VENTILATION TECHNOLOGIES IN URBAN AREAS

19TH ANNUAL AIVC CONFERENCE
OSLO, NORWAY, 28-30 SEPTEMBER 1998

A SEMI EMPIRICAL FLOW MODEL FOR LOW-VELOCITY AIR SUPPLY IN DISPLACEMENT VENTILATION

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A Semi Empirical Flow Model for Low-Velocity Air Supply in Displacement Ventilation

ABSTRACT

Similar to supply air jets in mixing ventilation this paper describes a comprehensive flow model for displacement ventilation derived from the integrated Navier-Stokes differential equations for boundary layers. A new test method for low velocity diffusers in displacement ventilation is developed based on this new flow model. Contrary to jet flow, it is shown that the only independent variable in the new model is the buoyancy flux. In addition to this variable the calculations need a single empirical constant, which is determined from a limited number of full-scale tests of a limited number of similar shaped diffusers of different size. There are made a number of tests to try out the new model.

The results are promising. For plane (two-dimensional) flow the velocity accelerates to a constant value. For radial flow there is also an acceleration zone, after which the velocity decays. Both theoretical and empirical data predicts that for similar shaped diffusers the width of the near zone (distance from the centre of the diffuser to a chosen velocity depends only on the buoyancy flux, not the dimensions of the diffuser (radius and height). One consequence of this is, among others, that the width of near zone cannot, for a certain air flow rate, be shortened by choosing a larger radius and a lower height of the diffuser. The diffuser constant K for radial diffusers has, however, turned out to be more or less dependent on the difference in temperature between the supply air and the room air, probably due to that the outflow is not ideally radial, and the effect of the temperature difference is to make the flow become more radial. The new model also enables the designer to calculate the near zone for arbitrary airflow rate, supply air temperature and arbitrary supply diffuser size of similar shaped diffusers. Practical benefits are, among other things, improved test standards and design methods for displacement ventilation.

LIST OF SYMBOLS

g - Acceleration due to gravity
h - Coordinate of boundary layer border
H - Height of diffuser
I - Profile integral
K - Performance constant
L - The horizontal perimeter of a diffuser
P - Pressure
q_V - Flow rate
x - x-co-ordinate
y - y-co-ordinate
T - Absolute temperature
U - Velocity in x-direction
V - Velocity in y-direction
ρ - Density of air
β - Thermal expansion coefficient
ϕ - Angle of spread

Subscripts

0 - location of outlet
1 - location of maximum velocity, classification of profile integral
2, 3 - classification of profile integral
m - maximum
p - plane
r - radial, in the room
f - in the flow
s - supply
U - a fixed arbitrary velocity

1 INTRODUCTION

There are lacking aero- and thermodynamic models for the near zone of the air supply diffusers in displacement ventilation. Because of lacking flow models it is expensive to give valid near zone data for all combinations of supply airflows and supply air temperatures. Manufacturers of the diffusers supply test data, which may be difficult to extrapolate to the actual situation. The aim of this paper is to supply new knowledge into this field and to present a more scientific based, and less expensive, test method and design guide for low velocity air diffusers in displacement ventilation.

2 Theory

2.1 Basics

The type of flow is a kind of boundary layer flow, fig.1, but it does not exhibit the features of self-preservation. Because it is a boundary layer flow the boundary layer momentum equations, neglecting surface friction, apply:

$$\left.\begin{aligned} & U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{x^r}\frac{1}{\rho}\frac{d}{dx}\left(x^r P\right)\ \text{(Mom.eqn.)} \\ & \frac{1}{x^r}\frac{\partial\left(Ux^r\right)}{\partial x}+\frac{\partial V}{\partial y}=0\ \text{(Cont.eqn.)} \end{aligned}\right\} \qquad (1)$$

