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Introduction to the Design of Natural Ventilation Systems Using Loop Equations

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Synopsis

The design of natural, including passive, ventilation systems assumes one of two generic forms: the nasty design problem where the designer seeks to size ventilation openings given climatic conditions and thermal comfort criteria or the nice design problem where the designer seeks to size ventilation openings given climatic conditions, indoor temperature distributions, and specified airflow rates – presumably determined from separate thermal or air quality considerations. The nasty form of design demands consideration of the complex dynamic coupled interaction of a building's airflow systems, thermal characteristics and airflow and thermal excitations – a challenge that only the most advanced simulation programs have been able to address and one were few, if any, can claim real expertise at this time. The nice form of ventilation design, on the other hand, is quite tractable and may be approached using existing, relatively simple and intuitively direct theory. Yet it is commonly perceived to be of a nasty character demanding iterative and approximate techniques for its solution.

This paper will present an 'exact' approach to the *nice* design problem that may be considered to be a more complete formulation of the approximate approach recently published in the CIBSE Application Manual AM10:1997 [1]. The approach presented is based on so-called *loop equations* that are commonly used in flow network simulation in the hydraulics field but have been largely ignored in the building ventilation field. It allows direct sizing of a variety of airflow components and the direct and unambiguous consideration of both stack-driven and wind-driven flows without resorting to simplifying approximations. Yet, the approach is developed in such a way as to enable building designers to identify a full range of feasible design configurations so that other, nontechnical design constraints may be included in the process of seeking a design solution – an example of such a design scenario will be presented.

Introduction

The benefits of ventilation have been know to humankind for all time – as a means to displace stale or contaminated air and to cool advectively or convectively. We can easily imagine the design of ventilation systems for air quality control began when the first individual attempted to redirect the smoke from a cooking fire to avoid its personal impact and, for cooling, when another moved to a shaded hill top location to gain the benefits of cooling breezes. With this extraordinarily long history in mind one should reasonably ask why do we now consider the design of natural ventilation systems such a challenge? The short answer to this questions is that now the stakes are higher – there is a significant energy penalty, and the consequent environmental impact, associated with over-ventilation [2, 3] and, of course, comfort and potentially health problems with under-ventilating buildings. Hence we seek to ventilate buildings with greater control and precision than in the past.

How then does a building designer or architect approach the design of building ventilation systems to achieve these more demanding objectives? Five distinct design tasks can be identified:

- *Establish Global Geometry* The designer must initially set the global geometric configuration of the system – e.g., siting of the building and landscape configuration, overall building form, and positions of fresh air inlets and stale air exhausts.
- *Establish System Topology* The designer must layout the airflow paths from inlet to outlet that will achieve the desired airflow objective – e.g., cooling, moisture control, CO_2

control, etc. – and select the types of airflow components – e.g., windows, doors, vents, etc. – that will provide the control of airflow desired.

- *Component Sizing* The designer must then size the components of the airflow system considering reasonable and relevant climatic conditions – the *design conditions* – and appropriate *design criteria.*
- *Control Strategy* The designer must develop a strategy to control the ventilation flow rates to achieve the objective design criteria and select hardware and, possibly, software to implement the strategy.
- *Detail and Assembly* Finally, the designer must develop detail and assembly drawings so that the system can actually be built.

This paper addresses only Task 3 – the task often identified as "design" in the engineering community – up-to-date guidance for Tasks 1 and 2 for both domestic and non-domestic buildings may be found in the recent publications of the British Research Establishment (BRE) and the Chartered Institution of Building Services Engineers (CIBSE) [1, 4].

In the allied fields of building technology, component sizing methods are invariably based on the same theoretical principles used to predict building performance – i.e., to *simulate* building behavior – although, recognizing the time and financial constraints designers face, these methods are generally simplified inversions of the mathematical methods used for whole system simulation. In the simulation of building ventilation systems, two classes of analytical problems may be distinguished – simple *network airflow analysis* where ventilation airflows are predicted for selected climatic conditions given temperature distributions within the building and the more complex *coupled network airflow/thermal analysis* where both building temperature distributions and airflows are predicted for selected climatic conditions.

