Computational analysis of indoor air circulation and heat transfer in a house ventilated by wind-catch

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ABSTRACT

Considering that natural ventilation is a climatic factor, a renewable alternative, as well as an important source of comfort, it seems to be relevant to consider its great potential when applied to architecture, taking into account economical and environmental gains. It is difficult to think about studying air circulation without mentioning the north Africa vernacular architecture and its ability to create the best solutions to increase the ventilation without any energy consumption, therefore, adopting passive cooling. According to Hassan Fathy (1986) -- an Egyptian architect who developed a collection of fruitful works using natural ventilation and local materials -- in that region, ventilation, light and view could not be associated at the same time to a single window because each function demands a different window configuration. Criative systems of passive cooling appeared in these areas and among them the wind-catch or malqaf can be distinguished. It was created to improve ventilation requirements catching wind from high above, where the air is cooler, stronger and with less sand and other particles, and channeling it down into the building. In agreement with Fathy, another advantage of the malqaf is the possibility of solving the problem of screening that is the consequence of building settlements in a ordinary town plan. In Egypt the malqaf is very developed and its first usage dates back to ancient historical periods. In this work, based on the writings of Fathy (1986) we analyse numerically one case in which wind-catch is used. It is the Qã'a of Muhib AdDin Ash-Shãf'i Al-Muwaqqi, known as Othmãn Katkhudã, in Cairo, dated from the fourteenth century A.D. presented in Fathy's book, (Fathy 1986).

The analyses start by solving the air circulation problems to determine the wind fields, using a mixed stabilised finite element method applied to the full Navier-Stokes equations. With these wind fields, the heat transfer problem is solved and analysed. Numerical results are compared with the studies presented by Fathy (1986). The results obtained suggest the capacity of cooling the indoor air by improving the ventilation with a reasonable arrangement of openings only.

1. INTRODUCTION

The growing requirements of reducing environmental costs, led citizens and several professionals in an effort towards reduction of energy consumption and pollute emission. In the specific case of air circulation, it is difficult to think about studying it without mentioning the north Africa architecture and its ability to find solutions to increase ventilation without energy consumption, therefore, adopting passive cooling.

Considering that natural ventilation is a climatic factor, a renewable alternative, as well as an important source of comfort, it seems to be relevant to consider its great potential when applied to architecture, taking into account economical and environmental gains.

In the case of air circulation, it is difficult to think about studying it without mentioning the north Africa vernacular architecture and its ability to create the best solutions to increase the ventilation without any energy comsumption, therefore, adopting passive cooling. According to Hassan Fathy (1986), in that region, ventilation, light and view could not be associated at the same time to a single window because each function demands a different window configuration. The wind-catch, malqaf, was created to improve ventilation requirements catching wind from high above, where the air is cooler, stronger and with less sand and other particles, and channeling it down into the building, avoiding, therefore much work on the design of the common windows to ensure such requirements. In Egypt the malqaf is very developed and its first usage dates back to ancient historical periods.

In this work, based on the writings of Hassan Fathy (1986) -- an Egyptian architect who developed a collection of fruitful works using natural ventilation, proper orientation, local materials, traditional construction methods and energy-conservation techniques -- we analyse numerically one case in which wind-catch is used. It is the Q˜a'a of Muhib AdDin Ash-Shf'i Al-Muwaqqi, known as Othmn Katkhud, in Cairo, dated from the fourteenth century A.D. Figures 1-2, extracted from Fathy (1986), show the section throught the Q˜a'a of Muhib AdDin Ash-Shf'i Al-Muwaqqi and its plan, respectively. "The Q˜a'a is a central room, usually a living room in a residence or a meeting room in a formal hall" (1986).

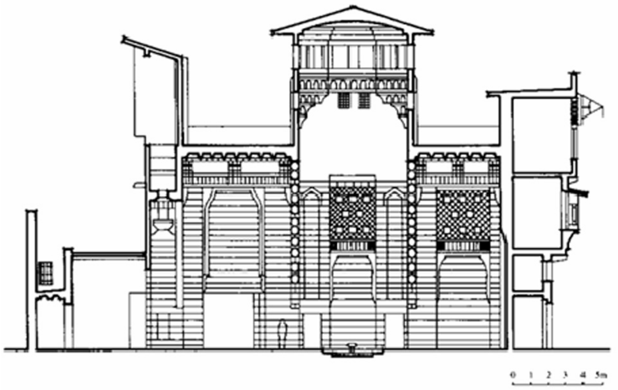

Figure 1: Section throught the Q˜a'a of Muhib AdDin Ash-Shf'i Al-Muwaqqi from Fathy (1986).