Plane flow: r=0

Radial flow: r=1

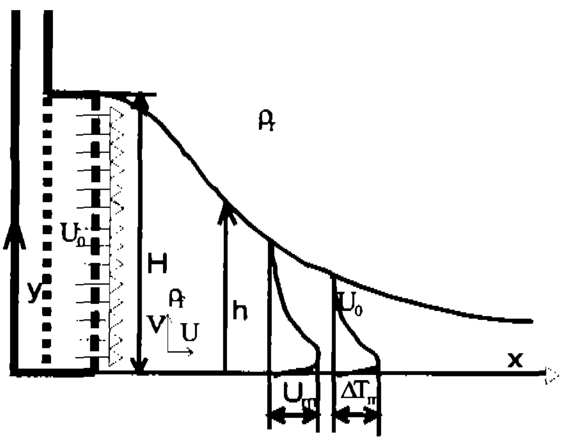

Fig.1 Flow pattern behind a low-velocity diffuser. Supply air temperature lower than room air temperature.

There is no external pressure gradient in the flow region so that the pressure gradient is totally governed by the internal (thermal buoyancy) forces. When the outflow has a lower temperature than the surrounding air, the equation for the static pressure difference between the location (x, y) and a point at the same level in the room outside the boundary region becomes:

$$\begin{aligned} \Delta P_{xy} &= g\int_{y}^{h(x)}\left(\rho_f(y)-\rho_r\right)dh(x) \\ &= g\rho_r h(x)\int_{y/h(x)}^{1}\frac{\left(\rho_f(y)-\rho_r\right)}{\rho_r}d\frac{y}{h(x)} \end{aligned} \qquad (2)$$

Further relations are:

$\rho_f(y)-\rho_r=\rho_r\beta\Delta T_f(y)$

$\Delta T_f = T_r - T_f$ *=Temperature difference between the boundary flow and the room air*

$\beta=\frac{1}{T_r}$ *=Volumetric thermal expansion coeff.*

In the following we substitute h(x) with the parameter h. The final equation for the pressure difference in the boundary layer at a location (x, y) then becomes:

$$\begin{aligned} \Delta P_{xy} &= g\rho_r h\int_{y/h}^{1}\left(\beta\Delta T_f(y)\right)d\frac{y}{h} \\ &= g\rho_r\beta\Delta T_m h\int_{y/h}^{1}\left(\frac{\Delta T_f(y)}{\Delta T_m}\right)d\frac{y}{h} \end{aligned} \qquad (3)$$

For the bottom streamline the pressure equation becomes:

$$\begin{aligned} \Delta P_{xy_{(y=0)}} &= g\rho_r\beta\Delta T_m h\int_{0}^{1}\left(\frac{\Delta T_f(y)}{\Delta T_m}\right)d\frac{y}{h} \\ &= g\rho_r\beta\Delta T_m h I_1(x) \end{aligned} \qquad (4)$$

$I_1(x)$ is the integral if the dimensionless temperature profile, in principle a function of x.

Just for information the energy equation can be used to express ΔT_m in terms of q_v. Differentiating equation 4 with respect to x gives the pressure gradient along the bottom streamline:

$$\left.\frac{d\Delta P_{xy}}{dx}\right|_{(y=0)} = g\rho_r\beta\frac{d}{dx}\left(\Delta T_m h I_1(x)\right) \qquad (5)$$

In most situations q_v will not vary much, but it is necessary to incorporate the initial induction when calculating ΔT_m. The energy equation tells us however that $(q_v \Delta T_m)$ is constant and equal to $(q_{v0}\ \Delta T_0)$ implicitly suggesting that the buoyancy flux, $g\beta\Delta T_m q_v$, does not vary with x.

