Numerical Analysis of Natural Convection in Insulating Porous Medium

V. Shankar, C.E. Hagentoft
Department of Building Physics
Chalmers University of Technology
S-412 96 Gothenburg, Sweden

1 Abstract

The Paper deals with numerical computations, carried out, in order to predict the effects of natural convection on the thermal performance of porous material. In this paper the effect of natural convection in a horizontal porous layer will be discussed. The study of the above configuration is essential to understand the functioning of insulation of the type used in attics. The influence of natural convection on the thermal properties of mineral wool, loose-fill insulation, insulation made of small and large polystyrene balls are studied. The results are presented in terms of dimensionless numbers and the temperature distribution across the insulation.

2 Introduction

Insulating materials are porous in nature and contains a large quantity of air in it. In case of mineral wool, the heat transfer and thereby the thermal conductivity can increase due the air movement in materials with high permeability. Even though a structure is fully "wind protected " , the process of heat transfer is bound to occur due to the difference in temperature. Natural convection in an insulating material is due the fact that density of the warm air is lesser than that of the cold air. The hot air movement from the insulations 'hot side' to its 'cold side', increases the thermal conductivity of the material. Apart from the proper design of the ventilation system, synchronization of the same with building design is important to decrease the energy consumption and thus, reducing the energy costs. The asthetic, economic and functional energy requirements must be accurately estimated. In order to achieve the above mentioned goal, the heat transfer processes, i.e., conduction, convection and radiation within the building and building and the surroundings should be accurately predicted.

2.1 The Code

The standard three-dimensional finite volume computer program CALC-BFC [4] (**B**oundary **F**itted Coordinates) for three-dimensional complex geometries has been used. This program uses Cartesian velocity components, and the pressure-velocity coupling is handled with the SIMPLEC procedure.

3 Governing Equations of Motion

3.1 Horizontal Insulating Medium

A schematic diagram of two-dimensional horizontal porous medium along with the boundary conditions is shown in the Fig. 1. In the case of horizontal porous layer the vertical walls are assumed to be impermeable. In porous medium, Darcy's law is assumed to hold and the fluid is assumed to be of normal Boussinesq fluid, i.e., the variations of density is given a great importance. Most analytical studies are based on Darcy's law. However, Darcy's law cannot account for the no-slip boundary condition either at the interface between porous and solid media, nor the continuity of velocity at the interface of porous and air cavity. Brinkman extension is therefore used in order in order to account for the boundary effect. The inertia and viscous drag are neglected. The same is valid for low Darcy and particle Reynolds number. Since the inertial term is neglected, velocity slip at the boundary is necessary. The governing equations of motion i.e., the conservation equation

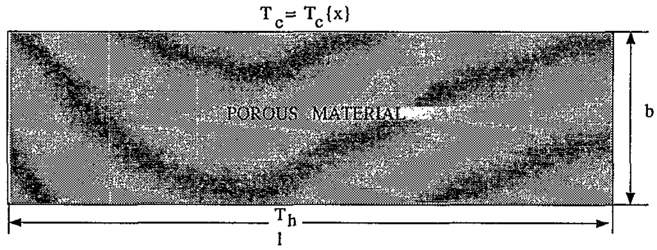

Figure 1: *horizontal porous material.*

for mass, momentum and the energy for steady, two-dimensional flow in an homogeneous, porous medium are given by a. Continuity:

$$\frac{\partial}{\partial x}(U_D) + \frac{\partial}{\partial y}(V_D) = 0 \tag{1}$$

b. Momentum:

$$\frac{\partial}{\partial x}(P) + \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial}{\partial x}U_D\right) + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial}{\partial x}U_D\right) - \frac{\mu_{eff}}{K}U_D = 0 \tag{2}$$

$$\frac{\partial}{\partial y}(P) + \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial}{\partial x}V_D\right) + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial}{\partial y}V_D\right) - \frac{\mu_{eff}}{K}V_D + \rho g\beta(T - T_c) = 0 \tag{3}$$

c. Energy:

$$U_D\frac{\partial}{\partial x}(T) + V_D\frac{\partial}{\partial y}(T) = \alpha\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}T\right) + \alpha\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}T\right) + \frac{S}{\rho c_p} \tag{4}$$

where S is the rate of volumetric heat generation. The above equations are non-dimensionlized.

