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# A METHOD FOR THE ECONOMICAL OPTIMIZATION OF DESIGN TEMPERATURES AND THE CONNECTING FLOWS OF A COOLING SYSTEM

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### **SYNOPSIS**

The planning parameters of a cooling system for ventilation, for example the vaporization and condensing temperatures, heat capacity flow rates, design temperatures and design temperature differences have a strong influence on the investment and operating costs.

The target of this research is to find economically optimized design parameters by minimizing the present value of investment and the operating costs of the cooling system. The cooling system may have several series-connected heat exchangers on the vaporization and condensing sides of the cooling machine.

When the optimization is carried out, such factors as the price of electricity, maintenance costs, operating time, interest rate, annual running time, the coefficient of performance, marginal costs of heat transfer surface areas and the overall heat transfer coefficients of the vaporizer, the condenser and the other heat exchangers which are connected to the same system, are taken into account.

This new method is useful for the economical optimization of the design parameters of ventilation and other cooling systems.

### SYMBOLS

A	heat transfer surface area, m <sup>2</sup>	Greek letters	
Ċ	heat capacity flow rate, W/K	Е	effectiveness of a heat exchanger
COP	coefficient of performance	${\it \Phi}$	design cooling power, W
J	economical efficiency of a heat	η	efficiency of a total process
	exchanger, W/US\$ K	$\eta_{cd}$	ratio of process efficiencies of real
K	present value of costs, US\$		ideal process and Carnot process
<i>K</i> <sub>0</sub>	constant part of the investment costs, US\$	$\eta_i$	indicated efficiency which takes into account differences between the real
Р	electric power of compressor motor, W		indicated process and the real ideal
S	economic number, K		process
Т	temperature, K	$\eta_m$	efficiency of driving motor and gear
а	factor of present value of periodic	$\eta_{mk}$	mechanical efficiency of compressor
	payment during the operating time	$\theta$	temperature difference, K
c <sub>p</sub>	specific heat, J/kgK	$\sigma$	dimensionless economy number
е	unit price of electricity, US\$/kWh		·
$q_m$	mass flow rate, kg/s	Subscripts	
h	marginal cost of a heat transfer surface	R	reduced
,	area, US\$/m <sup>2</sup>	а	ambient
ĸ	overall heat transfer coefficient,	с	condenser
	W/m <sup>2</sup> K	ln	logarithmic
r	ratio of annual maintenance cost and	v	vaporizer
*	investment cost	r	room
S A	dimensionless optimization parameter		
l	annual peak-load power time, n		

### 1. INTRODUCTION

Cooling systems have usually been optimized by using thermodynamic or economical methods. There are examples of the recent research in references  $\frac{1}{2}$  and  $\frac{3}{2}$ .

In this research a new technical and economical method to optimize a cooling system is presented. The cooling system may have several series-connected heat exchangers on the vaporization and condensing sides of the cooling machine (fig. 1). Planning parameters like vaporizing temperature  $T_v$ , condensing temperature  $T_c$  (fig. 2) as well as heat capacity flows between heat exchangers and their design temperatures have big influence on the economy of the cooling system.



Figure 1. Optimized cooling system.

The target of the research is to determine economically optimal dimensioning parameters for the cooling system presented in fig. 1, where the vaporizer and the condenser are thought to be counterflow heat exchangers in which the other side temperatures are approximately constant and the values of effectivenesses are  $\varepsilon_v$  and  $\varepsilon_c$ , fig. 2. The following factors are taken into account in the optimization: overall heat transfer coefficient, k; marginal costs of heat transfer surface area, h; maintenance costs, which are estimated by the ratio of annual maintenance costs and investment costs, r; the factor of present value of periodic payment during the operating time, a; annual peak-load power time, t; efficiency of the process,  $\eta$ ; and unit price of electricity, e.

Remetry As a result, when the system is optimized by the new method, economically optimal values of effectivenesses and surface areas of the condenser and the vaporizer are obtained. If the cooling system has several series-connected heat exchangers on the vaporization and condensing sides of the cooling machine (fig. 1) also economically optimal surface areas and connecting heat capacity flows of heat exchangers and temperatures in corresponding conditions are obtained as a result.