Before solving the differential equation for the bottom stream layer it is convenient to change the continuity equation in equation 1 somewhat. We assume insignificant induction in the acceleration section and, integrating the continuity equation across the boundary layer, we can express the velocity at the bottom plane through the total flow rate, q_v, of the boundary layer. We get:

$$\frac{1}{x^r}\int_0^h \frac{\partial(Ux^r)}{\partial x}dh + \int_0^h \frac{\partial V}{\partial y}dh = \frac{1}{x^r}\frac{\partial}{\partial x}\left(U_m h x^r\right)\int_0^1 \frac{U}{U_m}d\frac{y}{h} = 0$$

For a constant flow rate, qv, V=0 both at the bottom plane and at the outer boundary. Then:

$$\frac{\partial}{\partial x}\left(U_m h x^r\right)\int_0^1 \frac{U}{U_m}d\frac{y}{h} = \frac{\partial}{\partial x}\left(I_2(x)U_m h x^r\right) = 0$$

$$\left(I_2(x)U_m h x^r\right) = const. = \frac{q_v}{\phi^r} \qquad (6)$$

$I_2(x)$ is the integral of the dimensionless velocity profile and is in principle a function of x

U_m is then given by:

$$U_m = \frac{q_v}{I_2(x)\phi^r h x^r} \qquad (7)$$

The momentum equation in equation 1 can now be transformed to be valid for a stream layer at the bottom plane:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{x^r}\frac{1}{\rho}\frac{d}{dx}\left(x^r \Delta P_{x0}\right)$$

$$(8)$$

$$x^r U\frac{\partial U}{\partial x} = -\frac{d}{dx}\left(x^r g\beta I_1(x)\Delta T_m h\right)$$

The justification for the expressions above is that $V\frac{\partial U}{\partial y} \approx 0$ at the bottom plane.

U is substituted by U_m in equation 7.

$$x^r \frac{q_v}{I_2(x)\phi^r h x^r}\frac{\partial}{\partial x}\left(\frac{q_v}{I_2(x)\phi^r h x^r}\right) = -\frac{d}{dx}\left(x^r g\beta I_1(x)\Delta T_m h\right)$$

Which further expanded yields:

$$\frac{1}{2}\frac{q_v^2 x^r}{\left(\phi^r\right)^2\left(I_2(x)hx^r\right)^3}\frac{d}{dx}\left(I_2(x)hx^r\right) = g\beta\frac{d}{dx}\left(I_1(x)\Delta T_m h x^r\right)$$

(9)

ΔT_m is a function of x because the profiles change with x. However through the energy equation we can link it to the inflow conditions:

$$\Delta T_m = \frac{q_{v0}}{q_v}\frac{I_2(x)}{I_3(x)}\Delta T_0$$

$I_3(x)$ is the integral of the product of the dimensionless velocity- and temperature profile and is in principle a function of x.

Substituting this expression in equation 9 we get:

$$\frac{1}{2}\frac{q_v^2 x^r}{\left(\phi^r\right)^2\left(I_2(x)hx^r\right)^3}\frac{d}{dx}\left(I_2(x)hx^r\right)$$
$$= g\beta\frac{d}{dx}\left(\frac{q_{v0}}{q_v}\frac{I_1(x)}{I_3(x)}I_2(x)\Delta T_0 h x^r\right)$$

Here q_v is the flow rate after the primary induction when for instance a perforated plate is used. Later the induction is small so we can put $\frac{q_{v0}}{q_v}$ constant. Another legal approximation is that $\frac{I_1(x)}{I_3(x)}$ may be considered constant.

Altogether an acceptable simplification will be that: $\frac{q_{v0}}{q_v}\frac{I_1(x)}{I_3(x)}$=*constant.*

Finally we can rewrite equation 9 in the following way after gathering the derivatives:

$$\frac{d}{dx}\left(I_2(x)hx^r\right)\left(\frac{1}{2}\frac{q_v^2 x^r}{\left(\phi^r\right)^2\left(I_2(x)hx^r\right)^3} - \frac{(I_1)(x)}{(I_3)(x)}\frac{q_{v0}}{q_v}g\beta\Delta T_0\right) = 0 \qquad (10)$$

