Improvement of Thermal Comfort in a Naturally Ventilated Office

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SYNOPSIS

In natural ventilation systems, fresh air is often provided through opening of windows, and there is a wide range of possibilities with regard to selection of window type and position in the facade. Different window types have quite different characteristics and thereby different impact on the thermal comfort conditions in the occupied zone. The thermal comfort is also very dependent on the selected natural ventilation strategy, the outdoor conditions and the available pressure difference across the window opening. A combination of different window types and windows in different positions in the same natural ventilation design should, by using their strong sides, be capable of both improving ventilation capacity, thermal comfort and IAQ.

The paper describes the results of laboratory investigations in a mock-up of an office space with the purpose of investigating the impact of different opening strategies on thermal comfort conditions in the occupied zone. The results show that different window opening strategies result in quite different airflow and thermal comfort conditions. The conditions are a result of a multivariable impact, and detailed descriptions of the flows involved are complex.

NOMENCLATURE

A _{win} :	Minimum cross-sectional opening area of window opening (m ²)		
Ar:	Archimedes' number (n.d.)	\dot{V}_{w} :	Volumetric flow rate through window opening (m ³ /s)
C_D :	Discharge coefficient (n.d.)	Δp_{win} :	Pressure difference across window/facade (Pa)
		ΔT_{w} :	Temperature difference across facade (K)
Lopen	Opening length (mm)	ρ:	Density of air (kg/m ³)
$\mathbf{K}_{\mathbf{p}}$	Velocity coefficient for plane jet (n.d.)	β:	Thermal expansion coefficient for air $\approx 1/T (K^{-1})$
K_{dp}	Velocity coefficient for plane stratified flow (m ⁻²)	T:	Absolute temperature of air (K)
Ka	Velocity coefficient for 3-D. jet (n.d.)	$u_{\text{max}}, u_{\text{x}}$ u_0 :	Maximum, local, and inlet velocity
\mathbf{K}_{sa}	Coefficient for penetration depth $(s \cdot (m/K)^{0.5})$	\mathbf{x}_0 :	offset of virtual source
g	gravitational acceleration (m/s ²)	x :	distance from real source

INTRODUCTION

Natural ventilation of a building can be defined as fresh air supplied through openings in the building envelope. The driving forces are "natural", insofar that they consist only of pressure differences due to either wind pressure or thermal buoyancy. The forces acting on buildings are extensively described in literature (e.g. Andersen 1996). The openings are usually windows, but other types are also used, namely in the form of vents. Windows are usually placed close to the occupied zone, and the characteristics of the airflow from the window are thus critical to comfort conditions. Nielsen et al. (2000), Svidt et al. (2000), Heiselberg et al. (1999).

The overall aim of this research is to establish a scientific basis on which to develop intelligent strategies for the regulation of window and other openings in the building envelope. This paper describes several series of full-scale experiments performed in an office mock-up (with real windows), based on a real-life test case.

There are several experimental goals:

- To observe flow patterns and to define these as a function of a characteristic number, presumably the Archimedes number, and possibly of other influences, namely the surrounding geometry.
- To obtain characteristic values for the window openings, which allow the flow to be calculated by existing ventilation theory.
- To observe differences between different opening strategies, for instance opening windows in different positions or in different numbers. It is hoped that it will prove possible to use the advantages of different configurations in appropriate combinations.
- To estimate thermal comfort conditions in the occupied zone under different conditions and with different opening strategies.

Pressure difference across window

The driving force behind the flow through a window is the pressure difference across the window opening. This can come about in four different ways: 1) Thermal buoyancy in several stories building leading to "stack effect". 2) Cross ventilation on same floor, wind pressure driven. 3) Thermal buoyancy in single room working via single-sided ventilation 4) Turbulent fluctuations in the wind. See Andersen 1996 for a discussion of the forces involved. In practice, combinations are often used, and natural forces are often assisted with mechanical ventilation, known as hybrid ventilation (see hybvent.civil.auc.dk for references).

In the present experiments, only double-sided ventilation is considered, since this is known to be predominant in the real-life test case (an office facing a large atria with exhaust openings in the roof). This is simulated by a mechanical ventilation system.

