VENTILATION AND COOLING

18 TH ANNUAL AIVC CONFERENCE
ATHENS, GREECE, 23-26 SEPTEMBER, 1997

Methode die sneller gaat dan CFD, maar minder precies ook !!

Ergens tussen CFD en nodenetwork

Zonal models : presentation and proposal of new expression of balance equations applied to the study of air flow and heat transfer in buildings.

MUSY M.*, WURTZ E.*, NATAF J.M.✧

* L.E.P.T.A.B. , Université de La Rochelle, 17042 La Rochelle, FRANCE.
✧ S.G.R. L.N.B.L., University of California, Berkeley 94720, U.S.A.

SYNOPSIS

This paper presents an analysis of different possibilities of representing mass transfers in zonal models.
In this aim, formulations derived from the Navier-Stokes equations or from Euler's theorem are obtained. The models which result from them and empirical models are compared so that to define the best compromise between simplicity, accuracy and easy convergence.

LIST OF SYMBOLS :

a : coefficient (s^{-2})
b : coefficient (s^{-2})
C : empirical constant ($m/s^1 Pa^{1/2}$)
$\vec{F}_S$: surface forces (N)
$\vec{F}_V$: volume forces (N)
g : gravitation constant (m/s^2)
h : height of the cell (m)
h_c : convection transfer coefficient (W/m^2K^1)
$\vec{i}$: unit vector relative to the axe X
$\vec{k}$: unit vector relative to the axe Z
l : width of the cell (m)
n : empirical exponent (-)
$\vec{n}$: perpendicular unit vector
P : air pressure (Pa)
qm : mass flow rate (kg/s)
r : air molar constant (m^2/s^2K)
S : surface (m^2)
T : air temperature (K)
u : velocity along the axe X (m/s)
$\vec{V}$: velocity vector (m/s)
w : velocity along the axe Z (m/s)
X : horizontal axe
Z : vertical axe
ΔP : pressure difference (Pa)
ρ :air density (kg/m^3)

Subscripts :

0 : centre
Bottom : bottom neighbour
i : studied cell
North : north neighbour
South : south neighbour
Top : top neighbour

1. INTRODUCTION

The zonal method is a simplified tool which allows to study the air flow and heat transfers in buildings. Intermediate between one-node models, which results do not permit to predict accurately the thermal comfort or the air quality in a local, and CFD models, which are very slow and require large amount of memory, especially three dimensional, this approach is based on the partitioning of a room or group of rooms into a small number of sub-zones or cells. In these cells, energy and mass balance apply while the exchanges between cells are described at their interfaces.

In our study, SPARK environment is used to develop the zonal model. SPARK is based on object oriented environment and is designed to solve large systems of non-linear equations. The modularity of SPARK permit to test successively the different models without having to rebuild the whole simulation each time.

2. REDUCTION OF THE NAVIER-STOKES EQUATIONS

To make our approach clearer, we will carry out our calculations in two-dimensional cases only.
The purpose of this part is the description of mass flows which occur to the interfaces of a standard cell in a local (cell « i » in figure 2.1).

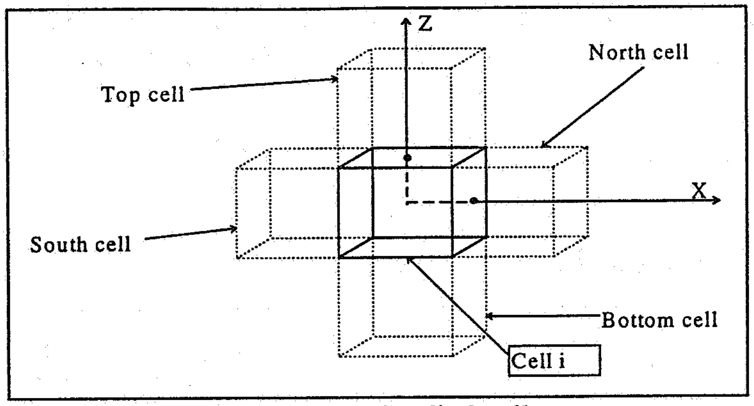

Figure 2.1 : Studied cell.