Eq. 10 has three possible solutions. Either the derivative is zero or the expression within the parenthesis is zero or both are zero. The first alternative means that $\left(I_2(x)hx^r\right)$ is constant. But this has no meaning in the acceleration section because this implies that the velocity is constant. Letting the value of the parenthesis expression be zero we obtain the lowest value of $\left(I_2(x)hx^r\right)$ or the value of $\left(I_2(x)hx^r\right)$ which gives the maximum value of the velocity. The derivative and the parenthesis may as a third alternative be zero simultaneously. Anyway we get:

$$I_2(x)hx^r = \left(\frac{1}{2} \frac{q_v^2 x^r}{\frac{I_1(x)q_{v0}}{I_3(x)q_v} \left(\phi^r\right)^2 g\beta\Delta T_0} \right)^{\frac{1}{3}}$$

Which substituted into eq. 7 gives: (11)

$$U_m = \left(2\frac{I_1(x)}{I_3(x)} \frac{g\beta\Delta T_0 q_{v0}}{\phi^r x^r} \right)^{\frac{1}{3}}$$

Equation 11 gives the maximum value of the maximum velocity at the bottom of the boundary layer. For a radial flow pattern this is a function of the distance x. The procedure outlined here does not allow us to establish an expression for calculating the distance x where the maximum value occur.

The result shows that the maximum velocity is uniquely determined by the buoyancy flux of the inflowing air $(q_{V0} g\beta\Delta T_0)$. The relevance of the buoyancy flux is among others discussed by Sandberg (1991).

Index zero refers to the outlet of the diffuser for which: $\Delta T_o = T_r - T_0$

T_0 = *Supply air temperature*

2.2 Plane flow

Plane flow is two-dimensional without any radial spread, fig 2, and is seldom realised in practice. In eq. 11 we put r=0 and we get the following expression for $I_2(x)h$ and U_m (from eq. 6 and 11). The buoyancy flux is given per unit length of the diffuser.

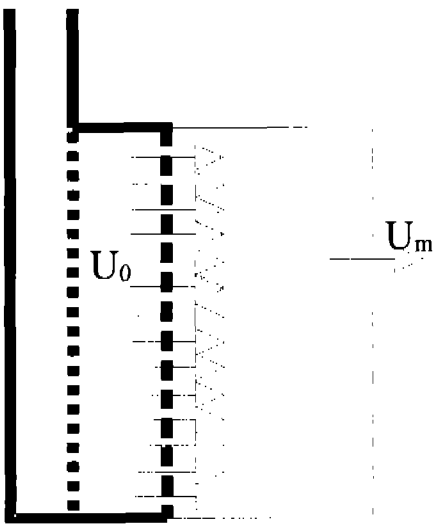

Fig.2 Plane, two-dimensional flow.

$$I_2(x)h = \left(\frac{1}{2} \frac{q_v^2}{\frac{I_1(x)q_{v0}}{I_3(x)q_v} g\beta\Delta T_0} \right)^{\frac{1}{3}} \quad (12)$$

$$U_m = \left(2\frac{I_1}{I_3} g\beta\Delta T_0 q_{v0} \right)^{\frac{1}{3}} = K_p \left(g\beta\Delta T_0 q_{v0} \right)^{\frac{1}{3}} \quad (13)$$

The order of magnitude for the constant K_p in equation 13 is found by putting actual numerical values into the expression:

$$K_p \approx \left(2\frac{I_1}{I_3} \right)^{1/3} = \left(2\frac{0,6}{0,368} \right)^{1/3} \approx 1,5$$

(We have used relevant values calculated for ordinary jet flows, Abramovich (1963).)

The analysis shows that the maximum velocity is uniquely determined by the buoyancy flux of the supply air. The result also shows that the velocity is constant in the far region, quite similar to what is found for buoyant plane plumes. The extension of the acceleration region has no analytical solution and consequently needs to be determined experimentally or numerically.

