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(Title)

PROBABILISTIC MODEL OF HEAT LOSS THROUGH THE BUILDING ENVELOPE

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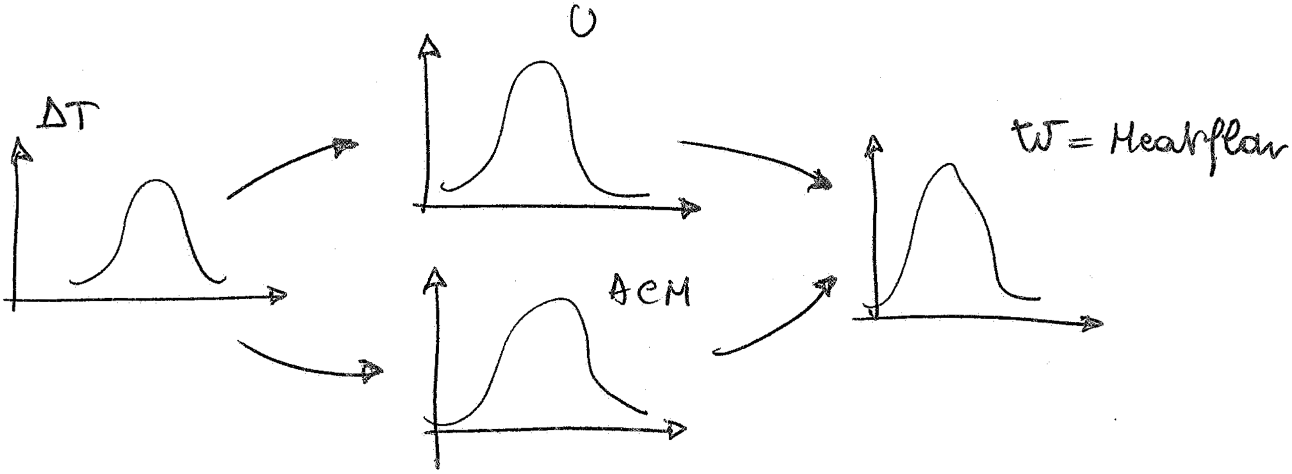

SYNOPSIS

A model for the application of probabilistic methods in the estimation of heat loss caused by convection and heat conduction through the material is developed. Temperature difference (ΔT) between inside and outside of a building, air change rate (ACH) and coefficient of thermal transmittance (U-value) of the building structure are treated as random variables. The mean value and the standard deviation of heat loss are estimated for different parameters of distributions for temperature difference, air change rate and thermal transmittance. The interaction between the above three quantities is described for different cases of tight and leaky envelopes by assuming certain degree of correlation between the random variables.

1. INTRODUCTION

The total heat loss in a building is generally calculated by adding the contributions from the heat loss due to natural ventilation and transmission through the materials, where the material properties, thermal transmittance and convection are treated as deterministic quantities. Influence of variation of leakage properties of the building, climatic conditions and the material properties is not catered for in the assessment of heat loss.
Thermal performance of a building is associated with two main quantities selected from several parameters of interest, which can be defined as:

heat loss due to air flow through the construction
heat loss due to transmission

The heat loss due to natural ventilation and transmission through the building structure is determined in a complex interaction, by local climatic conditions and the factors related to the building and their surroundings. The heat loss due to transmission is governed by the temperature difference across the envelope, properties of material and the type and quality of construction. Air infiltration is governed by the pressure difference due to wind and temperature and the leakage characteristics of the house. Examination of the effects of air infiltration and heat transmission on the thermal performance of the building indicates that these are interdependent and climate is the common parameter, which is random in space and time.

2. DETERMINISTIC MODEL FOR HEAT LOSSES IN A BUILDING

2.1 Heat loss due to ventilation

Heat loss caused by ventilation or infiltration (called later ventilation heat loss) forms a significant part of the energy consumption in buildings. To keep the internal temperature at a constant level, fresh air supplied to the house must be heated. The common measure of the amount of exchange of air is air change rate (ACH).
The energy needed for heating of the supplied air can be described by following equation:

$$W_v = \Delta T \rho c V \frac{ACH}{3600} \tag{2.1}$$

where

W_v - heat loss (kW)
ΔT - temperature difference between inside (T_{int}) and outside (T_{ext}) the building (K)
V - volume of the house (m^3)
ρ - air density (kg/m^3)
c - specific thermal capacity (kJ/kgK)