Consequently, we may distinguish two generic classes of component sizing problems – the *nasty* and the *nice* "design" problems as articulated above in the **Synopsis**. The difficult *nasty* "design" problem was considered in an earlier paper [5]. The more tractable *nice* form of ventilation "design", on the other hand, is the object of this paper.

Irving and his colleagues have developed an approach to the *nice* "design" problem [1, 6, 7] that is coincidentally similar to an approach I've presented to architectural design students at MIT and Yale. Both approaches are based on the same fundamental theory – the basis of methods of network airflow analysis – and may be considered to be formulated as simplified *loop equations* for these networks. For the professional building designer, however, there is little penalty and much to be gained through the use of the more general and complete method based on an <u>exact</u> formulation of *loop equations*. This paper will present this formulation and, building on the work of Irving and his colleagues, demonstrate its application and utility.

Basic Theory

A building system may be idealized as a collection of control volumes (e.g., rooms, zones, or joints of a duct network system) linked by discrete airflow components (e.g., windows, doors, cracks, or duct segments of a duct network). For the representative *macroscopic idealization* illustrated below, a single room is modeled with an inlet opening at a, a room outlet grill at b, and a ducted stack running from b to c terminated by a stack terminal device at c – the control volumes considered in this case are the room and the stack duct and the discrete flow components: the inlet, outlet, and stack terminal devices.

Discrete locations or nodes are identified – e.g., the black dots above – with which values of temperature and pressure are specifically defined and the form of the variation of temperature and pressure within the control volumes is assumed and directly related to the nodal values – e.g., most commonly, but not necessarily, the temperature is assumed to be uniform and the pressure assumed to vary hydrostatically within the control volume. One of several pressure-flow relations is then associated with each discrete flow component to complete the building idealization task.

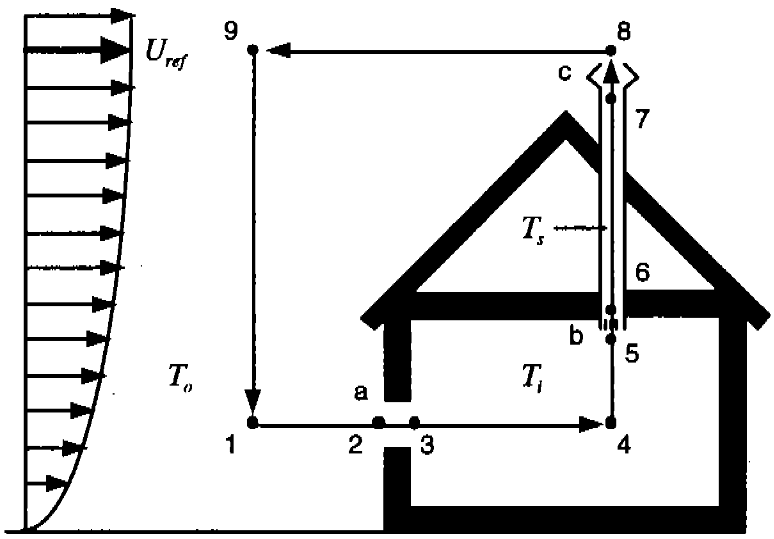

Figure 1 Representative macroscopic idealization of a building.

With such a *macroscopic idealization* in hand, systems of nonlinear algebraic equations – the *node equations* – can then be formed, by demanding the conservation of air flow in-to and out-of each control volume, and solved to determine nodal pressures and, subsequently, the airflows in each of the linking discrete airflow components.

Alternatively, one may approach the analytical problem by summing equations describing the changes of pressure as one traverses a continuous loop that follows possible airflow paths from node to node in the building idealization returning to the original starting node. With reference to Figure 1, one such loop is possible following the nodal path 1-2-3-4-5-6-7-8-9 and back to 1. These changes of pressure must, of course, add up to zero upon completion of the loop. While these *loop equations* are a bit tricky to form automatically for simulation purposes, they are well-suited to design investigations since the designer must design the loops – i.e., specify the global geometry and topology of the ventilation airflow paths that define the loops and size the discrete components along the path.