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The heat transfer surface areas of a vaporizer and a condenser are /4/

$$A_{\nu} = \frac{\dot{C}_{\nu}}{k_{\nu}} ln \left(\frac{1}{1 - \varepsilon_{\nu}}\right), \qquad (1) \qquad A_{c} = \frac{\dot{C}_{c}}{k_{c}} ln \left(\frac{1}{1 - \varepsilon_{c}}\right), \qquad (2)$$

where  $C_{\nu}$  is the cooled heat capacity flow and  $C_c$  is the heat capacity flow which cools the condenser.  $k_{\nu}$  and  $k_c$  are the overall heat transfer coefficients of the vaporizer and the condenser.  $\varepsilon_{\nu}$  and  $\varepsilon_c$  are the effectivenesses of a vaporizer and a condenser.

In optimization the design cooling power  $\Phi_v$  is constant (je helst een zehere  $\phi_v = q_{mv}c_{nv}(T_{r1} - T_{r2}) = \dot{C}_v \varepsilon_v \theta_v = constant$ . (3)

The cooling power of the condenser is

$$\phi_c = \dot{C}_c \varepsilon_c \theta_c = \phi_v + P = \phi_v \left( 1 + \frac{1}{COP} \right) = \phi_v \left( 1 + \frac{1}{\eta} \frac{T_{a1} - T_{r1} + \theta_c + \theta_v}{T_{r1} - \theta_v} \right), \quad (4)$$

where P is the electrical power of compressor, COP is the coefficient of performance.

The COP of the refrigerator is

$$COP = \frac{\Phi_{\nu}}{P} = \eta \frac{T_{\nu}}{T_c - T_{\nu}} = \eta \frac{T_{r_1} - \theta_{\nu}}{T_{a_1} - T_{r_1} + \theta_c + \theta_{\nu}},$$
(5)

where  $\eta = \eta_m \eta_{mk} \eta_{cd} \eta_i$ , (6)  $\theta_v = T_{r1} - T_v$ , (7)  $\theta_c = T_c - T_{a1}$ . (8)

Efficiencies are:  $\eta_m$  efficiency of the driving motor and the gear,  $\eta_{mk}$  mechanical efficiency of the compressor and  $\eta_{cd}$  ratio of process efficiencies of the real ideal process and the Carnot process and  $\eta_i$  indicated efficiency.

The economical efficiency of the heat exchanger is determined by equation

$$J = \frac{k}{h(1+ra)},\tag{9}$$

where h is the marginal costs of the heat transfer surface area and r is the ratio of annual maintenance costs and investment costs and a is the factor of present value of periodic payment

# 3. OPTIMAL SOLUTION FOR THE PRESENT VALUE OF INVESTMENT COSTS AND OPERATING COSTS OF THE COOLING SYSTEM

The present value of investment costs and operating costs are determined by discounting all costs to the present time. The cost function is

$$K = K_{0}(1+r_{0}a) + A_{c}h_{c}(1+r_{c}a) + A_{v}h_{v}(1+r_{v}a) + Ptea$$
  
$$= K_{0}(1+r_{0}a) + \frac{\dot{C}_{c}h_{c}}{k_{c}}(1+r_{c}a)ln\left(\frac{1}{1-\varepsilon_{c}}\right) + \frac{\dot{C}_{v}h_{v}}{k_{v}}(1+r_{v}a)ln\left(\frac{1}{1-\varepsilon_{v}}\right) + \left[\frac{\phi_{v}tea}{\eta}\frac{T_{a1}-T_{r1}+\theta_{c}+\theta_{v}}{T_{r1}-\theta_{v}}\right], (10)$$

where  $K_0$  is the piecewise continuous constant part of the investment costs of the cooling system and *e* is the unit price of electricity. The cost function is piecewise continuous, but between discrete points it is the function of two independent variables, the effectivenesses of the vaporizer and the condenser,  $\varepsilon_v$  and  $\varepsilon_c$ .