2.3 Radial flow

In practice most low velocity diffusers develop/exhibit a radial flow pattern, fig 3, 4 and 5. In eq.11 we put r=1 and we get the following expression for $I_2(x)h$ and U_m (from eq. 6 and 11). The buoyancy flux is given per unit length of the diffuser.

$$I_2(x)hx = \left(\frac{1}{2} \frac{q_v^2 x}{\frac{I_1(x)q_{v0}}{I_3(x)q_v} (\phi)^2 g\beta\Delta T_0} \right)^{\frac{1}{3}} \quad (14)$$

$$U_m = \left(2\frac{I_1}{I_3} \frac{g\beta\Delta T_0 q_{v0}}{\phi x} \right)^{\frac{1}{3}} \quad (15)$$

By multiplying the numerator and denominator in expression 15 with x_O, eq. 15 is transformed to:

$$U_m = \left(2\frac{I_1}{I_3} \frac{q_{V0} g\beta\Delta T_0}{\phi x_0} \right)^{1/3} \left(\frac{1}{\frac{x}{x_0}} \right)^{1/3} \quad (16)$$

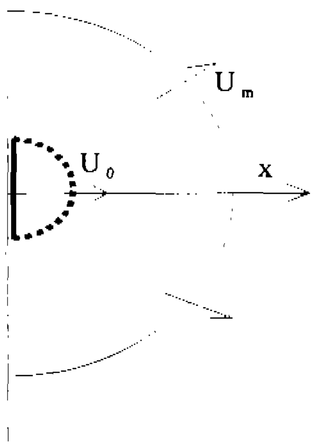

Fig. 3 Radial flow pattern.

Here $\frac{q_{V0} g\beta\Delta T_0}{\phi x_0}$ is the buoyancy flux per unit horizontal length of the diffuser front, because ϕx_0 is the circumference of the diffuser, which we may choose to denote L. When the radial flow has expanded to the side walls or symmetry boundary for neighbouring diffusers, x / x_0 acquires its largest value and the flow become two-dimensional (constant velocity for larger distances).

The solution gives the relation between buoyancy flux, distance from the diffuser and the maximum velocity (acceleration velocity) for radial flow. The solution does not give any information about the length of the acceleration region and no information about the rate at which the velocity decrease after the initial acceleration.

It is a problem in radial flow, contrary to two-dimensional flow where the x-value does not appear, that the x-value for maximum velocity cannot analytically be determined. However, tests have shown that the value for x / x_0 at maximum velocity does not vary very much for a set of similar shaped diffusers. The value seems to be within a span of 2,5 - 3,5. This implies that the maximum velocity can be determined by an expression of the type:

$$U_m = K_r \left(\frac{q_{V0} g\beta\Delta T_0}{L} \right)^{1/3} \qquad (17)$$

The order of magnitude for the constant K_r is found by putting relevant numerical values in eq.16:

$$K_r \approx \left(2 \frac{I_1}{I_3 \frac{x_m}{x_0}} \right)^{1/3} = \left(2 \frac{0,6}{0,368 \cdot 3} \right)^{1/3} \approx 1 .$$

We have here in the same way as for two-dimensional flow used values for the profile integrals from jet flow.

The buoyancy flux, $q_V g\beta\Delta T_m$, is however constant and can be set equal to: $q_{Vo} g\beta\Delta T_o$, where subscript o refers to the supply outlet where: $\Delta T_o = T_o - T_s$

T_s = *supply air temperature*

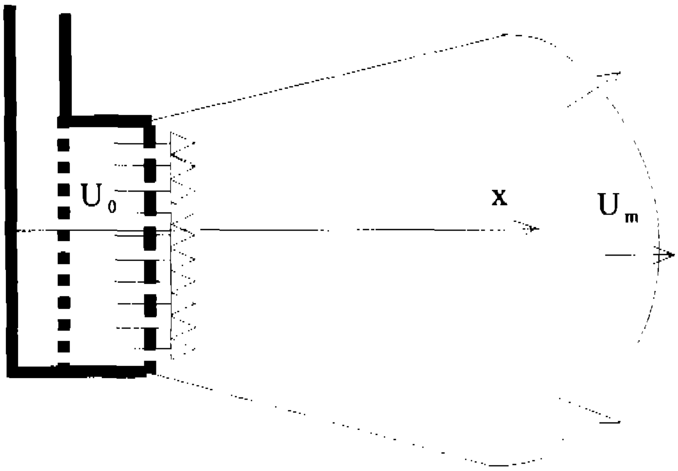

Fig 4. Horizontal flow pattern behind a plane diffuser.