3.2 Boundary Conditions

To solve the above set of equations, following are the thermal boundary conditions often used in heat transfer problems: a. Dirichlet condition (imposed temperature)

$$T = T_h \tag{5}$$

for 'warm' surface. b. Nuemann condition (imposed heat flux)

$$T = T(x) \tag{6}$$

for 'cold' surface.

3.3 Heat Transfer Results

The overall heat transfer rate across the enclosure is expressed in terms of Nusselt's number which is defined as

$$Nu = \frac{hl}{k_f} \tag{7}$$

In a steady state, for a global system, the Nusselt's number is a characteristic of the heat exchange. Nusselt's number is a function of Rayleigh's number and for horizontal porous layer and is evaluated at any horizontal plane and is calculated as follows [7].

$$Nu = \frac{\frac{1}{l}\int_0^l (k\frac{\partial T}{\partial Y} - \rho_f c_f VT)\partial X}{-\Delta k/K} \tag{8}$$

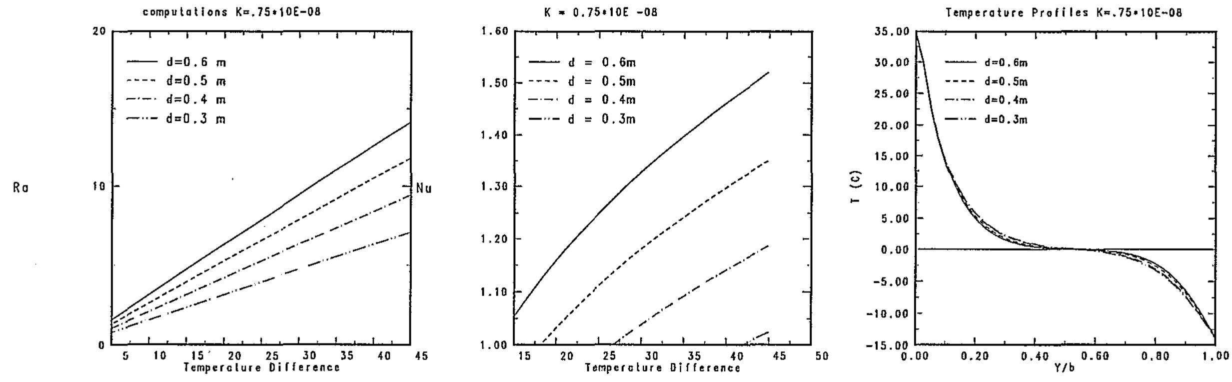

Figure 2: *The influence of temperature difference on Rayleigh's number (2.a) , Nusselt's number (2.b) and temperature profiles (2.c)in Loose - Fill insulation (permeability,* $K = 0.75 * 10E - 08$*).*