2.1. REDUCTION OF THE EQUATIONS

In steady state, the Navier-Stokes equations combined to the mass conservation equation in two-dimensional Cartesian coordinates (X,Z), applied to the air of a local, can be expressed as follows :

$$\begin{cases} -\dfrac{1}{\rho}\dfrac{\partial P}{\partial x} = u\dfrac{\partial u}{\partial x} + w\dfrac{\partial u}{\partial z} \\ g - \dfrac{1}{\rho}\dfrac{\partial P}{\partial z} = u\dfrac{\partial w}{\partial x} + w\dfrac{\partial w}{\partial z} \end{cases}$$

In these equations, air is assumed as an inviscid flow, only submitted to gravity forces.
To describe the mass flows that cross the frontiers of the cell i that are perpendicular to the axe X, the first Navier-Stokes simplified equation will be studied. In this equation, w is assumed to be equal to zero, it means that flow lines are considered parallel to the axe X. Furthermore, perfect gas law is supposed to apply to the air. The first Navier-Stokes equation becomes :

$$-\frac{rT}{P}\frac{\partial P}{\partial x} = u\frac{\partial u}{\partial x}.$$

By integrating this equation between the frontiers that separate the « cell i » from the « South cell » and the « North cell », with considering that temperature is homogeneous in the cell i and equal to à T_i, the following equation is obtained :

$$-rT_i \ln\left(\frac{P\left(\frac{l}{2},Z\right)}{P\left(-\frac{l}{2},Z\right)}\right) = \frac{1}{2}\left(u^2\left(\frac{l}{2},Z\right) - u^2\left(-\frac{l}{2},Z\right)\right).$$

Where P(l/2, Z) (respectively P(-l/2,Z)) is the pressure to the points included in the North (South) frontier which ordinate is Z.
The pressure is supposed to be hydrostatic in the cell and because of this in the frontiers too :

$$\begin{cases} P\left(\frac{l}{2},Z\right) = P_0\left(\frac{l}{2}\right) - \rho_i g Z \quad , \\ P\left(-\frac{l}{2},Z\right) = P_0\left(-\frac{l}{2}\right) - \rho_i g Z \end{cases}$$

where $P_0(-l/2)$ and $P_0(l/2)$ are the pressures in the centre of the frontiers.

Air density has been assumed constant and equal to ρ_i in the cell. This can be justified by the fact that it varies more with temperature (assumed to be constant) than with pressure.

Introducing the new relations $\begin{cases} P_0\left(\frac{l}{2}\right) = \frac{1}{2}\left(P_{0North} + P_{0i}\right) \\ P_0\left(-\frac{l}{2}\right) = \frac{1}{2}\left(P_{0South} + P_{0i}\right) \end{cases}$, where P_{0North}, P_{0South} and P_{0i} are respectively the pressures in the centre of the North, South and i cells, the final equation is :

$$u^2\left(\frac{l}{2}, Z\right) = u^2\left(-\frac{l}{2}, Z\right) - 2rT_i \ln\left(\frac{\frac{1}{2}\left(P_{0North} + P_{0i}\right) - \rho_i g Z}{\frac{1}{2}\left(P_{0South} + P_{0i}\right) - \rho_i g Z}\right). \quad (1)$$

Knowing the velocity profile on the south frontier (X=-l/2) of the cell and the centre pressure of North, South and i cells, the velocity of the flow can be calculated in every point of the North frontier. The boundary conditions permit to solve the problem. Once the velocity profiles evaluated, flow rates to the interfaces can be determined.
A similar study allows to establish the equation for Z direction :

$$w^2\left(\frac{h}{2}\right) = w^2\left(-\frac{h}{2}\right) - 2gh - 2rT_i \ln\left(\frac{\left(P_{0Top} + P_{0i}\right)}{\left(P_{0Bottom} + P_{0i}\right)}\right). \quad (2)$$

2.2. MODEL

The discretization of the vertical interfaces is made with a small number of iso-altitude on which velocity is calculated. The velocity profiles are then approximated by linearization and the mass flows calculated from these profiles.
At the frontier separating a cell and a wall, the mass flow is set to zero (impermeable wall).

2.3. RESULTS AND DISCUSSION

This model did not permit to obtain 2D or 3D convergent simulations, but gives good results in trivial 1D cases as those presented in figure 2.2.

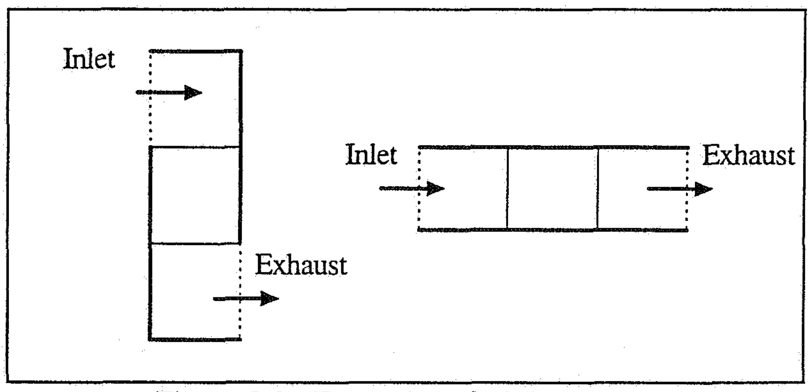

Figure 2.2 : Examples of trivial cases.