Discharge coefficient C_D

The volumetric flow rate through a window \dot{V}_{win} is defined by the pressure drop Δp_{win} across the window (which is the driving force), together with the geometrical opening area A_{geo} . To this must be added the influence of contraction and friction, both of which are a result of the geometry of the window opening together with the properties of the air itself. The contraction coefficient and the friction coefficient are difficult to measure separately, and they are combined to form the discharge coefficient C_D . See Andersen (1996) for a closer, more analytical discussion.

$$CD = \frac{\dot{V}win}{Awin} \sqrt{\frac{\rho}{2 \cdot \Delta pwin}}$$
 (1)

In this paper, discharge coefficients are calculated for three different window opening configurations, each for 5-7 different opening degrees, and for 4 different sets of temperatures, all at varying pressure differences. The values are calculated on the basis of simultaneous measurements of volumetric flow rate through window and pressure difference across facade, together with the measured geometrical areas.

Flow pattern in room

In the course of the present research, it has become clear that in the case of two-sided ventilation with fresh air entering through a bottom-hung window placed just below the ceiling, there are basically 5 possible flow patterns, see figure 1. Depending on the flow pattern, measurements must take place at different locations to determine the most critical velocities with regard to thermal comfort. The flow is depicted as two-dimensional, but is in reality often (if not always) three-dimensional, which must also be taken into account.

Thermal Comfort

Determinative for thermal comfort in the occupied zone are the velocities and temperatures at entry into occupied zone, assuming that overall heat balances and temperature conditions are acceptable. The fresh air will enter the occupied zone in two different ways: along floor, or at 1.8 m height.

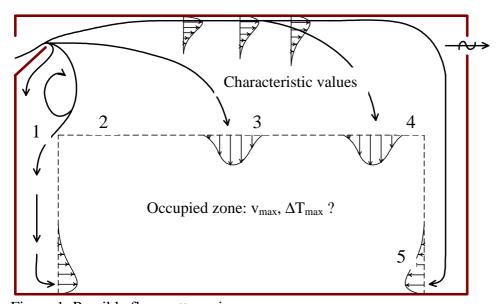


Figure 1: Possible flow patterns in room.

Pattern 1: This only occurs at very low pressure drops (0.1 - 0.2 Pa), where the flow is low turbulent, a so-called "creeping" flow.

Pattern 2: As turbulence is introduced with increasing flow rate, a recirculation bubble is created below the window. This is very similar in nature to the flow described in Heiselberg et al. 1996 (cold boundary layer flow meeting obstacle), where it is shown that the size of the recirculation zone is a function of the Achimedes number. It is conceivable that an obstacle,

for instance the windowsill or a table, will disturb the flow and possibly create local discomfort.

In the cases of patterns 1 and 2, cold, fresh air flows along the wall, hits the floor, and moves horizontally across the room in a stratified, displacement-like flow, see Nielsen (1994), (2000), Heiselberg (1994), Gan (1999)

Pattern 3: The flow pattern from the window establishes itself as a free jet, and flows into the room following a downward trajectory, see Koestel (1955), Etheridge and Sandberg (1999).

Pattern 4: Flow rate still increasing. Free jet attaches to the ceiling due to coanda effect. The penetration depth is of interest, as is maximum velocity at the entering point of occupied zone. Again, this is a well-described phenomenon, see Nielsen et. al. (1987), but the specific characteristics for these windows are not known.

Pattern 5: At a certain point, the jet attaches to the ceiling in its entire length, follows the back wall, and enters the occupied zone along the floor. Also in this case, it is of interest to know the characteristics of the jet.

Jet theory

The mechanisms of isothermal and buoyant jets are relatively well known. In the case of non-isothermal jets, the flow characteristics are determined by a balance between buoyancy and momentum forces, described by the Archimedes number Ar. Trajectory, maximum velocities and temperatures can be calculated, provided the characteristics of the inlet opening are known. The Ar number is described by:

$$Ar = \frac{\beta \cdot g \cdot \sqrt{A_{\text{win}}} \cdot \Delta T}{u_0^2}$$
 (2)

The characteristic behavior of a 3 dimensional wall jet is defined as, Rajaratnam (1976):

$$\frac{\mathbf{u}_{\mathbf{x}}}{\mathbf{u}_{0}} = \mathbf{K}_{\mathbf{a}} \cdot \frac{\sqrt{\mathbf{A}_{\mathbf{win}}}}{\mathbf{x} + \mathbf{x}_{0}} \tag{3}$$

Planar (2-dimensional) wall jet:

$$\frac{\mathbf{u}_{\mathbf{x}}}{\mathbf{u}_{0}} = \mathbf{K}_{\mathbf{p}} \cdot \sqrt{\frac{\mathbf{h}_{0}}{\mathbf{x} + \mathbf{x}_{0}}} \tag{4}$$

The Ar-number describes the forces of buoyancy divided by the forces of inertia, and therefore the penetration length will be described as a function of this parameter.