Wind & Hydrostatic Pressures

The equations needed to form the loop equations for a given building idealization are familiar. At surface locations external to a discrete flow component "*e*" wind-driven pressures P_{eo} are related to the ambient pressure $P_o(z_e)$ at the component level z_e and the dynamic pressure of the oncoming wind defined in terms of a wind pressure coefficient C_{Pe} and an associated dynamic pressure of a reference wind speed U_{ref}:

$$P_{eo} = P_o(z_e) + C_{Pe}\frac{\rho U_{ref}^{\ 2}}{2} \tag{1}$$

Pressure changes $\Delta P_i(\Delta z_{ij})$ due to elevation changes Δz_{ij} are defined by the discrete form of the hydrostatic equation for the usual assumption of uniform temperature distributions:

$$\Delta P_i(\Delta z_{ij}) = -\rho_i g \Delta z_{ij} \tag{2}$$

or by the integral form for nonuniform temperature distributions:

$$\Delta P_i(\Delta z_{ij}) = -\int_{z_1}^{z_2} \rho_i(z) g dz \tag{3}$$

Flow Component Relations

Finally, the pressure change, ΔP_e, along a discrete flow component "*e*" is related to the volumetric air flow rate Q_e and a characteristic design variable associated with the component ϕ_e as – in general, functional notation:

$$\Delta P_e = f(Q_e, \phi_e) \tag{4}$$

Here, four pressure-flow relations will be considered, the first, useful for larger openings such as windows and doors experiencing unidirectional flow, is based on the orifice equation:

Orifice Relation $$\Delta P_e = \frac{\rho Q_e^2}{2C_d^2 A_e^2} \text{ where } \phi_e \equiv A_e \tag{5}$$

where the discharge coefficient C_d may be expected to have a value close to 0.60 for flow intensities of interest in most situations. The second relation, often empirically fitted to measured behavior of adventitious openings, is based on the so-called power law relation:

Power Law Relation $$\Delta P_e = \frac{Q_e^{1/n}}{C_e^{1/n}} \text{ where } \phi_e \equiv C_e \tag{6}$$

The third relation, appropriate for very narrow crack openings, is based on a linear relation:

Linear Relation $$\Delta P_e = \frac{Q_e}{C_e^*} \text{ where } \phi_e \equiv C_e^* \tag{7}$$

Finally, the fourth relation is based on the familiar relation for flow in ducts:

Duct Flow Relation $$\Delta P_e = \frac{1000 fL}{D_h} \frac{\rho Q_e^2}{2A_e^2} \text{ where } \phi_e = D_h A_e^2 \tag{8}$$

where f is a friction factor that varies from 0.01 to 0.05 for likely flow intensities encountered, L is the length of the duct, D_h is the hydraulic diameter, and A_e is the duct cross-sectional area. Details relating to these relations may be found in Awbi [8] and the ASHRAE Handbook of Fundamentals [9].

Bousinesq Approximation

The subscript on the air density variable ρ in Equations 1 and 5 has been omitted to acknowledge that the uncertainty associated with the use of these equations does not warrant precision in the specification of air density. When applying Equation 1 and 5 sufficient accuracy, especially for design calculations, may be obtained by using an air density representative of the range of values associated with the problem at hand (e.g., a mean value). On the other hand, precision is required in the hydrostatic equations, Equations 2 and 3, thus the subscript is retained. These approximations, which are a discrete form of the Bousinesq assumption used in *microscopic* analysis, are not theoretically required but simply ease the burden of computation.