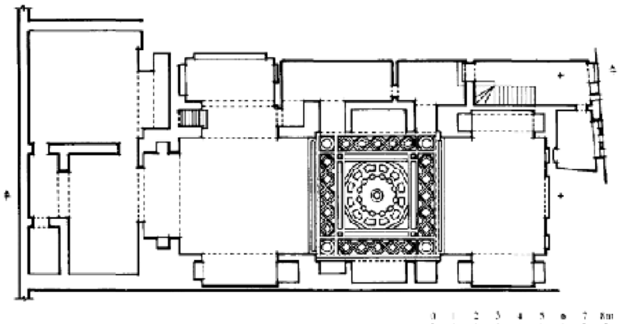

Figure 2: Plan of Muhib AdDin Ash-Shf'i Al-Muwaqqi from Fathy (1986).

The complete climatization system from which the malgaf is part it is shown in Figure3. This wind field mesurements were made by students from the Architectural Association School, London in 1973 (Fathy 1986).

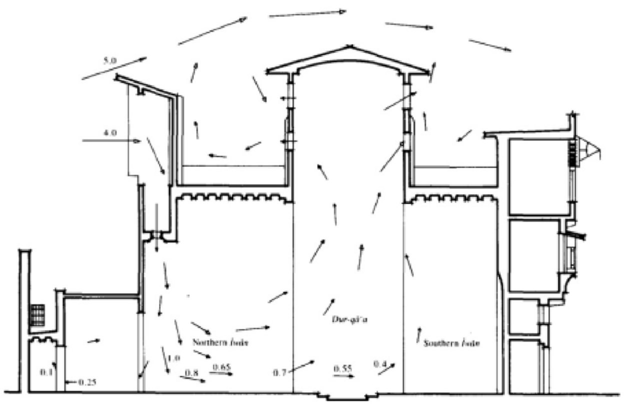

Figure 3: Section throught the Q˜a'a ofMuhib AdDin Ash-Shf'i Al-Muwaqqi - mesurements were made by students from the Architectural Association School, London in 1973 (Fathy 1986).

The analyses start by solving the air circulation problems to determine the wind fields, using a Petrov-Galerkin mixed stabilised finite element method (Drach and Karam F. 2004), applied to the full Navier-Stokes equations written in velocity and pressure variables (Brooks and Hughes 1982). With these wind fields, the thermal problem is analysed through a SUPG – Streamline Upwind Petrov-Galerkin stabilised finite element method. At the end, numerical results are compared with the studies presented in Fathy (1986) and some conclusions are done. The results suggest that the utilization of wind capture as well as optimal location of openings to improve ambient conditions deserve more attention and research, since these strategies show good potential in achieving significant quality with reduced environmental and economical costs.

2. METHODS

The problem of air circulation and heat transfer can be modelled through mass, momentum and energy conservation equations. Assuming incompressibility, the mathematical formulation for the geral problem can be written as: Find $\mathbf{u}$, p and θ satisfying the following system,

$$div\ (\mathbf{u}) = 0,\ \text{in}\ \Omega \times [0,\mathrm{T}], \quad (1)$$

$$\rho \frac{\partial \mathbf{u}}{\partial \mathrm{t}} + \rho\ (\nabla \mathbf{u})\mathbf{u} - 2\mu\ div\ \varepsilon\ (\mathbf{u}) + \nabla \mathrm{p} + \rho \mathbf{g} \beta (\theta - \theta_\infty) = 0,\ \text{in}\ \Omega \times \left[0,\mathrm{T}\right], \quad (2)$$

$$\rho c_p \frac{\partial \theta}{\partial t} + \rho c_p \mathbf{u}.\left(\nabla \theta\right) - k\ div\ \nabla \theta = 0,\ \text{in}\ \Omega \times \left[0,\mathrm{T}\right], \quad (3)$$