4 Influence of Thickness of Insulation and Temperature Difference

Fig. 2a, shows the influence of thickness of insulation and temperature difference on Rayleigh's number in horizontal insulation made of mineral wool. The magnitude of Rayleigh's number increases with the increase in the temperature difference, which in turn increases the 'risk' of natural convection. We can also infer from Fig. 2a, that when the thickness of the insulation is decreased the Rayliegh's number also decreases for a given temperature difference. Also the effect of natural convection (Nusselt's number is a measure of natural convection) decreases by 10 to 12 percent as shown in Fig. 2b, when the thickness of the insulation is decreased by 10 cm, at a temperature difference of 45^oC. Furthuremore, we can conclude from Fig. 2b, that the effect of natural convection on the thermal properties increases with the increase in temperature difference over the insulation. The temperature distribution across the horizontal porous medium made of mineral wool is shown in Fig. 2c. The influence of temperature difference and the thickness of insulation on Rayleigh's number in horizontal loose-fill insulation is shown in Fig. 3a. Once again the value of Rayleigh's number increases with the increase in the temperature difference and the thickness of the insulation. Comparing Fig. 2a and Fig. 3a, we can note that when the permeability of the insulation is increased from $0.75 * 10e - 08$ (mineral wool) to $1.0 * 10e - 08$ (losse-fill), the 'risk' for increase in air movement is higher. From the same, we can infer that the value of Rayleigh's number increases by 10 to 15 percent when the permeability of the insulation is increased from $0.75 * 10e - 08$ (mineral wool) to $1.0 * 10e - 08$ (losse-fill), at a temperature difference of 45^oC. Also when Comparing Fig. 2b and Fig. 3b, the value of Nusselt's number (natural convection) increases, by 10 to 15 percent when the permeability of the insulation is increased from $0.75*10e-08$ (mineral wool) to $1.0*10e-08$ (losse-fill) at a temperature difference of 45^oC. A similar behaviour is observed (see Figs. 4a, 4b, 5a and 5b) for insulation made of small and large polystyrene balls. The temperature distribution across Loose - Fill insulation and insulation made of small and large polystyrene balls are shown in Figs. 3c, 4c and 4d. In short we can conclude that the 'risk' for air movement and in turn the effect of natural convection on the thermal properties of the insulation increases with the increase in permeability, temperature difference between the 'hot' and 'cold' surface of the insulation and the thickness of the insulation.

5 Results and Discussion

The work presented in this report deals with the influence of covection on the thermal properties in case of insulating porous medium. Numerical investigations are carried out for mineral wool, loose - fill insulation, insulation made of small polystyrene balls and insulation made of large polystyrene balls. In general air permeability and the thickness of the insulation are the most significant parameters for the influence of natural convection on the global heat transfer rate in porous medium (insulation). The main conclusions of the results obtained are summarized below.

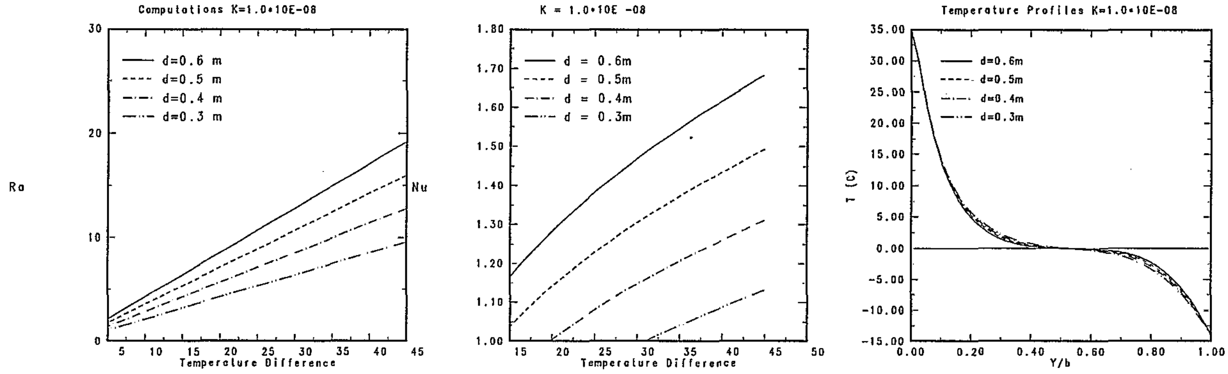

Figure 3: *The influence of temperature difference on Rayleigh's number (3.a) , Nusselt's number (3.b) and temperature profiles (3.c)in Loose - Fill insulation (permeability,* $K = 1.0 * 10E - 08$*).*