The conditions of the optimization are

$$\frac{\delta K}{\delta \varepsilon_c} = \frac{\dot{C}_c}{J_c} \frac{1}{(1 - \varepsilon_c)} - \frac{\dot{C}_c tea((\eta - 1)T_v + T_{a1})}{\left(\eta T_v \frac{\dot{C}_c \varepsilon_c}{\phi_v} - 1\right)^2} = 0, \qquad (11)$$

$$\frac{\delta K}{\delta \varepsilon_{\nu}} = \frac{\dot{C}_{\nu}}{J_{\nu}} \frac{1}{(1-\varepsilon_{\nu})} - \frac{tea}{\eta T_{\nu}} \frac{\phi_{\nu}^{2}}{\dot{C}_{\nu} \varepsilon_{\nu}^{2}} \left( \frac{\eta \left( \frac{C_{c} \varepsilon_{c}}{\phi_{\nu}} T_{a1} + 1 \right) - 1}{\left( \eta T_{\nu} \frac{\dot{C}_{c} \varepsilon_{c}}{\phi_{\nu}} - 1 \right)^{2}} + \frac{T_{c}}{T_{\nu}} \right) = 0.$$
(12)

The optimization parameters  $s_v^*$  and  $s_c^*$  are determined by equations (13) and (14)

$$s_{\nu}^{*} = 0.5(T_{r1} - T_{r2})^{2} \frac{teaJ_{\nu}}{\eta T_{\nu}} \left( \frac{\eta \left( \frac{\dot{C}_{c} \varepsilon_{c}}{\phi_{\nu}} T_{a1} + 1 \right) - 1}{\left( \eta \frac{\dot{C}_{c} \varepsilon_{c}}{\phi_{\nu}} T_{\nu} - 1 \right)^{2}} + \frac{T_{c}}{T_{\nu}} \right),$$
(13)

$$s_{c}^{*} = 0.5 (T_{a2} - T_{a1})^{2} \frac{teaJ_{c}}{(\eta - 1)T_{v} + T_{a1}}.$$
(14)

The economically optimal effectivenesses of the vaporizer and the condenser are

$$\varepsilon = \sqrt{s^{*2} + 2s^*} - s^*.$$
(15)

## 4. OPTIMAL SOLUTION OF SERIES-CONNECTED HEAT EXCHANGERS ON THE VAPORIZATION AND CONDENSING SIDES

If series-connected heat exchangers are of the counterflow type they can be reduced to one counterflow heat exchanger /5/, which is furthermore optimized by the method presented in chapter 3, where  $J_v$  and  $J_c$  in equations (13) and (14) are substituted for their reduced values in equations (16) and (17).

$$J_{\nu R} = \frac{J_{\nu 1}}{1 + \sigma_{\nu}}, \qquad (16) \qquad J_{cR} = \frac{J_{c1}}{1 + \sigma_{c}}. \qquad (17)$$

Generally

and for the system in fig. 1  $\sigma_c = \sqrt{\frac{J_{c1}}{J_{cn}}}$ , (19)  $\sigma_v = \sqrt{\frac{J_{v1}}{J_{vn}}}$ . (20)

 $\sigma = \sum_{i=2}^{n} \sqrt{\frac{J_1}{J_i}}$ (18)

Furthermore for the system in fig. 1, the optimal heat capacity flows between heat exchangers are

$$\dot{C}_{c12} = \dot{C}_{c1} \left( 1 + \frac{1}{\sigma_c} \right),$$
 (21)  $\dot{C}_{v12} = \dot{C}_{v1} \left( 1 + \frac{1}{\sigma_v} \right).$  (22)

The heat transfer surface areas  $A_{c1}$  and  $A_{v1}$  in fig. 1 are obtained from equations

$$A_{c1} = \frac{\dot{C}_{c1}}{k_{c1}} (1 + \sigma_{c}) ln \left(\frac{1}{1 - \varepsilon_{cR}}\right), \quad (23) \qquad A_{v1} = \frac{\dot{C}_{v1}}{k_{v1}} (1 + \sigma_{v}) ln \left(\frac{1}{1 - \varepsilon_{vR}}\right). \quad (24)$$

