebraic Reynolds stress turbulence models

In the literature a various number of algebraic Reynolds stresses turbulence models and additional constitutive (Shih et al., 1993) equations are available. In principle they all assume that the anisotropy tensor depends on the local velocity gradients and the turbulent time scale Eq. (6). Therefore this approach is also named nonlinear eddy viscosity concept.

$$b_{ij} = \sum_{\lambda} G_{\lambda} T_{ij}^{\lambda} \qquad T_{ij} = T_{ij}\left(S_{ij}, \Omega_{ij}, T\right) \qquad T = \frac{k}{\varepsilon} \tag{6}$$

Because of limited numerical stability and the individual adjustment of the model constants a reliable use of the existing nonlinear eddy viscosity models is often difficult. Therefore a new algebraic Reynolds stress turbulence model is proposed.

2.2.1. Basic concepts

For two dimensional flows Gatski and Speziale (1992) show that three linear independent tensors are sufficient to reproduce the Reynolds stresses.

$$b_{ij} = -C_{\mu} T S_{ij} - 2 C_{\mu} C_1 T^2 \left(S_{ik} S_{kj} - \frac{1}{3} S_{kl} S_{kl} \delta_{ij} \right) - 2 C_{\mu} C_2 T^2 \left(\Omega_{ik} S_{kj} - \Omega_{jk} S_{ki} \right) \tag{7}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \qquad \Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{8}$$

For simple flows (e.g. boundary layer flows) it can be shown that the coefficient C_2 is proportional to the normal stress difference b_{22} - b_{11} and the value (C_1 - $2C_2$)/4 is proportional to the normal stress difference b_{22} – b_{33}. That means that the coefficient C_1 and C_2 allow an individual adjustment of the turbulent normal stresses.

2.2.2. Development of a new algebraic Reynolds stress model

For simple shear flows the coefficients C_1 and C_2 in Eq. (7) can be determined directly from experimental and DNS data. In that case the dimensionless invariant

$$\eta = \frac{k}{\varepsilon}\sqrt{2 S_{ij} S_{ij}} \tag{9}$$

is a reasonable parameter to adapt the model constants to the local turbulence structure. For example the anisotropy behaviour and the invariant η of different experimental and DNS investigations are summarized in Table 2.

Table 2. Anisotropy tensor for different shear flows

	eta	b_{11}	b_{22}
author	-	-	-
Kim et al. (1987)	3.30	0.179	-0.127
TK (1989)	4.30	0.220	-0.160
TC (1981)	6.08	0.202	-0.145
TC (1981)	6.25	0.200	-0.150
SNT (1995)	7.70	0.180	-0.110
Laufer (1951)	3.10	0.220	-0.150

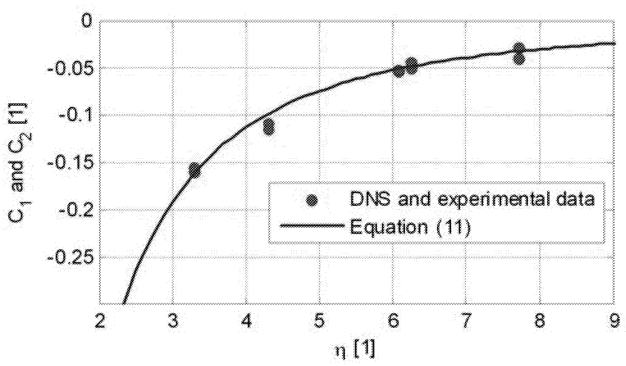

Figure 1. From experimental and DNS data determined model constants for equation 7 in dependence on the dimensionless invariant.

TK, TC and SNT are abbreviations for the investigations of Tavoularis and Corrsin (1981), Tavoularis and Karnik (1989) and de Souza et al. (1995).

Based on the above investigations of homogeneous and channel shear flows the coefficients C_1 and C_2 can be determined. In dependence on the non-dimensional invariant the distribution of the coefficients are shown in figure 1.

It is obviously that an accurate determination of the anisotropy factors requires variable coefficients. Because for homogeneous turbulent flows and for boundary layer flows b_{12} is proportional to η and b_{ii} is proportional to η^2 following approach is suggested:

$$C_i = f\left(\eta^2\right) \tag{10}$$

Based on the experimental and DNS data of Table 2 and the standard $k-\varepsilon$ turbulence model ($C_\mu = 0.09$) the following approximation is derived.

$$C_1 = C_2 = \frac{-1.9}{0.9+\eta^2} \tag{11}$$

For two-dimensional boundary layer flows this formulation ensures positive turbulent normal stresses so that higher numerical robustness can be expected. The model was implemented with UDF (User Defined Functions) in the commercial CFD-Code Fluent 6.3.26 and the obtained results are labeled in this paper as SKE-ARSM.