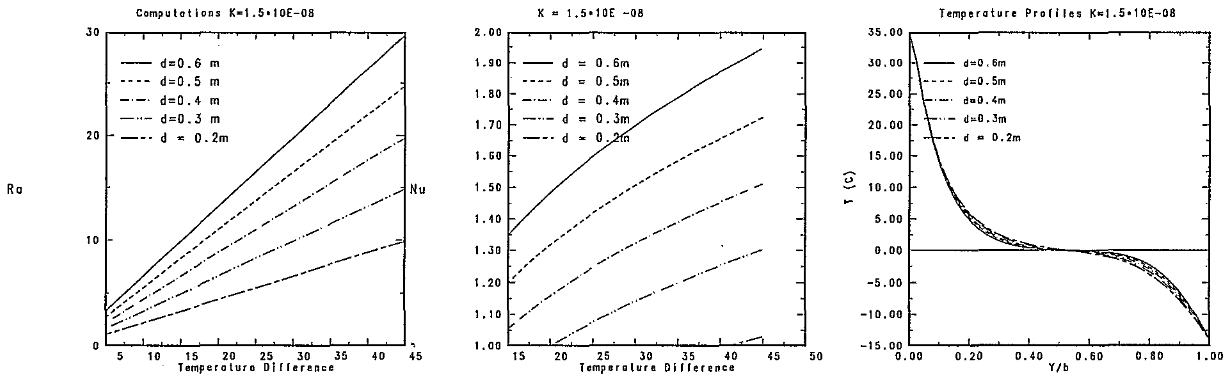

Figure 4: *The influence of temperature difference on Rayleigh's number (4.a) , Nusselt's number (4.b) and temperature profiles (4.c)in Loose - Fill insulation (permeability,* $K = 1.5 * 10E - 08$*).*

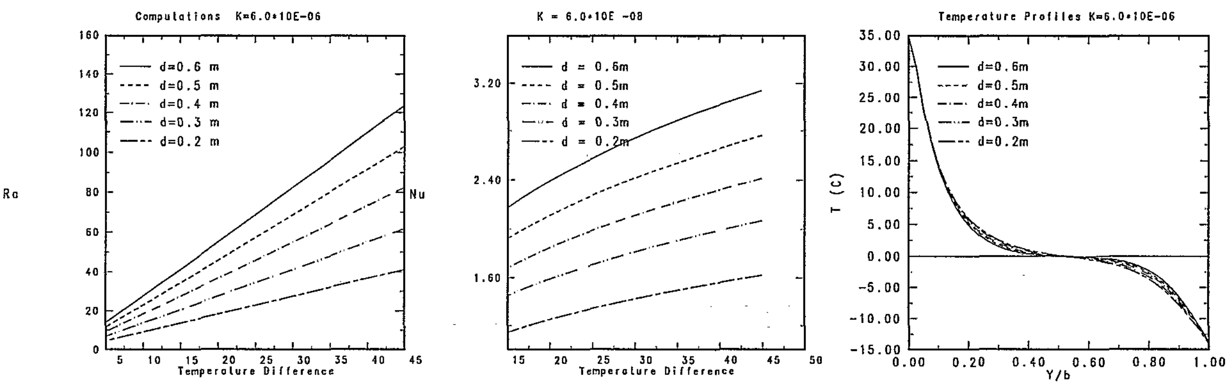

Figure 5: *The influence of temperature difference on Rayleigh's number (5.a) , Nusselt's number (5.b) and temperature profiles (5.c)in Loose - Fill insulation (permeability,* $K = 6.0 * 10E - 08$*).*

1. An increase in the temperature difference and thickness of insulation leads to an increase in the influence of natural convection on thermal properties of the insulation as shown in Figs. 2a, 3a, 4a and 5a.

2. When the thickness of insulation is decreased from 0.6m to 0.5m the influence of natural convection decreases by approximately 10 percent as shown in Figs. 2b, 3b, 4b and 5b for a temperature difference of 45^oC.

3. An increase in permeability of the insulation leads to an increase of air movement in the insulation and therefore an increase in the influence of natural convection on the thermal properties of the insulation.

4. Furthure more the temperature profiles shown in Figs. 2c, 3c, 4c and 5c, indicates the presence of boundary layer. These boundary layer diffuse yielding the pure conduction temperature distributions.