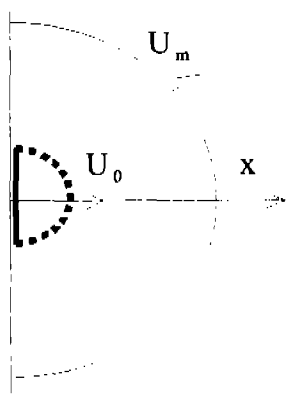

Fig.5. Horizontal flow pattern behind a radial diffuser.

3 VERIFICATION

Table 1

q_{v0} [m3/h]	q_{v0} [m3/s]	Δt [K]	L [m]	X_0 [m]	X_m [m]	T_s [K]	H_0[m]	Ar_0	Um [m/s]	K_r
200,	0,05556	3,0	0,785	0,25	0,85	292	1,0	20,1	0,300	1,56
311,	0,08639	3,1	0,785	0,25	0,56	292	1,0	8,6	0,440	1,95
420,	0,11667	3,2	0,785	0,25	0,47	292	1,0	4,9	0,640	2,54
310,	0,08611	5,2	0,785	0,25	0,77	290	1,0	14,6	0,455	1,70
309,	0,08583	8,2	0,785	0,25	1,09	286	1,0	23,5	0,510	1,63
309,	0,08583	3,1	0,785	0,25	0,59	292	0,8	4,5	0,450	2,00
310,	0,08611	3,2	0,785	0,25	0,57	292	0,5	1,1	0,515	2,26
179,	0,04972	3,1	0,393	0,125	0,38	293	1,0	6,5	0,620	2,63
292,	0,08111	5,1	0,393	0,125	0,37	291	1,0	4,0	1,020	3,10
399,	0,11083	8,1	0,393	0,125	0,50	287	1,0	3,5	1,430	3,35
309,	0,08583	7,3	0,785	0,25	1,03	282	1,0	21,2	0,480	1,59

Table 1 shows the result of laboratory tests, Wulff (1995), on a series of similar diffusers where K_r is calculated from eq. 17 based on test-data. As we can see the constant is not very much "constant". A comprehensive parameter study was performed. The Π-theorem in dimensionless analyses suggests that the Archimedes number is a significant dimensionless parameter if the length scale matters. Our results did not verify this assumption. The parameter study resulted in the following: The flow pattern is not strictly radial but is influenced by the airflow per unit length and the temperature difference. Increasing flow rates (inertial effect) reduces the radial effect, and an increased temperature difference (buoyancy effect) has an opposite effect. The constant K_r showed to be a function of the parameter $(\Delta T_0/(q_{v0}/L)^2)$ only. In fig. 6 is shown an example from a test series where airflow rates, diffuser height, diffuser radius and temperature difference were varied. The result could be uniquely correlated in the following function:

$$K_r = 25{,}4\left(\frac{\Delta T_0}{q_{v0}^2/L}\right)^{-0{,}44} \qquad (18)$$

Increased airflow rate increases the constant while increased temperature difference has the opposite effect.

For two-dimensional flow we found that the velocity is constant in the far region analogue to two-dimensional convection plumes. If in radial flow there is an analogy to three-dimensional convection plumes the velocity decay in the far region should obey the following relation:

$$U_m \approx (x_0 + x)^{-1/3}$$

Our test data, fig. 7, indicate that the decay exponent is closer to -1. Work carried out by Peter V. Nielsen et al in Denmark (1998, 1991 and 1992) also indicates that the exponent for x is more like -1 in the far region. We therefore adopt the exponent -1 in the far region for radial flow.

