VENTILATION AND COOLING

18TH ANNUAL AIVC CONFERENCE
ATHENS, GREECE, 23-26 SEPTEMBER, 1997

A MODIFICATION OF THE POWER-LAW EQUATION TO ACCOUNT FOR LARGE SCALE WIND TURBULENCE

$q = c \cdot \Delta p^n$ → aanpassen voor turbulentie

Kai Sirén

Helsinki University of Technology
Finland

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SYNOPSIS

Existing infiltration and exfiltration calculation methods are mainly based on the stationary approach, where long term mean values are used for wind input data. The real wind speed is, however, varying continuously with time. Because the process of the crack flow is non-linear, using mean wind speed values will give erroneous results for the air flows. The goal of the research has been to develop a simple method to account for the effect of large scale wind turbulence on the calculated air flows.

A modification of the power-law equation has been derived based on the assumption of sinusoidal wind speed fluctuation. The equation is integrated over time to form a new turbulent power-law equation. The integration is carried out numerically using the simple trapezoidal rule approximation. This equation can be used in flow computation in the place of the ordinary power-law equation. The additional input data needed for such a computation is the wind turbulence intensity.

The performance of the turbulent power-law equation is tested computationally by comparing its results against the results of a theoretically far more detailed calculation method, which takes into account the dynamics of the air in the cracks and the capacity of the building space. The computations are carried out using Simnon, a program specially designed for simulation of non-linear systems. Real, with one second time interval measured wind speed has been used as input data. A simple building model with two floors and eight cracks in the walls has been used as a test case. The results show, that the error in the flow rates caused by the stationary approach is mainly dependent on the flow exponent and the turbulence intensity and varies roughly between 0 - 20 %. The higher the turbulence intensity and the more laminar the crack flow, the higher is the error. The turbulent power-law equation performs well and is capable to reduce this error by roughly one decade.

LIST OF SYMBOLS

a	amplitude	m/s
C_p	pressure coefficient	-
K	flow coefficient	$(m^3/s)/(Pa)^n$
I_t	intensity of turbulence	-
n	flow exponent	-
p_i	internal pressure	Pa
p_w	wind pressure	Pa
Δp	pressure difference	Pa
q	volume flow rate	m^3/s
q_t	total infiltration volume flow rate	m^3/s
q_{tr}	total infiltration reference volume flow rate	m^3/s
R_w	proportionality factor	-

t	time	s
T	period of integration	s
v	wind velocity	m/s
v_m	mean wind velocity	m/s
V	volume	m^3
ρ	density	kg/m^3
σ_v	standard deviation of velocity	m/s
ω	angular velocity	rad/s
$\Im$	integral term	$(m/s)^{2n}$

INTRODUCTION

The usual procedure to compute the infiltration and exfiltration air flows implies fixed wind velocity values. This implies, that an average velocity representing one hour or a longer period of time is used as input value. The nature of crack flow through the building envelop is non-linear, which means that using average wind velocity as an input does not give correct average flow rate values and air exchange rates. The larger is the turbulence, the larger are the errors expected when applying the steady-state approach. The effect of wind velocity fluctuations on air exchange has been studied and different kinds of approaches to tackle the problem have been made [1-6]. In the following a modification of the power-law equation is derived, which is capable to account for the wind turbulence. The performance of the equation is shown by comparing the results with the results of a detailed dynamical infiltration/exfiltration computation.

POWER-LAW EQUATION

The non-linear interdependence in crack flow between the pressure difference and the volume flow rate is usually described either by the quadratic equation or by the power-law equation [7]. Both equations have their advantages and disadvantages. In general they both, however, describe the crack flow very satisfactorily, as long as the coefficients included in the equations are chosen according to the properties of the crack or cracks the equation is representing. Here the power-law equation forms the basis for a further development. The power-law equation is usually presented in the following form:

$$q = K\,\Delta p^n \tag{1}$$

where q is the volume flow rate, K is a flow coefficient, Δp is the pressure difference and n is the flow exponent. The flow coefficient is related to the size of the opening and the flow exponent is dependent of the type of the flow. For a fully laminar flow n=1.0 and for a fully turbulent flow n=0.5. In real buildings the flow paths leading air through the building shell are usually combinations of several cracks and material layers. This means, that nor fully laminar neither fully turbulent flows computationally exist and effective values between the lower bound and the upper bound for the flow exponent in equation (1) have to be chosen. The non-linearity of equation (1) also clearly explains, why using mean velocity of some time period gives erroneous results for air flows during that period, when the wind velocity is temporally fluctuating.