$$\mathbf{u}(\mathbf{x},\mathrm{t}) = \bar{\mathbf{u}}(\mathbf{x},\mathrm{t}), \text{in}\ \Gamma_u \times \left[0,\mathrm{T}\right], \mathbf{u}(\mathbf{x},0) = \mathbf{u}_0, \text{in}\ \Omega \times \left[0,\mathrm{T}\right]$$
$$k \nabla \theta . \vec{\mathbf{n}} = 0,\ \text{in}\ \Gamma_d \times \left[0,\mathrm{T}\right],\ \theta(\mathbf{x},t) = \bar{\theta}, \text{in}\ \Gamma_c \times \left[0,\mathrm{T}\right]$$
$$\text{and}\ \theta(\mathbf{x},0) = \theta_o(\mathbf{x}),\ \text{in}\ \Omega$$

where: $\mathbf{u} = \mathbf{u}(\mathbf{x},t)$ is the velocity vector, p=$(\mathbf{x},t)$ is the pressure, $\theta = \theta(\mathbf{x},t)$ is the temperature, μ is the viscosity, ρ is the density, k is thermal condutivity, θ_∞ is the reference temperature, $\vec{\mathbf{n}}$ is the normal vector, $\varepsilon(\mathbf{u}) = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T\right)$, c_p is the specific heat, β is the coefficient of thermal expansion, $\mathbf{g}$ is the gravity vector, Ω is the bounded domain with boundary $\Gamma = \Gamma_c \bigcup \Gamma_d$ and the time $t \in [0,\mathrm{T}]$.

The term $\rho \mathbf{g} \beta (\theta - \theta_\infty)$ allows the coupling of the air circulation and the heat transfer problems.

The numerical solutions are here obtained by a stabilyzed mixed finite element method that allows to deal with the difficulties that come from the first equation system, Equations (1) and (2): the difficulty in constructing approximation spaces for problems with internal constraint; non–linearities of the convective type and numerical instabilities when advection effects are dominant. Here, a Petrov-Galerkin type method was implemented and applied to analyse indoor air circulation cases, ensuring stability for dominant advection and for the internal constraint (Karam F. and Loula 1992).

Being L^2 and H^1 the usual Hilbert spaces and R_l^h the Lagrange polynomial space of the degree l and class C^0. Then, defining the following aproximation spaces

$$V^h=\left\{\mathbf{u}_h \in (H_0^1(\Omega)\cap R_l^h(\Omega))^2\right\} \subset V=\{\mathbf{u}\in (H^1(\Omega))^2\}$$

$$P^h=\left\{p_h \in (L^2(\Omega)\cap R_l^h(\Omega)); \int_\Omega p_h \partial\Omega = 0\right\} \subset P=\{p\in (L^2(\Omega))\}$$

$$S^h=\left\{\theta_h \middle| \theta_h(\mathbf{x},t)\in H^1(\Omega)\cap R_l^h(\Omega)\right\} \subset S=\{\theta\in (H^1(\Omega))\}$$

with the usual norm

$$\|\mathbf{u}\|_1^2 = \|\mathbf{u}\|_0^2 + \|\nabla\mathbf{u}\|_0^2 \text{ of } H^1 \text{ and } \|p\| = \|p\|_0 \text{ of } L^2.$$

The wind field can be determined by solving the following formulation:

Find $\{\mathbf{u}^h, p^h\} \in V^h \times P^h$ satisfying the following system

$$B(\mathbf{u}_h, p_h; \mathbf{v}_h, q_h) = F(\mathbf{v}_h, q_h), \ \forall \ (\mathbf{v}_h, q_h) \in V_h \times P_h$$

where:

$$B(\mathbf{u}_h, p_h; \mathbf{v}_h, q_h) = \left(\frac{\mathbf{u}_h^n - \mathbf{u}_h^{n-1}}{\Delta t}, \mathbf{v}_h\right) + \left((\nabla\mathbf{u}_h)\mathbf{a}_h, \mathbf{v}_h\right) +$$
$$2\nu\left(\varepsilon(\mathbf{u}_h), \varepsilon(\mathbf{v}_h)\right) - \left(p_h, div(\mathbf{v}_h)\right) + \left(q_h, div(\mathbf{u}_h)\right) +$$
$$\left(div(\mathbf{u}_h), \delta_2 div(\mathbf{v}_h)\right) + \mathbf{g}\beta(\theta-\theta_\infty) + \delta_1\sum_{e=1}^{Nel}\left(\frac{\mathbf{u}_h^n - \mathbf{u}_h^{n-1}}{\Delta t} +\right.$$
$$(\nabla\mathbf{u}_h)\mathbf{a}_h - 2\nu \ div \ \varepsilon(\mathbf{u}_h) + \nabla p_h,$$
$$\left.\left((\nabla\mathbf{v}_h)\mathbf{a}_h - 2\nu \ div \ \varepsilon(\mathbf{v}_h) + \nabla q_h\right)\right)_h + \gamma(p_h, q_h),$$
$$\forall \ \mathbf{v}_h \in V_h \ e \ q_h \in P_h.$$

with $\gamma << 1$ and δ_1 and δ_2 stabilyzed parameters suggested by (Franca and Frey 1992).