Then the following interesting relations can be derived for the far region:

$$\frac{U_x}{U_m} = \frac{x_1}{x} \qquad (19)$$

x_1 is the x-value where the decay curve crosses the ordinate value 1 ($\frac{U_x}{U_m} = 1$)

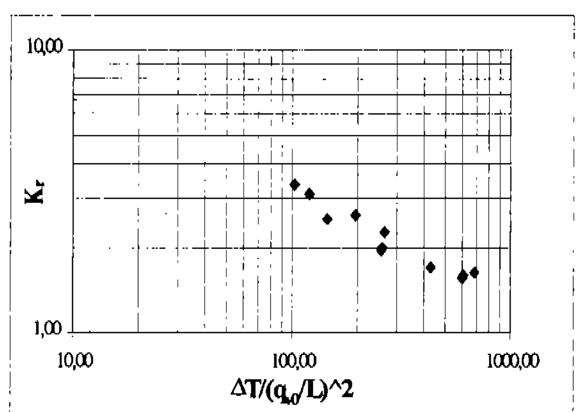

Fig. 6 Lab tests where K_r is corrected by the parameter $\Delta T_0/(q_{v0}/L)^2$

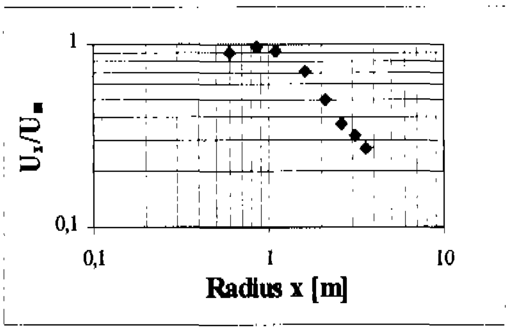

Fig. 7 Velocity as a function of x.

For a certain, arbitrary chosen, velocity the following relation is valid in the far region, based on eq. 16 and 19:

$$U_{x,U=konst} = \left(2\frac{I_1}{I_3}\frac{q_{V0}g\beta\Delta T_0}{L}\right)^{1/3}\left(\frac{1}{\frac{x_m}{x_0}}\right)^{1/3}\frac{x_1}{x_U}$$

$$= \left(\frac{x_1}{x_U}\right)^{\frac{2}{3}}\left(\frac{x_1}{x_m}\right)^{\frac{1}{3}}\left(2\frac{I_1}{I_3}\frac{q_{V0}g\beta\Delta T_0}{L}\right)^{1/3}\left(\frac{1}{\frac{x_U}{x_0}}\right)^{1/3} \quad (20)$$

When the chosen constant velocity is f.ex. 0,2 m/s we can put $x_U = x_{0,2}$.

For a series of similar shaped diffusers, tests have indicated that the following relation holds:

$$\left(\frac{x_1}{x_U}\right)^{\frac{2}{3}}\left(\frac{x_1}{x_m}\right)^{\frac{1}{3}} \approx Constant$$

Then we can postulate that test data should obey the following relation in the far region for a set of similar shaped diffusers:

$$U_{U=konst} = K_{rU}\left(\frac{q_{V0}g\beta\Delta T_0}{L}\right)^{1/3}\left(\frac{1}{\frac{x_U}{x_0}}\right)^{1/3} \quad (21)$$

For a product series of similar shaped diffusers it can be expected that test data should obey the following relation in the region with decreasing velocity between the distance from the diffuser and an arbitrary chosen constant velocity:

$$\frac{U_x}{\left(\frac{q_{V0}g\beta\Delta T_0}{L}\right)^{1/3}} = K_{rU}\left(\frac{1}{\frac{x_U}{x_0}}\right)^{1/3} \quad (22)$$

The constant K_{rU} must be experimentally (empirically) determined.