The penetration length can be described by the following equation, see Grimitlin (1976) and Nielsen and Möller (1987):

$$\frac{x_s + x_0}{\sqrt{A_{win}}} = K_{sa} \cdot K_a \cdot \sqrt{\frac{u_0^2}{\Delta T \cdot \sqrt{A_{win}}}}$$
 (5)

where K_{sa} is a coefficient dependent on room dimensions, locations of thermal loads etc.

During pattern 1 and 2 (see Figure 1), stratified flow occurs. When the air reaches the windowsill it deflects into the room and from here it falls down to the floor, and after a short distance it turns into a stratified flow with an insignificant rate of entrainment.

An indication of a stratified flow is a constant thickness (δ) of the flow parallel to the floor, (Nielsen, 1994). This situation arises because of the density difference between the air in the flow and the surrounding air, which causes the rate of entrainment to be insignificant. In a plane-stratified flow the velocity level will also have a constant level due to this, and therefore the velocity level can be described by (see Nielsen 1994):

$$\frac{\mathbf{u}_{\mathbf{X}}}{\dot{\mathbf{V}}_{\mathbf{W}}} = \mathbf{K}_{\mathbf{Dp}} \tag{6}$$

METHODS

Arrangement of experimental series

Combination of windows

The windows [opening configuration] is arranged in three different ways, namely two separate windows in different positions alone, in the following named W1 and W2, and all four windows open together, named W4. This will determine the first level of sub-division of the experimental series. W1 is defined by being the outmost in row of windows. This places the window in a corner, where one side is blinded. W2 is a window that has other windows on both sides, in this case the window next to W1. This could simulate an arbitrary window in a long row. W4 in this case covers the full width of the room.

Temperature differences

Both isothermal and non-isothermal experiments are performed. Temperature differences between the inlet flow from the window and the ambient temperature influences the flow pattern, and are also important for draft sensations in the occupied zone. Temperature differences of 0 K (isothermal) and 10 K are used.

Opening Length and minimum opening area

The opening of the window is defined by the "opening length" L_{open} , which is the distance from the inner side of the (movable) window frame, where this meets with the outer window construction. In each window configuration (W1, W2 & W4), this corresponds to a minimum opening area, measured as the minimum cross-sectional area the air must pass trough..

The following opening lengths and corresponding geometrical areas are used:

Table 1: Opening lengths in experiments

Tuble 1. Opening lengths in experiments							
$L_{open} (mm)$	23	43	48	98	148	198	248
$A_{win} (m^2)$							
W1			0.0292	0.0647	0.0922	0.129	0.173
W2			0.0285	0.0771	0.119	0.172	0.235
W4	0.0712	0.0867	0.0892	0.179	0.232	0.316	0.420

Pressure differences

Often, natural ventilation systems have to operate at very low pressure differences, though srong wind can at times create pressure differences of 50 Pa or even more.

Experiments were performed, for each of the above combinations, at different pressure drops across the facade. The pressure difference across the window opening will determine the flow rate through the window. Pressure differences as low as 0.1 Pa were used, and as high as 60 Pa (with a small opening). It was found, however, that unacceptable noise was generated from the windows at pressure drops larger than approx. 20 Pa, which corresponds to a high wind velocity, for which reason this was set as the upper limit for the experiments.

Laboratory setup.

The experiments were conducted in a test room placed in a laboratory. The test room was partitioned with plywood walls, see figure 2, to create a mock-up of an office with dimensions W = 4.9 m by L = 4.9 m by H = 3.8 m. and an insulated "cold room". Heating was supplied to the office room at the floor, through evenly spaced heating cables. The ventilation inlet was placed in the cold section. Care was taken that the inlet did not have a direct influence on the airflow through the windows. An exhaust was placed in the back of the office room. This arrangement makes it possible to control the pressure difference across the window facade. Each of the bottom-hung windows has a width of 1 m and a height of 0,45 m.