Loop Equations

Using the equations enumerated above, we may directly form the loop equations for a given building idealization. For the representative idealization shown in Figure 1, we begin at the ambient pressure node 1, P_{1o}, and move forward around the loop adding first the increase due to the wind acting on the wall at node 2, then the pressure drop ΔP_a along the flow link a, the hydrostatic decrease resulting from the elevation change from 4 to 5, and so on to obtain:

$$+C_{P2}\frac{\rho U_{ref}^2}{2} - \overbrace{\Delta P_a}^{\text{window}} - \rho_l g \Delta z_{45} - \overbrace{\Delta P_b}^{\text{room outlet}} - \overbrace{\Delta P_{bc}}^{\text{stack duct}} - \rho_s g \Delta z_{58} - \overbrace{\Delta P_c}^{\text{stack terminal}} - C_{P8}\frac{\rho U_{ref}^2}{2} + \rho_o g \Delta z_{91} = 0 \quad (9)$$

where ρ_l and ρ_s are the air densities in the room and the stack respectively. Substituting the flow relations for each of the components – e.g. here using the orifice relation for the window, the power law relation for the room outlet and the stack terminal, and the duct relation for the stack duct – yields the final explicit form of the loop equation. For a more complex building additional flow paths and, hence, loop equations may be formed.

Presented as in Equation (9), the loop equations may seem rather formidable, but on closer examination it may be seen that these equations involve simply a) a summation of hydrostatic changes that define a stack-driven pressure difference ΔP_s, b) windward and leeward pressures that define the wind-driven pressure difference ΔP_w, and c) a summation of pressure drops of each of the flow component ΔP_l along the loop that has the general form:

General Form

$$\overbrace{\sum f(Q_l)}^{\Delta P_l} = \overbrace{g\sum \rho_i \Delta z_{ij}}^{\Delta P_s} + \overbrace{(\Delta C_P)\frac{\rho U_{ref}^2}{2}}^{\Delta P_w} \quad (10)$$

where $f(Q_l)$ is positive for *forward* flow along the loop, Δz_{ij} is positive for increases in elevation along the loop, and ΔC_p is the algebraic sum of pressure coefficients with C_P summed positively when traversing the loop from the exterior to a wall surface location and negatively other wise.

For loops involving orifice flow links only, the loop equations assumes the following form:

Orifice-Only Loop

$$\overbrace{\sum \frac{\rho Q_l^2}{2 C_d^2 A_l^2}}^{\Delta P_l} = \overbrace{g\sum \rho_i \Delta z_{ij}}^{\Delta P_s} + \overbrace{(\Delta C_P)\frac{\rho U_{ref}^2}{2}}^{\Delta P_w} \quad (11)$$

This relation is essentially identical to that developed by Irving and his colleagues although their model was limited to control volumes with single inlets and outlets [1, 6, 7].

At any given stage of design all variables in a loop equation will be known, with the exception, of course, of the flow component design parameters (e.g., the opening areas A_ℓ for orifice components), and may be substituted directly. In general, however, one will not be able to define a sufficient number of loop equations to determine a unique solution. In natural ventilation design many alternative *feasible design solutions* may be identified. This apparent problem, we shall see, presents the designer with the opportunity to include nontechnical considerations in the design process if the problem is approach correctly. This may be made clear by consideration of a more complex example.

CIBSE Example Application

Consider the design of the hypothetical three story building, based on the example considered by Irving [1, 6, 7], sketched below. In this sketch, discrete flow components are identified by alphabetic labels – i.e., components a, b, ... g are distinguished – and four zones are considered – zones G, 1, 2, and s (i.e., stack). In addition, relevant pressure nodes and elevation increments are indicated. This example differs from that presented in the CIBSE application manual on natural ventilation [1] in that internal flow resistances – i.e., components b, d and e – are included.