2.2 Heat loss due to transmission

Transmission loss through the building envelope is the sum of heat losses through the building components and can be described by the following equation [3]:

$$W_c = \Delta T \frac{1}{1000} \sum_{i=1}^{m} Nu_i U_i A_i \tag{2.2}$$

where:

W_c - transmission heat loss (kW)
ΔT - temperature difference between inside and outside the building (K)
m - number of building components
U_i - overall average thermal transmittance of i:th component (W/m^2K)
A_i - area of the i:th component (m^2)
Nu_i - Nusselt's number for the i:th component describing the effect of convection flows (leakage and interstitial convection) on the thermal performance of a structure

In order to include the changes in the average thermal transmittance of the components caused by the influence of the convective flow on conduction heat losses, one can define the overall average thermal transmittance of a building envelope U_m as:

$$U_m = \frac{1}{A_m} \sum_{i=1}^{m} Nu_i U_i A_i \tag{2.3}$$

where: A_m - Area of the building envelope (m^2)

Equation 2.2 can be rewritten by including equation 2.3 as:

$$W_c = \Delta T \; U_m \; A_m / 1000 \tag{2.4}$$

3. PROBABILISTIC MODEL OF HEAT LOSSES THROUGH THE BUILDING ENVELOPE

A probabilistic model for the estimation of heat loss is developed by including the variations in the temperature difference ΔT, air change rate ACH and average heat transfer coefficient

U. Influence of variation in the climatic parameters and the material properties on the heat loss is studied for two types of buildings with tight and leaky envelope. It is assumed in the analysis that description of permeability of an envelope is related to the outer surface of the building. Total heat loss in a building at a time t can be described as a sum of ventilation and transmission heat losses:

$$W(t) = W_V(t) + W_C(t) \qquad (3.1)$$

where:

$W(t)$ - total heat loss in a building (kW)
$W_V(t)$ - component due to ventilation heat loss (kW)
$W_C(t)$ - component due to transmission heat loss (kW)

Substituting expressions 2.1 and 2.4 into 3.1 gives:

$$W(t) = \frac{\rho c V}{3600} \Delta T(t) ACH(t) + \frac{Am}{1000} \Delta T(t) Um(t) \qquad (3.2)$$

The main assumption is to include the randomness of the thermal transmittance and the climate in the model. The model should enable the estimation of the mean value and the standard deviation of the heat loss for the assumed statistical parameters of distributions of particular random variables. Four parameters from equation 3.2 are considered to vary with time.

Define

$\Delta T(t)$ = $X(t)$ inside-outside temperature difference
$ACH(t)$= $Y(t)$ air change rate
$U_m(t)$ = $Z(t)$ average overall thermal transmittance
$W(t)$ ventilation and transmission heat loss in a building

For a constant value of internal temperature of 20°C the air density becomes also constant. Equation 3.2 can now be written as:

$$W(t) = g(X,Y,Z) = a\, X(t)\, Y(t) + b\, X(t)\, Z(t) \qquad (3.3)$$

where

$$a = V\, c\, \rho \,/\, 3600 \qquad (3.4)$$

$$b = A_m \,/\, 1000 \qquad (3.5)$$

It is important to emphasis that the temperature difference across the building envelope is a common parameter for both air change rate and the thermal transmittance. Air infiltration is governed by the pressure difference due to wind and temperature while thermal transmittance depends on the temperature of the material and the leakage paths through the insulation Approximate mean and variance of the function W may be obtained by expanding g(X,Y,Z) in a Taylor series about the mean values of the variables X, Y and Z. For practical reasons the first-order approximations for mean μ_W as well as for variance σ_w^2 of the function W have been used.

$$\mu_W = a\mu_X\mu_y + b\mu_X\mu_Z \qquad (3.6)$$

$$\sigma_W^2 = (a\mu_y + b\mu_z)^2\sigma_x^2 + (a\mu_x)^2\sigma_y^2 + (b\mu_x)^2\sigma_z^2$$
$$+2(a\mu_y + b\mu_z)a\mu_x\rho_{xy}\sigma_x\sigma_y$$
$$+2(a\mu_y + b\mu_z)b\mu_x\rho_{xz}\sigma_x\sigma_z + 2ab\mu_x^2\rho_{yz}\sigma_y\sigma_z \qquad (3.7)$$

where:

μ - mean value
σ - standard deviation
ρ - correlation coefficient

Assume that the mean values of X, Y, Z are the design values and the variations from the mean are caused by the influence of random parameters (climate etc.). It has been found [4] that for a naturally ventilated house with the uniformly distributed leakage over the building envelope, the distribution of air change rate can be defined by Normal probability density function. Temperature difference [4] and thermal transmittance are also assumed as normally distributed. A joint normal distribution function for the variables X,Y and Z has been applied in the model. The assumption of Normal probability density function for the heat losses has been verified for all cases of tight and leaky envelopes by comparing the statistical moments obtained by two different methods [2]. Normal distribution is found to be reasonable for a tight building with balanced or exhaust ventilation. However , for a leaky building, a slightly skewed distribution function gives a better fit.

Correlation coefficients among variables X, Y, Z show linear dependence between them and they are specific for individual case (tight envelope, infiltration, exfiltration).
The coefficient of partial correlation of air change rate and thermal transmittance, if the influence of temperature difference is eliminated, is of the form [1]:

$$\rho_{yz.x} = \frac{\rho_{yz}-\rho_{xy}\rho_{xz}}{\sqrt{(1-\rho_{xy}^2)(1-\rho_{xz}^2)}} \qquad (3.8)$$

For a tight building the coefficient of partial correlation $\rho_{yz.x}$ is equal to zero. It is negative for the case of infiltration of the cold air entering the building and positive for the case of exfiltration of the warm air out of the building.
The range of values of the correlation coefficient ρ_{yz} can be estimated from the knowledge of ρ_{xy}, ρ_{xz}, and $\rho_{yz.x}$. Limits for ρ_{yz} are presented in table 1.

1	Tight Envelop	$\rho_{yz.x} = 0$	$\rho_{xy}>0.\ \rho_{xz}<0.\ \rho_{yz}<0.$ $\rho_{yz}=\rho_{xy}\rho_{xz}$
2	Leaky Envelope Infiltration	$\rho_{yz.x} < 0$	$\rho_{xy}>0.\ \rho_{xz}<0.\ \rho_{yz}<0.$ $\rho_{xy}\rho_{xz} - \sqrt{(1-\rho_{xy}^2)(1-\rho_{xz}^2)} \le \rho_{yz} < \rho_{xy}\rho_{xz}$
3	Leaky Envelope Exfiltration	$\rho_{yz.x} > 0$	$\rho_{xy}>0.\ \rho_{xz}>0.\ \rho_{yz}>0$ $\rho_{xy}\rho_{xz} \le \rho_{yz} \le \rho_{xy}\rho_{xz} + \sqrt{(1-\rho_{xy}^2)(1-\rho_{xz}^2)}$

Table 1: The range of values assumed for the correlation coefficients among random variables X, Y and Z

4. SENSITIVITY ANALYSIS OF HEAT LOSS IN A BUILDING

Sensitivity analysis of coefficient of variation of heat losses is carried out by considering certain ranges of values of the parameters ρ_{xy}, ρ_{xz}, ρ_{yz}, I_x, I_y, I_z and are estimated for the five cases shown in table 2

The coefficient of variation of heat loss is given by:

$$I_w = \sqrt{(A + B + C)} / (a + b\, s) \tag{4.1}$$

where $s = \mu_z/\mu_y$.

$A = (a + b\, s)^2 I_x^2$

$B = 2\,(a + b\, s)\, I_x\, [a\, I_y\, \rho_{xy} + b\, s\, I_z\, \rho_{xz}]$

$C = (a^2 I_y^2 + 2\, a\, b\, s\, I_y\, I_z\, \rho_{yz} + b^2 s^2 I_z^2)$

Case No.	*Building Type*	*Design Conditions*
1	Tight building	ACH=0, *Only transmission heat losses are considered and is a purely theoretical case.*
2	Tight building	ACH=const, *Balanced ventilation is provided by mechanically operated inlets and outlets.*
3	Tight building	ACH≠const , *Exhaust ventilation is provided by mechanically operated outlets. The air inlets are specified openings through ducts.*
4	Leaky building	*(a) Infiltration is provided through the building envelope and the outlets are placed above the neutral pressure layer (Nu = 1.0)* *(b) Infiltration is provided through the building envelope and the mechanical outlets. (Nu = 0.8)*
5	Leaky building	*(a) Exfiltration is provided through the building envelope and the inlets are placed below the neutral pressure layer. (Nu = 1.0)* *(b) Exfiltration is provided through the building envelope and the inlets are placed below the neutral pressure layer (Nu = 1.2)*

Table 2: Cases considered in the Analysis

Figure 1 shows the above fives cases considered in the analysis.