And find θ(x,t) satisfying the following system:

$$\left(\frac{\theta_h^n - \theta_h^{n-1}}{\Delta t}, s_h\right) + \left(k\nabla\theta_h, \nabla s_h\right) + \left(\mathbf{u}.\nabla\theta_h, s_h\right) +$$
$$\sum_{e=1}^{Nel}\left(\tau, \left(\left(\frac{\theta_h^n - \theta_h^{n-1}}{\Delta t}\right) - k\Delta\theta_h + \mathbf{u}.\nabla\theta_h\right)\right)_h = 0$$
$$\forall \ s_h \in S_h$$

With $\tau = \tilde{k}.\nabla s_h$, the stabilyzed parameters suggested by (Brooks and Hughes 1982).

The time discretization has been done by a backward Euler finite difference.

3. SIMULATIONS

Meshes were generated with linear triangular elements as in Figure 4. The whole meshes comprise areas bigger than the areas of the plans in order to impose the boundary conditions on their borders and to leave unknown the velocities at the entrances that are determined by the solution of the problem.

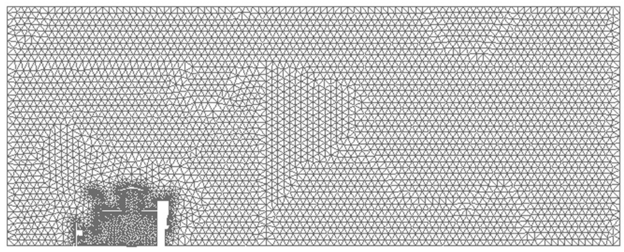

Figure 4: Q˜a'a Muhib AdDin Ash-Shf'i Al-Muwaqqi whole mesh.

Arrow in Figure 5 indicates the direction of the outside wind, providing the boundary conditions to the air circulation problem. The absolute value of 1m/s was adopted for the outside wind. According to the Beaufort Scale, used in time forecast, corresponding to breeze without wind perception. The initial temperature conditions considered were 28°C indoor and 24°C outdoor, selected from the average temperature in Cairo, Egypt, see Figure 5.

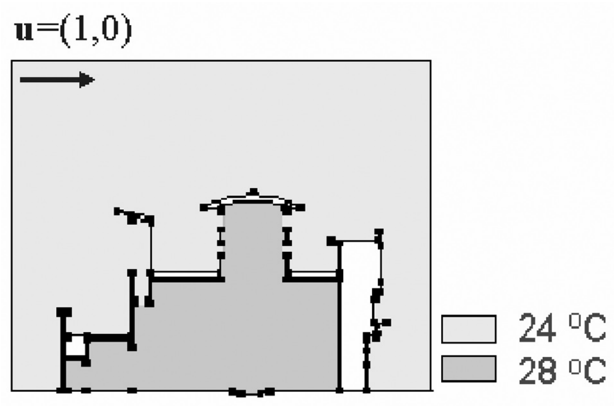

Figure 5: Initial temperature conditions. The arrow indicates the outside wind direction.

4. RESULTS

Results for the studied case are presented in Figures 6–

11 in terms of contour fill of temperature and Figures 12-16 in terms of velocity wind field.

From Figures 12 - 16 we can observe that the velocity results, that give the air circulation patterns, not only confirm the global circulation sketches in Fathy's book, but also show the details of the flow formed inside the Q˜a'a house even when the external wind is low (1m/s). These patterns promote favorable conditions to widely distribute the circulation effects.

The consequence of the circulation to the temperature field conditions can be observed in Figures 12-16. It can be estimated that even for the case of weaker outside wind there was a global reduction in the average temperature inside the Q˜a'a when compared with the average temperature considering the initial temperature.