2.3. Discussion about the anisotropy factors

The principal near-wall behaviour of the used turbulence models can be shown by comparison with DNS data of a two-dimensional channel flow. Fig. 2 shows the computed anisotropy factor b_{33}-b_{22} and DNS data of Moser et al. (1999). For computation a periodic condition with given pressure gradients in mean flow direction were used to ensure equivalent turbulent shear stress (b_{12}) distributions.

The RSM-IP-GL and the RSM-IP-GY models predict too large differences of the anisotropy factors. The RSM-IP-GY version gives slightly better results. In the core flow the new algebraic Reynolds stress model agrees well with the DNS data. Because of the calibration procedure (low Reynolds effects are not considered) an underprediction of the anisotropy factor difference b_{33}-b_{22} near the wall (y^+<130) can be observed.

The results show that a wall reflection term is necessary to predict the strong anisotropy behaviour near the wall. The model without a wall reflection term (RSM-IP-GL-WWR) thus underpredicts the strong anisotropy behaviour.

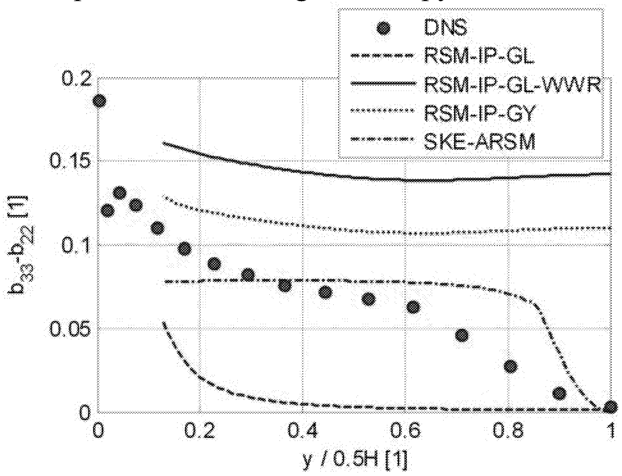

Figure 2. Comparison of the computed anisotropy factor b_{33}-b_{22} with DNS data of Moser et al. (1999). RSM-IP-GL-WWR means RSM-IP-GL model without wall reflection term ($C_1' = C_2' = 0.0$).

3. VALIDATION

All computational results are obtained with the commercial CFD code Fluent 6.3.26. The SIMPLE base segregated solver is used, for the convective terms the second order upwind discretization is applied.

The test case considered is the model room I investigated by Heschl et al. (2008). The flow in this room contains a free jet region, a wall affected area and strongly three dimensional effects which are induced by the turbulent shear stresses (entrainment) and turbulent normal stresses (3-D wall jet region). The main geometry of the computational domain is shown in Fig. 3.

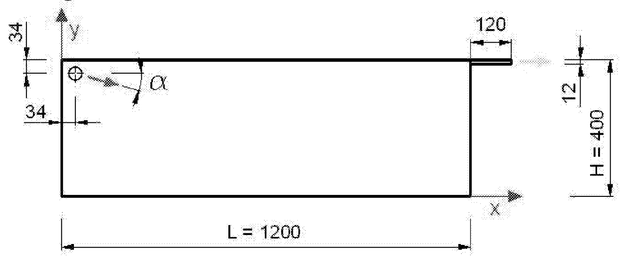

Figure 3. Computational domain of the investigated complex 3-D room airflow (dimensions in mm, α = 16°).

For the computations two different grids – a fine and a coarse grid – are used. The coarse grid was designed with a $y^+ \sim 15$ for turbulence models with standard wall functions (RSM-IP-GL and RSM-IP-GY). The number of grid cells for the whole computational domain is about 1.200.000. The fine grid was designed for near-wall low-Re turbulence models (standard $k-\varepsilon$ and the algebraic Reynolds stress turbulence model with enhanced wall treatment) with a $y^+ < 1$. The number of grid cells for the whole computational domain is about 3.500.000. All used turbulence models show good convergence behaviour.