TURBULENT POWER-LAW EQUATION

With the turbulent power-law equation a trial is made to account for the problem described above. In a simplified approach the wind velocity as a function of time is approximated by a sine curve in the following manner

$$v(t) = v_m + a\sin(\omega t) \tag{2}$$

where v_m is mean velocity, a is amplitude, ω is angular velocity and t is time. When such a wind strikes a building, the pressure on the building external surfaces follow the velocity and the pressure difference over a crack located in the envelop is

$$\Delta p(t) = p_w(t) - p_i(t) = C_p \frac{1}{2}\rho\, v^2(t) - p_i(t) \tag{3}$$

where p_w is wind pressure, p_i is internal pressure, C_p is pressure coefficient and ρ is air density. Substituting the wind velocity in equation (2) to equation (3) and further, substituting the pressure difference in equation (3) to the power-law equation (1) yields for the flow rate through a crack

$$q(t) = K\left[C_P \frac{1}{2}\rho\ (v_m + a\sin(\omega t))^2 - p_i(t)\right]^n \tag{4}$$

The temporal variation of the internal pressure inside the building $p_i(t)$ is unknown. Let's now set the following hypothesis: the internal pressure fluctuates proportional to the wind kinetic pressure

$$p_i(t) = R_w \frac{1}{2}\rho\, v^2(t) \tag{5}$$

where R_w is the proportionality factor having a constant value. Substituting (5) into (4) gives further for the flow rate as a function of time

$$q(t) = K\left[(C_P - R_w)\frac{1}{2}\rho\ (v_m + a\sin(\omega t))^2\right]^n \tag{6}$$

The conservation of mass for the zone j in the building is

$$\frac{d(\rho_j V_j)}{dt} = \sum_{k=1}^{N} \rho_k q_k(t) \tag{7}$$

where ρ_j and V_j are the density and the internal volume of zone j, and the sum is taken over all cracks k which are connected to the zone j. The density and the volume of the zone are now, as an approximation, set to be constant values, which yields for each zone a mass conservation equation of the type

$$\rho_1 q_1(t) + \rho_2 q_2(t) + \dots \rho_N q_N(t) = 0 \tag{8}$$

The aeraulic behaviour of the whole building is covered by a set of mass conservation equations of the type above, one written for each zone. This set of non-linear algebraic equations is usually solved by iterative methods. Now, however, the flow rates in equation (8) are not constant, but are varying with time. To be able to get a solution, we first integrate both sides of equation (8) over a period of time, which can be done term by term and then divide both sides by the period of integration

$$\frac{1}{T}\int_0^T \rho_1 q_1(t)\,dt + \frac{1}{T}\int_0^T \rho_2 q_2(t)\,dt + \dots + \frac{1}{T}\int_0^T \rho_N q_N(t)\,dt = 0 \tag{9}$$

If the period of time T is long enough, we get a new set of mass conservation equations, where the fluctuating, time depended flow rates q(t) are replaced by mean flow rates q_m

$$\rho_1 q_{m1} + \rho_2 q_{m2} + \dots \rho_N q_{mN} = 0 \tag{10}$$

According to this, equation (6) is now integrated for one period of the sine function

$$q_m = \frac{\omega}{2\pi} K \left[(C_P - R_w)\frac{1}{2}\rho \right]^n \int_0^{2\pi/\omega} (v_m + a\sin(\omega t))^{2n}\,dt \tag{11}$$

Because the flow exponent n is not an integer, there is no analytical solution for this equation. For this reason a numerical approach for integration is utilised. There are several alternatives available to do the numerical integration. Here the trapezoid rule with eight subintervals is used. Thus the integral term in equation (11), numerically integrated, yield

$$\Im = \frac{1}{8}(v_m + a)^{2n} + \frac{1}{4}(v_m + \frac{a}{\sqrt{2}})^{2n} + \frac{1}{4}v_m{}^{2n} + \frac{1}{4}(v_m - \frac{a}{\sqrt{2}})^{2n} + \frac{1}{8}(v_m - a)^{2n} \tag{12}$$

and the mean flow rate, when the time of integration is set to T=1, is

$$q_m = K\left[(C_P - R_w)\frac{1}{2}\rho \right]^n \Im \tag{13}$$

In computation of infiltration and exfiltration air flows, the normal power-law equation can be replaced by the above turbulent power-law equation. The computation procedure is exactly the same than in the mean velocity approach. The internal mean pressures, which now are the unknowns, correspond to the mean velocity