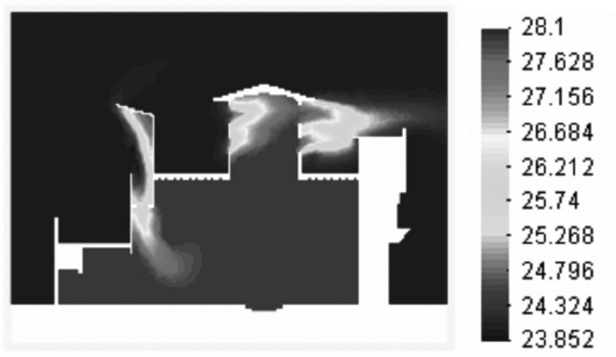

Figure 6: Step 10 - contour fill of temperature.

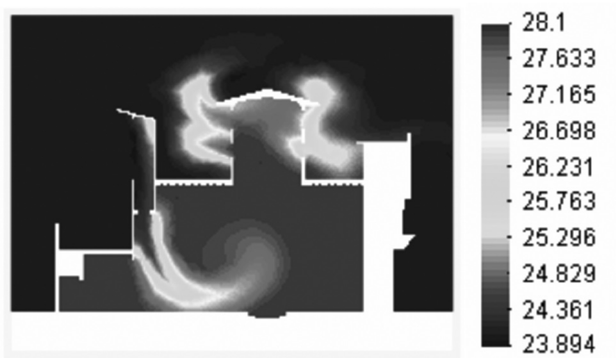

Figure 7: Step 30 - contour fill of temperature

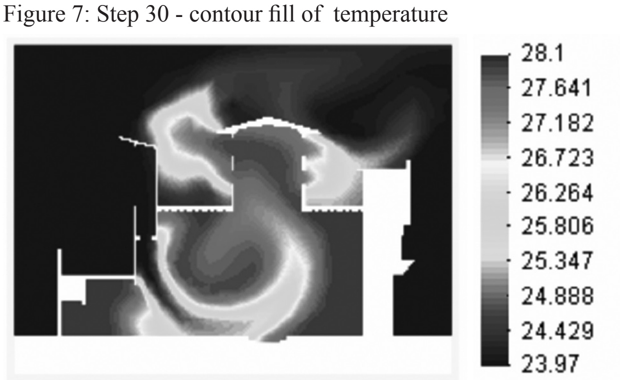

Figure 8: Step 60 - contour fill of temperature.

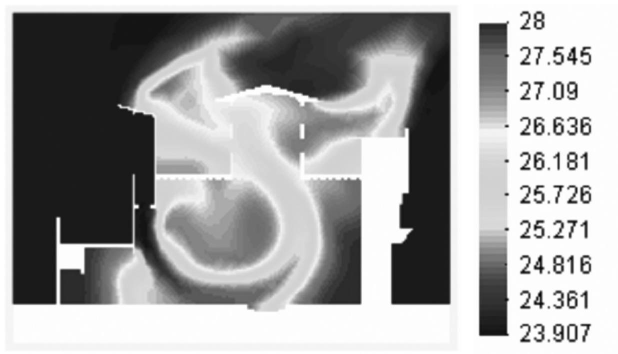

Figure 9: Step 120 - contour fill of temperature.

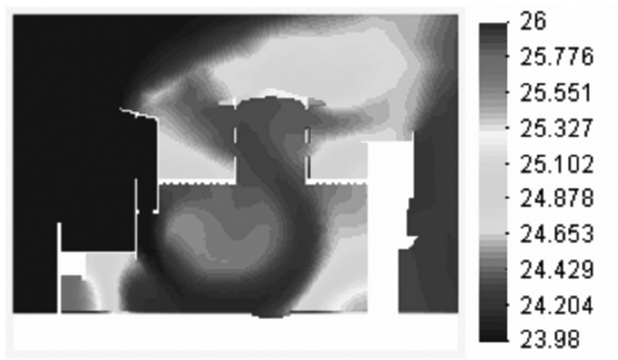

Figure 10: Steady state - contour fill of temperature.

In Figure 10 steady state results of contour fill of temperature are presented for a temperature scale with superior limit of 25 ºC. It makes possible a better visualization of the results.

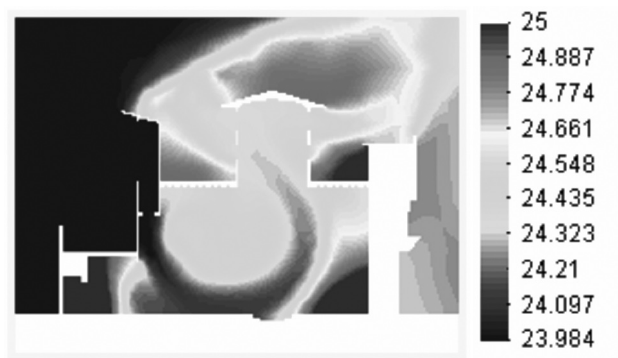

Figure 11:Steady state – contour fill of temperature.