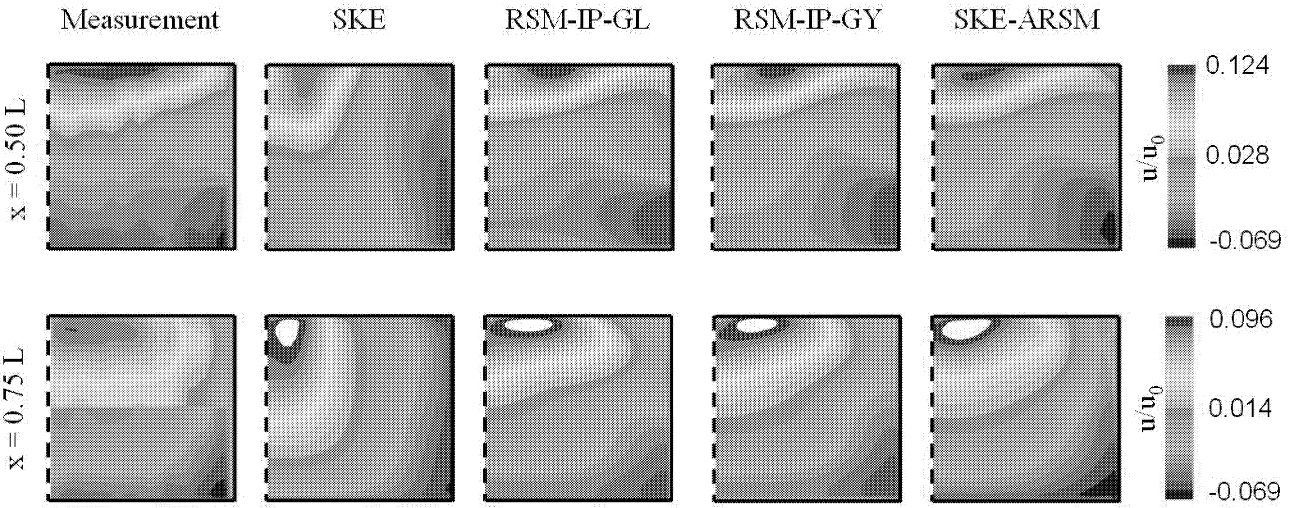

Figure 4. Measured and computed x-velocity distribution in the x= ½ L = 600mm and x= ¾ L = 900mm plane (values in m/s; the dashed line ----- corresponds to z=B/2)

The performance of the turbulence models are assessed by the x-velocity distribution in the x = ½·L and x = ¾·L plane. The results shown in Fig. 4 differ in the penetration of the jet into the room and its diffusion.

The differences in the velocity distributions between the linear eddy viscosity (SKE) and the anisotropy turbulence models (SKE-ARSM, RSM-IP-GL and RSM-IP-GY) are obvious. Especially the predicted wall jet formation on the ceiling differs from the measurements when the SKE turbulence model is used. The RSM-IP-GY model gives slightly better results than the RSM-IP-GL model. The new SKE-ARSM model gives again very satisfying results.

4. Summary and Conclusion

Linear eddy viscosity turbulence models are widely used to predict air flows in ventilated rooms. In many air supply configurations a dominant interaction between the air flow and the surrounding walls arises. Especially the anisotropic Reynolds stresses close to walls can produce secondary motions which influence the airflow. For this reason a new algebraic Reynolds stress turbulence model is proposed. Compared with the linear eddy viscosity turbulence models clearly better agreements with experimental findings are obtained. In comparison with differential Reynolds stress turbulence models the new model gives comparable results but needs less computational effort.

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Nomenclature

k	kinetic turbulence energy [m²/s²]
b_{ij}	Reynolds stress anisotropy tensor [1] $b_{ij} = \left(\overline{u_i'u_i'} - 2/3\,k\delta_{ij}\right)/2k$
$\overline{u_i'u_i'}$	Reynolds stress tensor [m²/s²]
η	non-dimensional invariant [1]
S_{Ω_1}	source term [1/s²]
S_{ij}	strain rate tensor [1/s]
Ω_{ij}	rotation rate tensor [1/s]
T	turbulent time scale [s]

u_i	time averaged velocity [m/s]
$\phi_{ij,1}$	slow pressure strain term [kg/m s³]
$\phi_{ij,2}$	rapid pressure strain term [kg/m s³]
$\phi_{ij,w}$	wall reflection term [kg/m s³]
δ_{ij}	Kronecker delta function [1]
μ_t	turbulent dynamic viscosity [Pa s]
ε	dissipation rate [m²/s³]
ε_{ij}	dissipation rate tensor [m²/s³]
P	production of k [kg/m s³]
P_{ij}	production of $\overline{u_i'u_i'}$ [kg/m s³]

ABBREVIATIONS

SKE: Standard $k-\varepsilon$ turbulence model

SKE-ARSM: Standard $k-\varepsilon$ based algebraic Reynolds stress turbulence model presented in this paper

RSM-IP-GL: Reynolds stress turbulence model with Gibson and Launder (1978) pressure strain model and with wall reflection term

RSM-IP-GL-WWR: Reynolds stress turbulence model with Gibson and Launder (1978) pressure strain model without wall reflection term

RSM-IP-GY: Reynolds stress turbulence model with Gibson and Younis (1986) pressure strain model

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