$$p_{im}(t) = R_w \frac{1}{2} \rho v_m^2(t) \qquad (14)$$

and are included in the turbulent power-law equation in the R_w factor. Further, the amplitude a of the approximative sinusoidal velocity (2) has to be known to be able to compute a value for the integral expression (12) and the flow rates (13). The real wind velocity, however, is not sinusoidal and has no constant amplitude. On the other hand, the real wind velocity has a mean value and a standard deviation. The sinusoidal velocity, presented by an analytical function, can also imagined to have a standard deviation, which quite easily can be shown to be

$$\sigma_v = \frac{a}{\sqrt{2}} \qquad (15)$$

where a is the amplitude of the fluctuation. The intensity of turbulence, which can be computed based on wind velocity measurements, is defined as the standard deviation divided by the mean velocity

$$I_t = \frac{\sigma_v}{v_m} \qquad (16)$$

According to equations (15) and (16) we get for the amplitude of the sinusoidal velocity

$$a = \sqrt{2}\, I_t\, v_m \qquad (17)$$

which now can be calculated by substituting the mean velocity and intensity of turbulence of the real wind into the equation (17).

COMPUTATIONAL VALIDATION

Because of the many simplifying approximations and one hypothesis included in the concept above, the results can not be precise, even if all other factors like flow coefficients, flow exponents, pressure coefficients etc. would have exactly correct values. To test the performance of the turbulent power-law equation, a computational validation was done. The demand is, that the reference results, which form the base of the comparison, must have essentially higher quality than the results produced by the method under the test. Here the reference computations decided to be carried out using Simnon [8], a program specially designed for simulation of non-linear systems described with differential or difference equations. It contains a macro level language to define the system and a solver which gives the choice between several algorithms of numerical integration. The integration is done using an automatic step size adjustment.

The case used for validation is a simple building with two floors, having one crack in each wall in both floors, together eight cracks. Momentum equations for each crack and the opening between the floors and a conservation of mass equation for both zones (floors), together eleven differential equations describe the temporal characteristics of the system.

Measured wind velocity was used as input data for the Simnon computations. The input velocity data was measured on the roof level of a two storey building located in a sub-urban single-family house area in southern Finland. This way the characteristics of the local wind speed, which strikes the buildings, could be monitored. Each input data set consists a ten minutes sample recorded with one second time interval, together 600 velocity readings.

For the usual mean velocity computation approach the mean velocity of the ten minutes data sets was used as wind input. For the turbulent power law approach the information of the intensity of turbulence of each data set was added to the input data. For the Simnon reference computations all recorded 600 wind velocity values of each data set were used to perform the dynamic computations. In Fig. 1 the mean total infiltration air flows of the mean velocity approach and the turbulent power law approach are compared with the results of the reference computations.