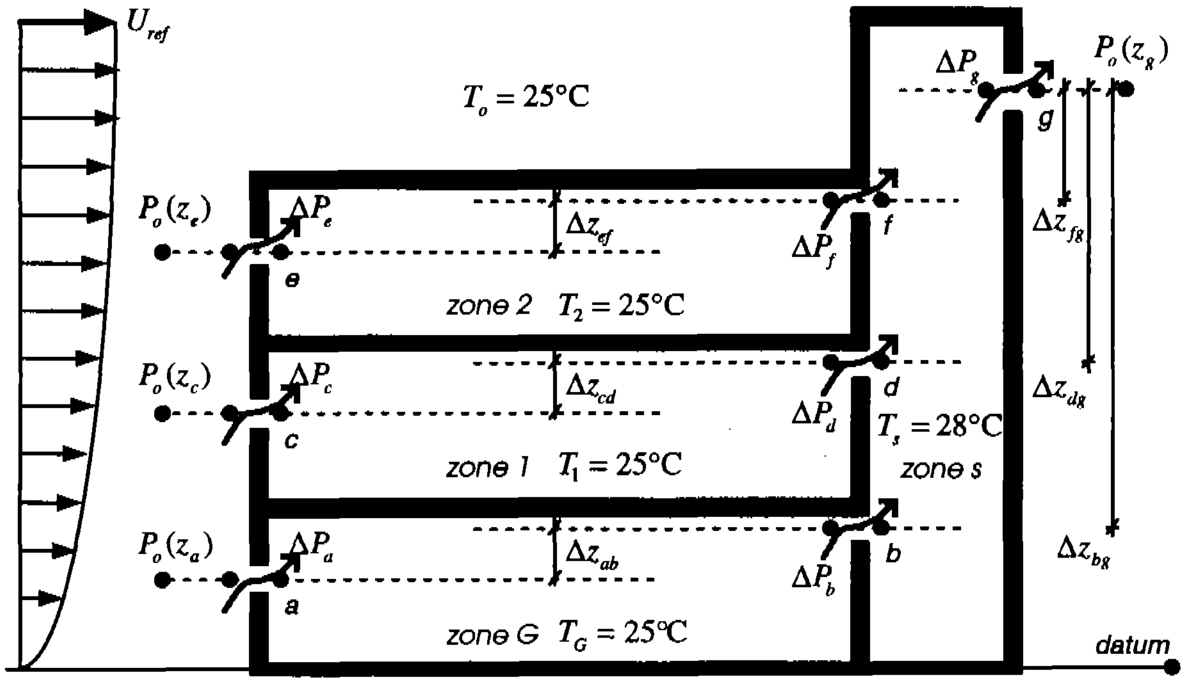

Figure 2Problem based on CIBSE example problem 1 [1]

Global Geometry & Topology

At this stage in design the global geometry and the ventilation system topology – i.e., the three flow paths, a-b-g, c-d-g, and e-f-g – have been defined by the designer. Regarding the former each story of the building is 3.25 m high, the inlet openings are centered 1.85 m above floor level, the room outlets at 2.85 m, the stack exhaust is centered 11.5 m above the ground, and each room has a volume of 322.9 m^3.

Climatic Conditions & Design Criteria

The climatic conditions and design criteria will be based on Irving's data [1]. The design is to provide a ventilation flow rate of 5 ACH for an outdoor ambient temperature of 25°C and a reference wind speed of 3 m/s. Windward wind pressure coefficients will be assumed to be +0.58 and leeward –0.20, internal room temperatures will be taken to be 25 °C and the mean air temperature of the stack will be assumed to be 28°C.

To transform the ventilation design objective into specific component airflow rates we simply demand continuity of flow – here, $\rho_o Q_a = \rho_G Q_b = \rho_o Q_c = \rho_1 Q_d = \rho_o Q_e = \rho_2 Q_f = \rho_s Q_g / 3$ – or, applying the discrete Bousinesq approximation, for the desired 5 ACH (0.448 m^3/s per room) ventilation flow rate the specific component flow rates required are:

$$Q_a = Q_b = Q_c = Q_d = Q_e = Q_f = 0.448 \text{ m}^3/\text{s} \;\; \& \;\; Q_g = 1.344 \text{ m}^3/\text{s} \qquad (12)$$