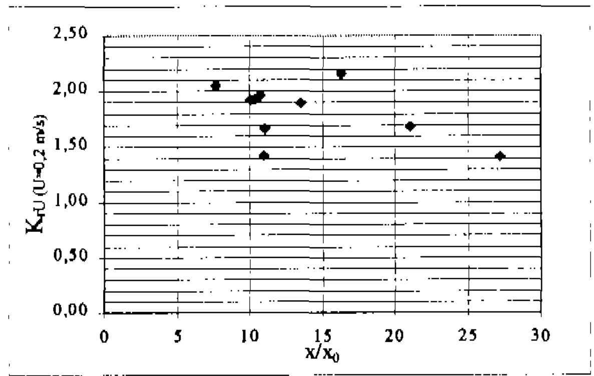

Fig. 8 $K_{r0,2}$ determined from tests using eq.22

In fig 8 the constant K_r is calculated from the same tests as shown in table 1 based on eq. 22. As we can see the constant is not very much "constant". A comprehensive parameter study showed that in this far region only the temperature difference matters. The explanation is the same as for the acceleration region that increasing thermal forces (buoyancy) increases the radial effect i.e. forces the flow more to the sides. The inertia forces have no effect in this region. Fig. 9 shows the temperature effect and the good correlation between the constant K_{rU} and the temperature difference. Again the height of the diffuser and, indirectly, the Archimedes number does not matter.

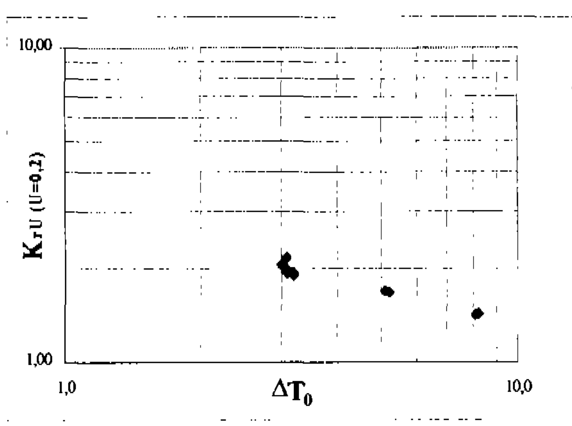

Fig 9. K_{rU} varies uniquely with ΔT_0.

The temperature function for this test series became:

$$K_{r0,2} = 3\Delta T_0^{-0,36} \quad (23)$$

This temperature function (temperature and flow rate function for the maximum velocity) is different for different diffuser design. Having ideal diffusers with ideal radial flow, the exponents in these functions are expected to become zero. Then the constants in equations 7 and 22 are really constant. Low K-factor is equivalent to good lateral spread.

The relations shown indicate that for a set of similar shaped diffusers it is not necessary to test more than a few sizes and loads to determine K_{rU}. Further it is sufficient to document a few velocities like 0,15, 0,2 and 0,25 m/s. The formulas make it possible to calculate the distance from a diffuser, $(x_U - x_0)$, where a given velocity occur, using arbitrary combinations of temperature differences and air flow rates.

4 Performance documentation of air diffusion devices

Performance documentation should contain the following information as a function of size and flow rate:

- Maximum velocity for the near zone of the diffuser as a function of distance and the temperature difference between the supply air and the air temperature 1,1 m above floor level, i.e. K_r or K_p i.e. to determine K_r or K_p as a function of ΔT_0. A few terminal velocities like 0,15, 0,2 and 0,25 m/s should be tested.
- Isovel envelopes to show the spread of the air flow.
- The relative temperature increase in the near zone.
- Pressure drop and noise generation data.

5 Summary

A semiempirical flow model is developed for displacement ventilation.

- The formulas make it possible to calculate the distance from a diffuser, $(x_U - x_0)$, where a given velocity occur, using arbitrary combinations of temperature differences and airflow rates.
- Utilizing the model the manufacturer has a cheaper and more useful and versatile method to perform performance documentation for low velocity diffusers compared to existing methods.
- The semiempirical model developed makes it possible for a designer to choose the right diffuser for a certain application if proper performance testing is made by the manufacturer.
- Existing testing standards should be revised according to the new method.

6 References

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