5. The insulation thickness and temperature difference play an important role on the magnitude of Rayleigh's number, and inturn on the global heat transfer rate.

References

[1] S.V. PATANKAR, Numerical heat transfer and fluid flow, McGraw-Hill, Washington, 1980.

[2] C.M. RHIE and W.L.CHOW, Numerical study of the turbulent flow past an airfoil with trailing edge separation, *AIAA J.*, **21**, pp. 1527-1532, 1984.

[3] V. SHANKAR, L. DAVIDSON and E. OLSSON, Ventilation by displacement: Calculations of the flow in vertical plumes, Vol.1, pp.59-74, ROOMVENT'92, Aalborg, Denmark, 1992.

[4] L. DAVIDSON and B. FARHANIEH, CALC-BFC: A finite-volume code employing colocated variable arrangement and Cartesian velocity components for computation of fluid flow and heat transfer in complex three-dimensional geometries, Rept., Dept. of Applied Thermodynamics and Fluid Mechanics, Chalmers University of Technology, Gothenburg, Sweden, 1991.

[5] V. SHANKAR: Numerical investigation of turbulent plumes in both ambient and stratified surroundings, Tek. lic., Rept.,Dept. of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg, Sweden, 1993.

[6] N.Z. INCE and B.E. LAUNDER, Computation of turbulent natural convection in closed rectangular cavities, Proc. 2nd U.K. Natn. Conf. on Heat Transfer, University of Strathclyde, Glasgow, Vol. 2, pp. 1389-1400, 1988.

[7] A.A. DELMAS and E.K. WILKES, Numerical analysis of heat transfer by conduction and natural convection in loose-fill fiber-glass insulation -Effects of convection on thermal performance, Rept., Oak Ridge National Laboratory, 1991.

[8] C.G. BANKVALL, Mechanisms of heat transfer in permeable insulation and their investigation in special guarded plate, ASTM STD, 554, American Society for Testing and Material, Philedelphia, 1975.

[9] C.G. BANKVALL, Natural convective heat transfer in insulated structures, Ph.D thesis, Division of Building Technology, Lund, Sweden, 1972.

[10] C.G. BANKVALL, Heat transfer in fibrous materials, Journal of testing and evaluation, Vol. 1, No. 3, 1973

[11] M. SERKITJIS, Egen och patvingad konvection i lösull, Rept., Division of Building Technology, Chalmers University of Technology, Gothenburg, Sweden, 1992.

[12] C. BECKERMANN, S. RAMADHYANI, and R. VISKANTA, Natural convection flow and heat transfer between a fluid and a porous layer inside a rectangular enclosure, ASME J. Heat Transfer, 109, 363-370, 1987.

[13] J.G. GEORGIADIS and I. COTTON, Prandtl number effect on Bernard convection in porous media, ASME J. of Heat Transfer, 108, 284-289, 1986

[14] M.R.M - DEHGHAN, A.J.W. SMITH and R.N. WALKER, A study of laminar air flow through cracks by numerical simulation of differential equations, International Journal of Numerical Methods in Fluids, 14, pp. 1485-1496, 1992.

[15] T.W. TONG and E. SUBRAMANIAM, Natural convection in rectangular enclosures partially filled with a porous medium, ASME - JSME, Thermal engineering joint conference, ASME, New York, Vol. 1, pp. 331-338, 1983

[16] C.L. TIEN and J.T. HONG 1985, Natural convection in porous media under non-uniform permeability conditions in: Natural Convection, S. KAKAC et.al., eds, Hemisphere, Washington DC, 1984

[17] V. PRASAD and F.A. KULACKI, Natural convection in a rectangular porous cavity with constant heat flux on one vertical wall, ASME Journal of heat transfer, vol.106, 1984, pp.152-157.

[18] M. HAAJIZADEH, A. OZGUC and C.L. TIEN, Natural convection in a vertical porous enclosure with internal heat generation, International Journal of heat and mass transfer, vol.27, 1984, pp 152-157.