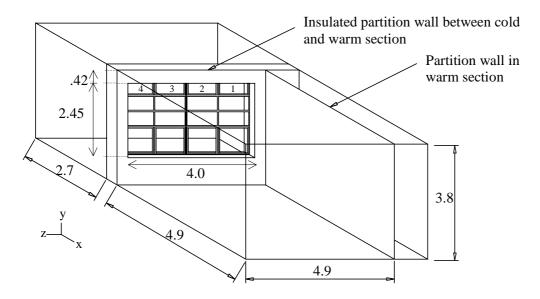


Figure 2: The structure of the full-scale model of the room with adjacent rooms.

Measurements

Velocity profiles were measured below the ceiling with hot sphere anemometers in order to characterize the inlet flow through the window openings. In cases W1 and W2, velocity profiles were also measured at an angle from the windows in order to characterize the inlet jets from the triangular side openings.

Smoke-tests of non-isothermal flow were conducted to approximate the penetration length of the inlet jet. The penetration length was defined as the length from the window to the place where the jet leaves the ceiling and flows down into the occupied zone, see also Svidt et.al (2000).

Measurements of the velocity in the downward airflow under non-isothermal conditions were carried out with hot sphere anemometers at 1.8 m above the floor, which was defined as the occupied zone. For these measurements, 21 anemometers were placed on a rack with a mutual

distance of 0.5 m. The rack was moved about between measurements to obtain more measuring points, in order to "catch" the maximum velocities.

Non-isothermal measurements of velocity profiles close to the floor were carried out at low volume flows and high temperature differences in order to analyze whether a stratified flow was created. These experiments were conducted with all 4 bottom-hung windows open and at 3 different opening areas, $A_{win} = 0.089$, 0.179 and 0.420 m².

RESULTS

Initially, C_D values were found for all opening lengths, window configuration, and for two temperature differences, 0°C (isothermal) and 10°C.

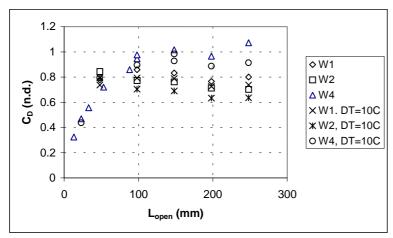


Figure 3: C_D values

Isothermal flow

Flow Patterns

Flow pattern 5 was observed in all isothermal experiments. For the 3 smallest opening areas, the jets were initially horizontal, and attached to the ceiling due to coanda effect. At the 2 larger openings the air was directed towards the ceiling. These effects were visualized by smoke tests, see Figure 4 (left and middle). The measurements also included velocity profiles measured at an angle to the windows because of the triangular side openings, Figure 4 (right).







Figure 4: Smoke visualization of the inlet jet, shown for opening areas of $A_{win} = 0.089 \text{ m}^2$ (left) and 0.420 m² (middle) for window 2, and smoke visualization of the jet from the triangular side openings, shown for an opening area of $A_{win} = 0.420 \text{ m}^2$ (right).

Jet characteristics, W1 and W2

The distance to the virtual origin (x_0) and the K_a -value were found from the reciprocal velocity decay as shown in Figure 5. It was not possible to calculate the distance to the virtual origin from the width of the jet (δ) since it was not unequivocally increasing but more assuming a constant level. The reference velocity u_0 is defined as the ratio between the volume flow and the smallest area that the air passes through.

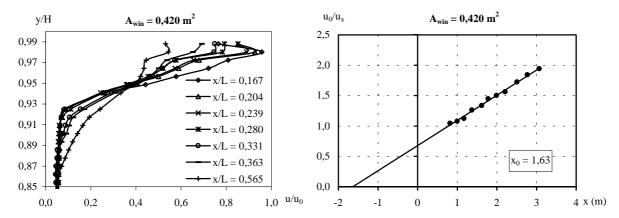


Figure 5: Velocity profiles from measurements of airflow from window 1 with an opening area of 0.420 m^2 (left), and the reciprocal velocity decay for the determination of the distance to the virtual origin (x_0) (right).

All measurements show the characteristic behavior of a 3 dimensional wall jet, see equation 3. The values of the distance to the virtual origin (x_0) and the K_a value are shown in the following Table 2.

Table 2: Distance to the virtual origin and K_a value for the 2 windows versus the opening area. Values account for both the measurements in front of the window and the measurements in the approximate centerline of the triangular side opening.