Loop Equations

To proceed, the designer now seeks to size the seven openings associated with the modeled flow components using the loop equations. Three loops are particularly relevant to the designer in this case: the first defined by the ventilation flow path a-b-g and the return loop to a; the second c-d-g-c; and the third e-f-g-e. Given the building geometry and topology, climatic conditions, and design criteria presented above, all loop equation parameters may be directly determined and the loop equations formed:

$$\text{Loop } a\text{-}b\text{-}g\text{-}a \qquad \frac{0.32893}{A_a^{\ 2}}+\frac{0.32893}{A_b^{\ 2}}+\frac{2.9604}{A_g^{\ 2}}=\overbrace{4.1418}^{\Delta P_w}+\overbrace{1.0003}^{\Delta P_g} \tag{13a}$$

$$\text{Loop } c\text{-}d\text{-}g\text{-}c \qquad \frac{0.32893}{A_c^{\ 2}}+\frac{0.32893}{A_d^{\ 2}}+\frac{2.9604}{A_g^{\ 2}}=\overbrace{4.1418}^{\Delta P_w}+\overbrace{0.62446}^{\Delta P_g} \tag{13b}$$

$$\text{Loop } e\text{-}f\text{-}g\text{-}e \qquad \frac{0.32893}{A_e^{\ 2}}+\frac{0.32893}{A_f^{\ 2}}+\frac{2.9604}{A_g^{\ 2}}=\overbrace{4.1418}^{\Delta P_w}+\overbrace{0.36427}^{\Delta P_g} \tag{13c}$$

Feasible Design Surfaces & Their Asymptotes

A combination of opening areas satisfying these loop equations will satisfy the design airflow objective for the assumed design conditions or, from a mathematical perspective, the loop equations define *feasible design surfaces*. Furthermore, *feasible design surfaces* are implicitly defined for both no-wind condition (i.e., $\Delta Pw = 0$) and with-wind conditions. For example, for loop *a-b-g-a* we obtain the surfaces shown below:

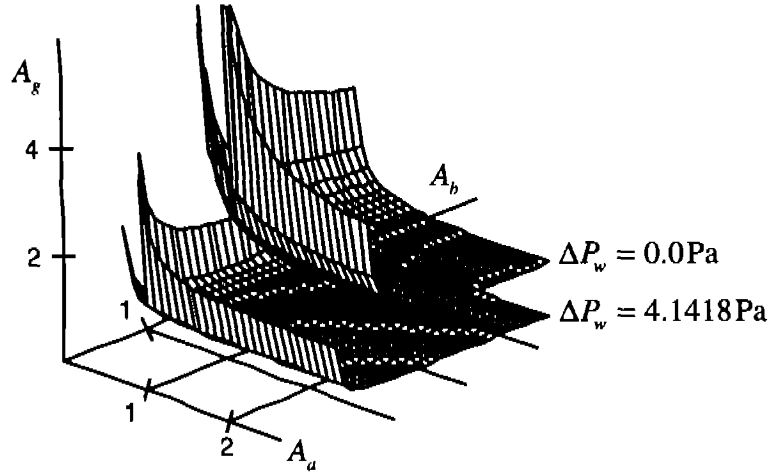

Figure 3 Feasible design surfaces for loop a-b-g-a for no-wind and with-wind conditions

For this example, and in general for *forward-flow loops*, the feasible design surface is hyperbolic in form and bounded by planar asymptotes oriented parallel to the principal axes. These aymptotes define the physically limiting conditions where the resistance of a single flow component governs the flow rate. As such, the asymptotes can prove very useful to the designer – e.g., while there is an unlimited number of feasible openings for this first loop, the opening areas must exceed these limiting values to achieve the design objective of 5 ACH under no-wind conditions. These asymptotes may readily be determined by systematically considering cases where all but one flow link is allowed to approach a negligible resistance (i.e., here an infinite opening area). For example, the A_a asymptote of the without-wind ($\Delta P_w = 0$) loop *a-b-g-a* plotted above is found as:

$$A_a \text{ asymptote} \quad \lim_{\substack{A_b \to \infty \\ A_g \to \infty}} \left(\frac{0.32893}{A_a^2} + \frac{0.32893}{A_b^2} + \frac{2.9604}{A_g^2} \right) = \frac{0.32893}{A_a^2} = \overbrace{1.0003}^{\Delta P_g} \tag{14}$$

$$A_a = 0.573 \ m^2$$

The with-wind surface is also limited by planar asymptotes that establish minimum openings for the second, with-wind, design condition. The *design surfaces* for the other two forward-flow loops will be of similar form and again minimum openings may be evaluated. The results of these straightforward evaluations are tabulated below.