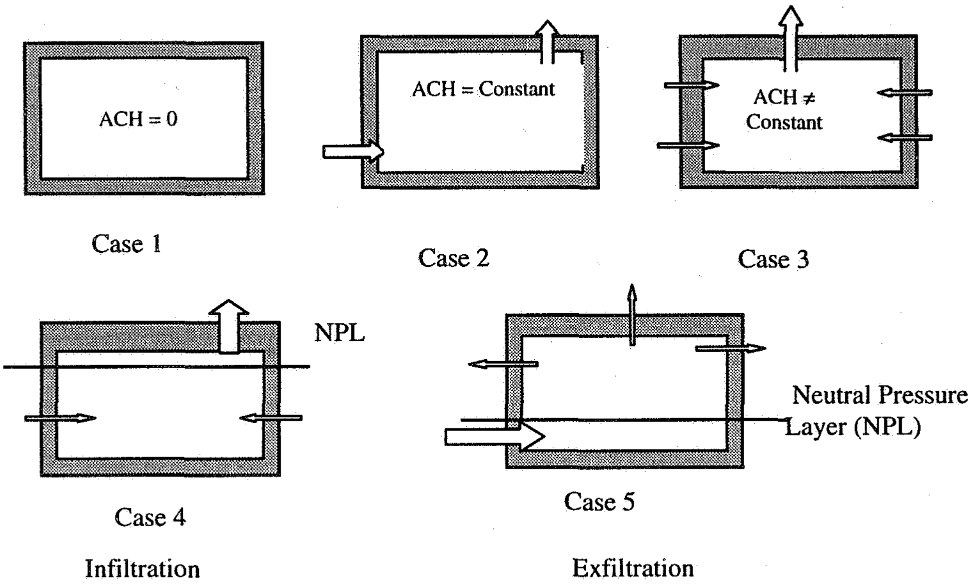

Figure 1 *Illustration for different cases.*

The influence of different parameters on the coefficient of variation of heat loss I_W is investigated for tight and leaky envelope, and table 3 presents the results of the investigation for a single family house.

	Case 1	*Case 2*	*Case 3*	*Case 4 (a)*	*Case 4 (b)*	*Case 5 (a)*	*Case 5 (b)*
Nu	-	-	-	1.0	0.8	1.0	1.2
I_x	0.25	0.25	0.25	0.25	0.25	0.25	0.25
I_y	0.00	0.00	0.01	0.25	0.01	0.25	0.25
I_z	0.025	0.025	0.025	0.05	0.10	0.05	0.10
ρ_{xy}	0.0	0.0	0.1	0.5	0.08	0.5	0.5
ρ_{xz}	-0.9	-0.9	-0.9	-0.9	-0.9	0.5	0.5
ρ_{yz}	0.0	0.0	-0.09	-0.5	-0.5	0.5	0.9
μ_w	1.32	3.60	3.60	3.60	3.33	3.60	3.86
σ_w	0.30	0.87	0.87	1.23	0.72	1.36	1.44
I_w	*0.23*	*0.24*	*0.24*	*0.34*	*0.22*	*0.38*	*0.37*

Table 3 *Estimation of transmission and ventilation heat flow in a single family house.*

Data for the numerical example used in the calculation is as follows.

Area of the house $A = 413\ m^2$, *Volume of the house* $V = 819\ m^3$
Specific thermal capacity $c = 1.0\ kJ / kg\ K$ *Air density* $\rho = 1.25\ kg / m^3$

Examination of the table shows that the influence of temperature is the governing factor in all cases. It should be noted that the influence of wind velocity is included in the evaluation of air change rate. Case 1 is a hypothetical case without any practical relevance. It is used as a reference case for transmission heat losses without the influence of ACH: Case 2 represents the influence of mechanically operated inlets and outlets. It can be noted that the coefficient of variation of the heat loss is approximately equal to the coefficient of variation of temperature difference and the influence of correlation is insignificant. The third case is similar to case 2 and again the variations in temperature are dominant.