Measurements in front of window				Measurements at an angle from window							
Window 1			Window 2			Window 1			Window 2		
A_{win} $[m^2]$	X ₀ [m]	K _a [-]	A_{win} $[m^2]$	x ₀ [m]	K _a [-]	A_{win} $[m^2]$	x ₀ [m]	K _a [-]	A_{win} $[m^2]$	x ₀ [m]	K _a [-]
0,029	-0,18	2,2	0,029	1,680	4	0,029	0,504	1,0	0,029	-0,080	0,6
0,065	1,03	3,3	0,077	2,840	4,8	0,065	0,446	1,7	0,077	0,479	1,6
0,092	1,56	4,7	0,119	1,280	3,6	0,092	0,660	2,7	0,119	0,316	2,0
0,129	1,99	6,1	0,172	0,740	3,5	0,129	1,790	4,0	0,172	0,693	2,7
0,173	1,63	5,8	0,235	0,790	3,7	0,173	4,123	5,5	0,235	1,991	3,3

Velocity profile measurements were carried out with opening areas of 0.089, 0.179 and 0.232 m². The measurements with all four windows opened show a behavior that could be characterized as a 2 dimensional wall jet, see equation 4:

Jet characteristics, W4

In Table 3 the area of the opening and the matching height is listed, as well as the found distance to the virtual origin and the K_p values from the measurements.

Table 3: The height of the o	pening, the distance to the virtu	al origin (x_0) and K_n value.
Table 5. The height of the o	pennig, the distance to the virte	an origin (Au) and Re varae.

$h_0[m]$	x_0 [m]	K _a [m]
0,016	0,543	2,6
0,051	3,692	4,3
0,097	9,524	6,0

Non-Isothermal flow

Penetration length from non-isothermal flow, W1 and W2

The smoke tests showed a highly unsteady flow with variations in the point of separation of up to 1 m during the 2 minutes observation period. The point of separation or the penetration length occurs when the forces of buoyancy become more influential than the forces of inertia.

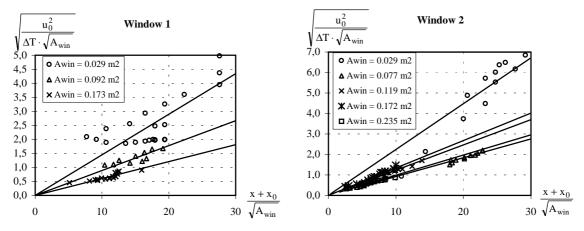


Figure 7: $\sqrt{u_0^2/\Delta T \sqrt{A_{win}}}$ versus the relative penetration depth. The depiction show values found for window 1 to the left and values for window 2 to the right.

It is seen from figure 7 that the penetration length is linear proportional with the square root of the reciprocal reduced Ar number for both windows. For each of the measurement series a linear regression line has been added, and Figure 7 shows a tendency that by reducing the opening size the penetration length is also reduced compared to the square root of the reciprocal reduced Ar-number.

The coefficient K_{sa} (see equation 5) has been found not to be constant. An increasing tendency is found if the total coefficient of equation 5 ($K_a \cdot K_{sa}$) is depicted versus the opening area, see Figure 8.

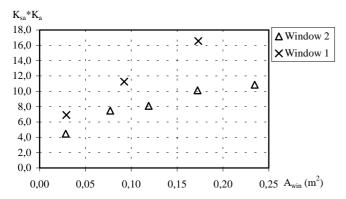


Figure 8: The total constant of equation 4 versus the area of the opening

There is a linear connection between the total constant of equation 4 and the opening area for both windows.

Flow patterns, W4

The flow was classified visually in each case as belonging to one of the flow types 1-5 (see figure 1). In some cases, intermediate flow types were observed; these are denominated by using the average, e.g. type 1.5. Figure 6 shows non-isothermal experiments at $\Delta T \approx 10^{\circ}$ C. It would appear that the Archimedes number is an appropriate dimensionless number to describe the flow type. A considerable variation is however seen, which indicates that other parameters has influence, namely the local geometry at the opening.