Table 1 Design surface asymptotes for the example problem

Opening	Without-Wind	With-Wind
A_a	0.573 m2	0.25 m2
A_b	0.573	0.25
A_c	0.63	0.26
A_d	0.63	0.26
A_e	0.94	0.27
A_f	0.94	0.27
A_g	1.53 loop a-b-g-a 1.90 loop c-d-g-c 2.82 loop e-f-g-e	0.74 loop a-b-g-a 0.77 loop c-d-g-c 0.81 loop c-d-g-c

The maximum value of these minimum openings establishes the minimum full-opened area of each opening needed to achieve the design airflows. As expected, the without-wind case and, for the stack exhaust, the upper loop, governs. (It is interesting to note that the without-wind design solution opening for the ground floor level presented in the CIBSE manual [1] – 0.579 m^2 – is very close to the limit of 0.573 m^2 tabulated above thus this single opening is critically designed in the CIBSE manual.)

Inclusion of Nontechnical Design Constraints

With these limits and the loop equations in hand, the designer can then proceed to select specific openings to serve both the airflow design objectives and other design constraints he or she faces. For example, the designer may have to select a stack opening from a limited set of off-the-shelf values, let's say a 4.0 m^2 full-opened opening is so selected. In addition, let's assume the designer prefers to use the same window for the windward wall and the room-to-stack openings (i.e., $A_a = A_b = A_{ab}$, $A_c = A_d = A_{cd}$ and $A_e = A_f = A_{ef}$). Recognizing the without-wind case is governing, the designer may then substitute these constraints into the loop equations to finalize design decisions – i.e., for the given constraints we solve:

$$\textit{Loop a-b-g-a} \quad \frac{0.32893}{A_{ab}^2} + \frac{0.32893}{A_{ab}^2} + \frac{2.9604}{4.0^2} = +\overbrace{1.0003}^{\Delta P_g} \text{ or } A_{ab} = 0.898 \ \text{m}^2 \tag{15a}$$

$$\textit{Loop c-d-g-c} \quad \frac{0.32893}{A_{cd}^2} + \frac{0.32893}{A_{cd}^2} + \frac{2.9604}{4.0^2} = \overbrace{0.62446}^{\Delta P_g} \text{ or } A_{cd} = 1.224 \ \text{m}^2 \tag{15b}$$

$$\textit{Loop e-f-g-e} \quad \frac{0.32893}{A_{ef}^2} + \frac{0.32893}{A_{ef}^2} + \frac{2.9604}{4.0^2} = \overbrace{0.36427}^{\Delta P_g} \text{ or } A_{ef} = 1.916 \ \text{m}^2 \tag{15c}$$

This feasible design solution establishes the full-opened opening areas needed for the with-out wind design condition – it is important to stress that other feasible solutions exist as well. The design solution for the with-wind case will, presumably, be achieved by closing down these openings. The with-wind loop equations may then be applied in a similar manner to determined the reduced size of openings to maintain the desired airflow objective.

Conclusion

This paper has outlined the basic tasks of natural ventilation design and presented an approach to the sizing of ventilation components based on the formulation of pressure loop equations that is:

- *exact* – i.e., based on fundamental theory without the need for simplifying assumptions,
- *complete* – i.e., allows the complete and unambiguous consideration of wind and buoyancy effects and enables the design of ventilation flow paths assembled from a variety of different flow components including but not limited to orifice, linear, power law, ducts and fittings, and, possibly, assist fans,
- *inclusive* – i.e., allows nontechnical considerations to be included in the selection of components recognizing ventilation design has no unique solution in general.

A more detailed and complete presentation of this approach will be presented in an AIVC Tech Note "A Practical Guide to Passive Ventilation Air Quality Control in Houses" presently being drafted by the author. This document will present additional worked examples, means to account for infiltration, recently developed self-regulating inlet vents [10-11], and assist fans, and present an approach to deal with the more challenging *nasty* "design" problem.

Acknowledgements

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