Cases 4 and 5 are for the leaky buildings where the location of neutral pressure layer (NPL) is considered in the analysis. There can be infiltration or exfiltration through the envelope of the structure depending on the height of the NPL. Influence of Nusslet´s number (Nu) is also considered in cases 4 and 5.

Comparison of probability density function for different cases were studied. Figure 2 shows two cases corresponding to tight building with balanced ventilation (Case 2) and leaky building with exfiltration (Case 5 a). The dotted lines indicate the results obtained by using normal density function and the solid lines show the results based on first order reliability model (FORM) as given in reference 2. It can be noted that normal distribution gives a good fit for the tight building while the probability density function is skewed to the right for the leaky building. However, assumption of normal distribution is reasonable even for leaky structures and will be used in further investigations.

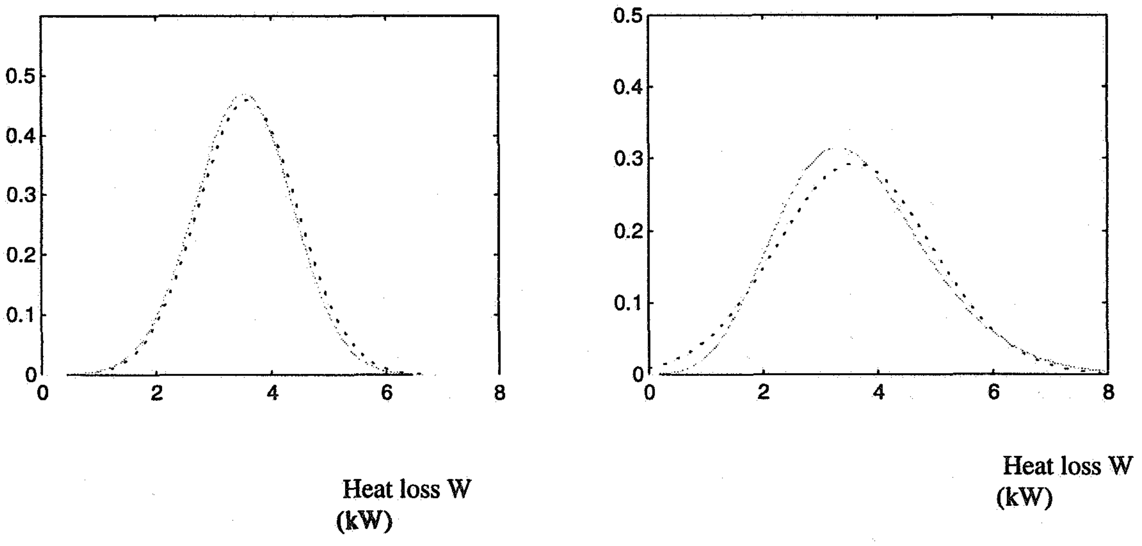

Figure 2 *Probability density functions for heat loss for tight and leaky building.*
(Cases 2 and 5)

5. CONCLUSIONS

Application of probabilistic model for estimation the heat losses in a building gives us the possibility of taking into account the interactions between ventilation and transmission heat losses. The main conclusions from the work are:

1. The model calculates the first two moments (mean value, standard deviation and correlation) for the heat transmittance coefficient, air change rate and the temperature variations over a specified period of time. It is found that Normal density function is reasonably representative of the heat loss caused by conduction and ventilation.
2. Influence of variations in temperature and air change rate is important for calculating the coefficient of variation of heat loss.
3. Influence of variations in the transmittance properties of the material is not significant for this particular example. This may be due to the fact that small variations in U are assumed in the model.
4. Influence of Nusselt´s number on the total variation in the heat loss is also non-significant due to small mean temperature differences and quasi static heat transmittance coefficient.
5. The analysis have shown that correlation between temperature and thermal transmittance does not have any significant influence, while correlation between temperature difference and air change rate is important and should be considered in the calculations.
6. Further work is needed to develop a model which considers the non-linearities inherent in the analytical model and examine their influence on the total heat loss.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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