And, finally, economically optimal heat transfer surface areas of the vaporizer and the condenser are

$$A_{cn} = \frac{k_{c1}}{k_{cn}} \sqrt{\frac{J_{c1}}{J_{cn}}} A_{c1}, \qquad (25) \qquad A_{\nu n} = \frac{k_{\nu 1}}{k_{\nu n}} \sqrt{\frac{J_{\nu 1}}{J_{\nu n}}} A_{\nu 1}. \qquad (26)$$

Usually different kinds of parallel fan units with different k and h values are installed to cooled rooms. They have to be reduced to one counterflow heat exchanger for the optimization. The reduced heat exchanger has the same cooling power, same heat capacity flow and same differential of price of the heat transfer surface area as the original fan units. If the fan units are designed so that the ratios between heat capacity flows in each unit are equal and optimal the optimization parameters for one reduced heat exchanger are

$$\dot{C}_{\nu 1} = \sum_{i=1}^{n} \dot{C}_{\nu 1i} , \qquad (27) \qquad \varepsilon_{\nu 1} = \sum_{i=1}^{n} \frac{\dot{C}_{\nu 1i}}{\dot{C}_{\nu 1}} \varepsilon_{\nu 1i} , \qquad (28)$$

$$J_{\nu_{1}} = \left(\sum \frac{\dot{C}_{\nu_{1i}}}{\dot{C}_{\nu_{1}}} \frac{1 - \varepsilon_{\nu_{1}}}{1 - \varepsilon_{\nu_{1i}}} \frac{1 - R_{\nu_{1}}\varepsilon_{\nu_{1i}}}{1 - R_{\nu_{1}}\varepsilon_{\nu_{1i}}} \frac{1}{J_{\nu_{1i}}}\right)^{-1}, \quad (29) \qquad \phi_{\nu_{1}} = \sum_{i=1}^{n} \phi_{\nu_{1i}}. \quad (30)$$

### 5.1 EXAMPLE

The cooling demand for an office building is 400 kW. The rooms are cooled by fan coil units. The number of fan coil units, cooling power, heat capacity flow of air and the ratio of overall heat transfer coefficient and marginal costs with installation costs for each unit are: 47 pc.,  $\phi_{v1}=1$  kW,  $\dot{C}_{v1}=85$  W/K,  $k_{v1}/h_{v1}=0,45$  W/(K US\$); 100 pc., 2,33 kW, 195 W/K, 0,685 W/(K US\$); 20 pc., 4 kW, 335 W/K, 0,905 W/(K US\$); 4 pc., 10 kW, 835 W/K, 1,09 W/(K US\$). Air is cooled in the fan coil unit from the temperature of +27 °C to + 15 °C. The effectivenesses of the fan coil units are equal. The ratio of overall heat transfer coefficient and marginal costs of the water cooler and the air condenser are 0,95 W/(K US\$) and 1,8 W/(K US\$). Air temperature difference in the condenser is 8 K and the inlet temperature of cooling air is 32 °C. The ratio of annual maintenance costs and investment costs in the fan coil units is  $r_{v1}=0,03$ , in the water cooler (vaporizer)  $r_{vn}=0,04$  and in the air condenser  $r_c=0,02$ . The interest rate is 10 % and operating time of the investment 15 years and thus the factor of present value of periodic payment a=12,39.  $\eta=0,55$ . Annual peak-load power time of the compressor is 2500 h. The unit price of electricity is 0,06 US\$/kWh.