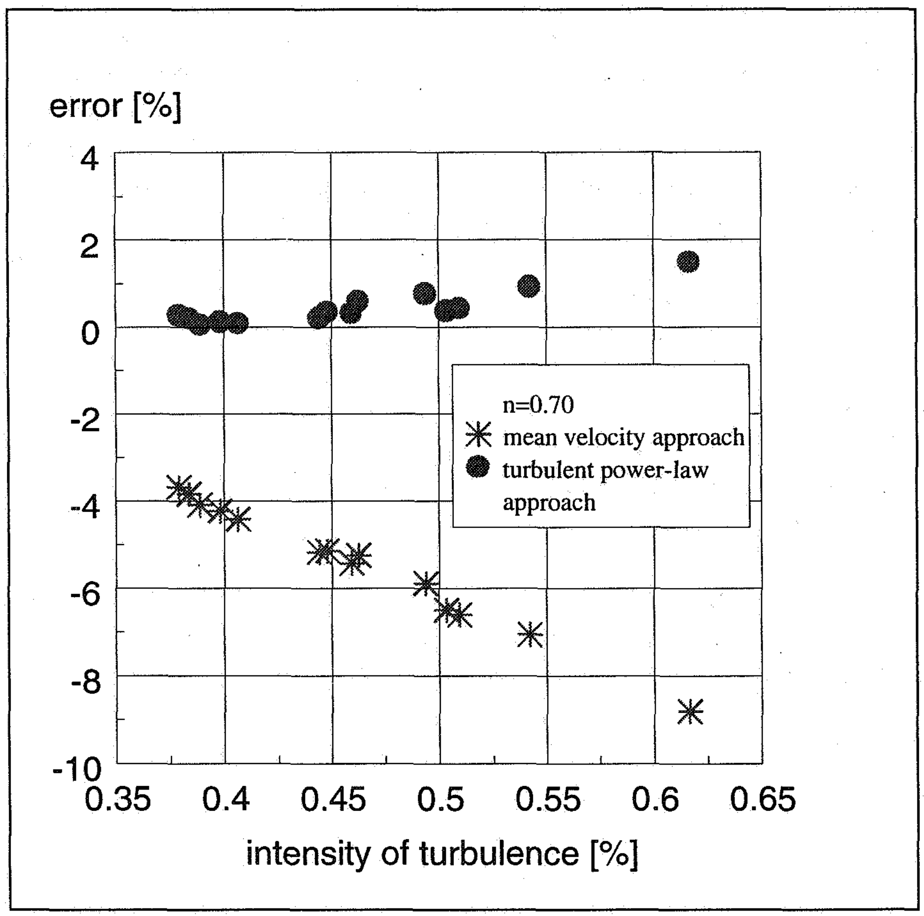

Fig. 1. Relative errors of the total infiltration air flows as a function of the intensity of wind turbulence.

The error terms in Fig. 1 are defined as follows:

$$error = 100 \frac{q_{mt} - q_{mtr}}{q_{mtr}} \tag{18}$$

where q_{mt} is the temporal mean total infiltration volume flow rate into the two floor building, computed with either the mean velocity approach or the turbulent power-law approach and q_{mtr} is the corresponding reference quantity, computed with SIMNON and representing the temporal mean of the time period of the wind input data set.

DISCUSSION

There are two main factors influencing the prediction error shown above. One is the intensity of wind turbulence, as clearly can be seen from Fig.1. The other is the flow exponent. The results shown are computed using a typical medium size value n=0.7 for the flow exponent. If the flow exponent is reduced, which means a more turbulent flow in the cracks, the error of both approaches is also reduced and vice versa.

The turbulent power-law approach shown, does not take into account any temperature difference between indoors and outdoors. This feature can, however, be added easily.

The validation described, is comparison between two computational results. Even though the dynamical SIMNON approach is very detailed and gives, without any doubt, sufficiently reliable results, a more convincing and final validation should be done by using measured reference values. The question is, however, of quite small differences and a measurement based validation is a very demanding task.

CONCLUSIONS

It has been shown, that the turbulent power-law approach performs well in the case of a simple computational exercise and is capable to reduce the error of the usual mean velocity approach by roughly one decade.

The error in computed results caused by wind velocity fluctuations, depends mainly on the intensity of wind turbulence and the nature of the crack flow. The higher the turbulence intencity and the more laminar the crack flow, the higher is the error.

According to the computations, the hypothesis of the indoor pressure to be proportional to the wind kinetic pressure seems to hold well.

REFERENCES

1. Hill J.E. and Kusuda T.
"Dynamic Characteristics of Air Infiltration"
ASHRAE Transactions 81 Part 1, 1975, pp 168-185.

2. Potter I.N.
"Effect of fluctuating Wind Pressures on Natural Ventilation Rates"
ASHRAE Transactions 85 Part 2, 1979, pp 445-457.

3. Sasaki T., Hayashi M. and Aratani N.
"On the Ventilating Characteristics of the Space under the fluctuating Wind Pressure"
Proceedings of Roomvent-87 "Air Distribution in ventilated Spaces", Session 4a, 1987.

4. Haghighat F., Rao J. and Fazio P.
"The influence of Turbulent Wind on Air Change Rates - a Modelling Approach"
Building and Environment, Vol. 26, No 2,1991, pp. 95-109.

5. Sharples S. and Palmer R.G.
"Modelling fluctuating Air Flows through Building Cracks"
Proceedings of the 15'th AIVC Conference, 1994, pp. 216-222.

6. Sharples S. and Thompson D.
"Experimental Study of Crack Flow with varying Pressure Differentials"
Proceedings of the 17'th AIVC Conference, 1996, pp. 343-351.

7. Liddament M.
"Air Infiltration Calculation Techniques - An Applications Guide"
Air Infiltration and Ventilation Centre, 1986.

8. Simnon for Windows, Version 2.0, May 1995. SSPA Maritime Consulting AB, Göteborg, Sweden.