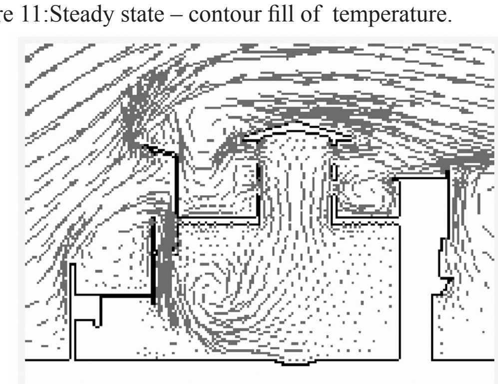

Figure 12: Step 10 – wind field.

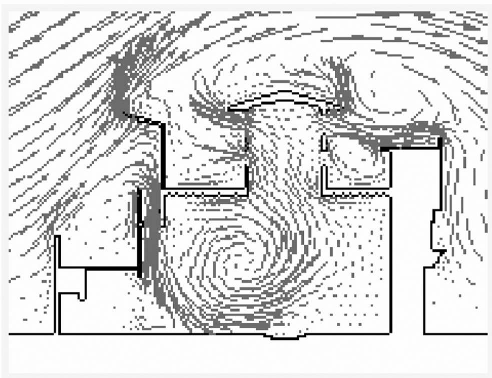

Figure13: Step 30 – wind field.

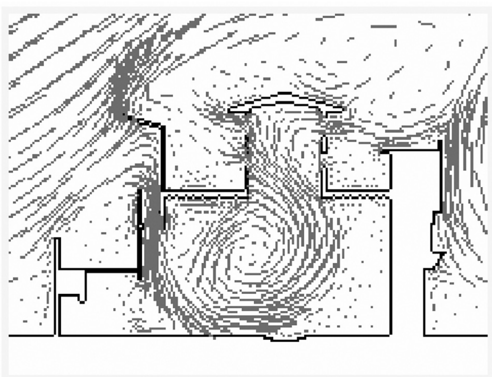

Figure 14: Step 60 – wind field.

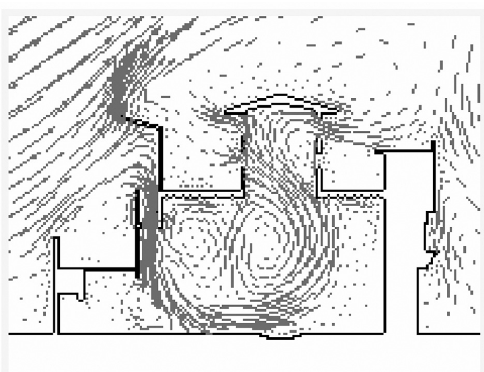

Figure 15: Step 120 – wind field.

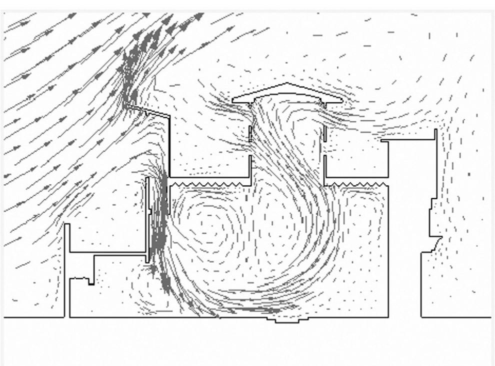

Figure 16: Steady state – wind field.

4. CONCLUSIONS

For the air circulation problem, the computational velocity results, that give the air circulation patterns, not only confirm the global circulation sketches in Fathy's book, but also show the details of the secondary flow formed inside the room. We can note the capacity of the system to promote the air circulation in the interior even when the outdoor wind is smooth, (1m/s).

The results obtained from the heat problem suggest the capacity of changing indoor temperature by improving the ventilation with a reasonable arrangement of openings. In Figures 10 and 11 it can be observed that the indoor temperature reached very close to the outdoor temperature values.

The computational results obtained here, confirming the Fathy's thoughts, suggest that the utilization of wind capture as well as optimal location of openings to improve ambient conditions deserve more attention and research, since these strategies show good potential in achieving significant quality with reduced environmental and economical costs.

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