## **5.2 SOLUTION**

Economical efficiency of the air condenser, eq. (9)  $J_{cn}=$  1,44 W/US\$K. Vaporizing temperature  $T_{\nu} = T_{r1} - (T_{r1} - T_{r2}) / \varepsilon_{\nu} = 276$  K. Dimensionless optimization parameter of the condenser, eq. (14)  $s_c^*=0,429$ . Optimal effectiveness of the condenser, eq. (15)  $\varepsilon_c=0,592$ . Heat capacity flow of the fan coil units, eq. (27)  $C_{\nu I}$ =33,5 kW/K. Condensing temperature  $T_c = T_{a1} + (T_{a2} - T_{a1}) / \varepsilon_c = 318$  K. Economical efficiency of the fan coil units, eq. (9) and eq. (29)  $J_{v1}$ =0,51 W/US\$ K. Economical efficiency of the water cooler, eq. (9) and eq. (29)  $J_{\nu n}$ =0,64 W/US\$ K. Dimensionless economy number of the vaporizer, eq. (20)  $\sigma_{\nu}$ =0,90. Reduced economical efficiency of the vaporizer, eq. (16)  $J_{vr}=0,27$  W/US\$ K. COP, eq. (5) COP=3,57.  $\phi_v / \dot{C}_c = (\phi_c / \dot{C}_c)(COP / (1 + COP)) = 6,25$  K. Dimensionless optimization parameter of the vaporizer, eq. (13)  $s_{\nu}^* = 1,17$ . Reduced effectiveness of the vaporizer eq.(15)  $\varepsilon_{vr}$ =0,76. The second iteration round:  $T_v$ =284 K;  $\varepsilon_c$ =0,59;  $T_c$ =318 K;  $\varepsilon_v$ =0,52. The third iteration round.  $T_v=277$  K;  $\varepsilon_c=0.59$ ;  $T_c=319$  K;  $\varepsilon_v=0.53$ . The fourth iteration round:  $T_v=277$ K;  $\varepsilon_c=0.59$ ;  $T_c=319$  K;  $\varepsilon_v=0.53$ . The heat capacity flow between the water cooler and fan coil units, eq. (22)  $\dot{C}_{\nu 12}$  =70,9 kW/K Mass flow of water  $q_m = \dot{C}_{\nu 12}/c_{pv}$  =16,9 kg/s. The sum of the fan coil unit conductances, eq. (24)  $G_{v1} = k_{v1}A_{v1} = 47,5 \text{ kW/K}$ , which consists of 47 pc. 0,119 kW/K, 100 pc. 0,276 kW/K, 20 pc. 0,475 kW/K and 4 pc. 1,19 kW/K. The conductance of the vaporizer of the water cooler, eq. (26)  $G_{vn}$ =42,6 kW/K. The conductance of condenser, eq. (2)  $G_c = k_c A_c = 188 \text{ kW/K}$ .

The sensitivities of effectivenesses  $\varepsilon_{\nu}$  and  $\varepsilon_{c}$  and logarithmic mean temperature differences  $\theta_{\text{lnv}}$ , eq. (33) and  $\theta_{\text{lnc}}$ , eq. (34) are presented in fig. 3 and fig. 4 as a function of the ratio of economic numbers  $S_{\nu}/S_{\nu0}$ , eq. (31) and  $S_{c}/S_{c0}$ , eq. (32) and efficiency  $\eta$ .

$$S_{\nu} = 0.5 (T_{r1} - T_{r2})^2 tea J_{\nu R}, \quad (31) \qquad S_c = 0.5 (T_{a2} - T_{a1})^2 tea J_c, \quad (32)$$



Figure 3. Sensitivity of effectivenesses



## 6. **DISCUSSION**

Ratio between the lifetime electricity costs and the marginal costs of the heat transfer areas are described by the optimization parameters  $s_{\nu}^*$  and  $s_c^*$ . The bigger the optimization parameters  $s^*$ , the bigger the economically optimal effectiveness of the heat exchanger. Annual operating time, price of electricity, interest rate of investment and operating life of the cooling system vary considerably in practice. Therefore economic design parameters should be chosen optimically especially for big cooling systems.

The optimization equations are solved by iteration. Iteration converges rapidly, and usually only four calculation rounds are needed, if  $\varepsilon_v = \varepsilon_c = 0.6$  is the first approximation in the beginning.

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