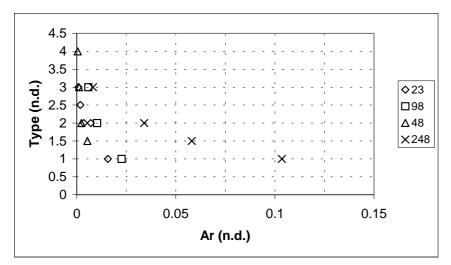


Figure 6: Experiments performed at $\Delta T \approx 10^{\circ}$ C organised according to Archimedes number Ar and flow type. Legends (right) show opening length L_{open}

Maximum velocity in the occupied zone from non-isothermal flow

To describe the maximum velocity as a function of both the volume flow as well as the temperature difference we define a parameter (Ar') which has the same structure as the non-dimensional Archimedes number with respect to temperature difference and volume flow:

$$Ar' = \left(\frac{\Delta T}{\dot{V}_w^2}\right) \cdot 10^{-3} \qquad \left(\frac{K \cdot s^2}{m^6}\right) \tag{7}$$

The following Figure 9 shows all of the measurements of the relative maximum velocity in the occupied zone as a function of the parameter Ar'.

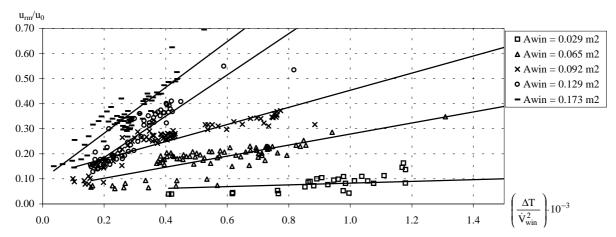


Figure 9: W1: Measurements of the maximum velocity in the occupied zone for 5 opening areas versus Ar'. Matching linear regression lines are shown for all 5 opening areas.

Figure 9 shows a linear proportionality between the relative maximum velocity in the occupied zone (u_{rm}/u_0) and the Ar'-number. It also shows that the slope of the linear regression lines increases with the opening area.

Plane stratified flow from 4 windows

On Figure 10 the width of the jet and the relative maximum velocity is depicted versus the distance from the window (x) at three different Ar'-numbers (see 5) with an opening area of $A_{\text{win}} = 0.089 \text{ m}^2$.

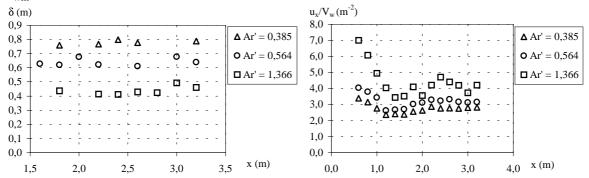


Figure 10: The width of the jet (δ) and the relative velocity level versus the distance x. Shown for 3 different Ar'-numbers with an opening area of $A_{win} = 0.089 \text{ m}^2$.

Figure 10 shows that the width of the jet has an approximately constant thickness (see also Wilkinson and Wood, 1971) for the 3 Ar'-numbers, it also shows that when the Ar'-number increases then the width becomes smaller. Figure 10 also shows a decreasing velocity level for x < 1,5 m, which is caused by the air being deflected into the room by the windowsill. For x > 1,5 m an approximately constant velocity level in the jet is found for each of the 3 Ar'-numbers. Figure 11 shows that K_{dp} increases with increasing Ar'-number for each of the 3 opening areas, which is due to an increasing vertical acceleration of the air down the window section because of an increase in the density difference. It also shows that K_{dp} seems to be independent of the opening area.

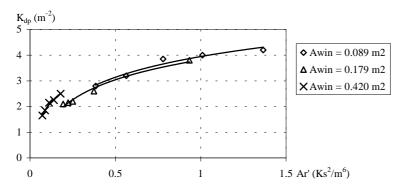


Figure 11: Kdp-values versus the Ar'-number for opening areas of $A_{win} = 0.089$, 0.179 and 0.420 m².

CONCLUSIONS

The results show that

- Different window opening strategies result in quite different airflow and thermal comfort conditions.
- The conditions are a result of a multivariable impact, and detailed descriptions of the flows involved are complex.
- The geometrical shape of the window opening is important for the flow characteristics
- The flow in the room will assume one of five main patterns. These can be described by traditional flow element theory for jets and for stratified flow by measuring flow characteristics.
- The Archimedes number describes the flow for a certain geometry.

More empirical data is required, especially with regard to:

- Different temperature conditions.
- The relationship between "main" flow and "side" flow in the case of single windows W1 and W2.
- Transient conditions, which must be expected to prevail most of the time in real life. Hysteresis effect was observed, but was not elaborated in these experiments.

Further research at this university will include studies on the dynamical behaviour of natural ventilation, and more detailed analysis of the consequences of the present findings on thermal comfort under different conditions.

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