The non convergence in 2D cases, encouraged us to re-examine the equations 1 and 2 so that to replace the logarithmic expression with a simpler expression.
In equation the logarithmic term can also be written as follows :

$$-2rT_i \ln\left(\frac{\frac{1}{2}(P_{0North}+P_{0i})-\rho_i gZ}{\frac{1}{2}(P_{0South}+P_{0i})-\rho_i gZ}\right) = -2rT_i\left(\ln\left(1+\frac{(P_{0i}-P_{0South})-2\rho_i gZ}{(P_{0North}+P_{0South})}\right)-\ln\left(1+\frac{(P_{0i}-P_{0North})-2\rho_i gZ}{(P_{0North}+P_{0South})}\right)\right)$$

The terms $\frac{(P_{0i}-P_{0South})-2\rho_i gZ}{(P_{0North}+P_{0South})}$ et $\frac{(P_{0i}-P_{0North})-2\rho_i gZ}{(P_{0North}+P_{0South})}$ are small behind 1, what permits to simplify the expression :

$$-2rT_i \ln\left(\frac{\frac{1}{2}(P_{0North}+P_{0i})-\rho_i gZ}{\frac{1}{2}(P_{0South}+P_{0i})-\rho_i gZ}\right) \approx -2rT_i\left(\frac{(P_{0i}-P_{0South})-2\rho_i gZ}{P_{0North}+P_{0South}}-\frac{(P_{0i}-P_{0North})-2\rho_i gZ}{P_{0North}+P_{0South}}\right).$$

$$\approx -2rT_i\left(\frac{P_{0North}-P_{0South}}{P_{0North}+P_{0South}}\right)$$

Admitting that $P_{0North}+P_{0South}\approx 2P_{0i}$, and using perfect gas law, a simple expression is obtained :

$$-2rT_i \ln\left(\frac{\frac{1}{2}(P_{0North}+P_{0i})-\rho_i gZ}{\frac{1}{2}(P_{0South}+P_{0i})-\rho_i gZ}\right) \approx -\rho_i\left(P_{0North}-P_{0South}\right).$$

This expression included in equation 1 gives :

$$\boxed{u^2\left(\frac{l}{2},Z\right)=u^2\left(-\frac{l}{2},Z\right)-\rho_i\left(P_{0North}-P_{0South}\right)}.$$

This is Bernoulli equation applicable along a horizontal flow line. The reduction of equation (2) leads to Bernoulli equation applicable along a vertical flow line.
These equations are therefore applicable to 1-D for which the flow lines that cross South (or Bottom) interface cross North (or Top) interface too. But this becomes false in 2-D and 3-D cases. It probably explains why the simulations did not converge.

3. EULER THEOREM APPLICATION
3.1. EQUATIONS DEVELOPMENT

In steady state, the expression of Euler's theorem is : $\iint_S (\rho\vec{V})(\vec{V}.\vec{n})dS=[\vec{F}_V]+[\vec{F}_S]$.

Applied in 2-D to the studied cell (figure 2.1), and projected onto X and Z axes it leads to :

$$\begin{cases}\iint_{South\_frontier}-\rho u^2 dS+\iint_{North\_frontier}\rho u^2 dS+\iint_{Bottom\_frontier}-\rho wu dS+\iint_{Top\_frontier}\rho wu dS=[\vec{F}_V.\vec{i}]+[\vec{F}_S.\vec{i}]\\ \iint_{South\_frontier}-\rho uw dS+\iint_{North\_frontier}\rho uw dS+\iint_{Bottom\_frontier}-\rho w^2 dS+\iint_{Top\_frontier}\rho w^2 dS=[\vec{F}_V.\vec{k}]+[\vec{F}_S.\vec{k}]\end{cases}.$$

Admitting that the force of gravity is the only volume force and surface pressures the only surface forces that act on the cell, the equations become :

$$\begin{cases} \iint\limits_{South\_frontier} -\rho u^2 dS + \iint\limits_{North\_frontier} \rho u^2 dS + \iint\limits_{Bottom\_frontier} -\rho wudS + \iint\limits_{Top\_frontier} \rho wudS = \iint\limits_{South\_frontier} -PdS + \iint\limits_{North\_frontier} PdS \\ \iint\limits_{South\_frontier} -\rho uwdS + \iint\limits_{North\_frontier} \rho uwdS + \iint\limits_{Bottom\_frontier} -\rho w^2 dS + \iint\limits_{Top\_frontier} \rho w^2 dS = \iint\limits_{Bottom\_frontier} -PdS + \iint\limits_{Top\_frontier} PdS + \iiint\limits_{cell\_i} \rho g dV \end{cases}$$

.(3)

3.2. APPLICATION TO A FLOW

To evaluate the integrals contained in equation (3), hypothesis on velocity profiles must be done. They are supposed to be plan :

$$\begin{cases} u_{South\_frontier}(Z) = u_{0\_South} + a_{South}Z \\ u_{North\_frontier}(Z) = u_{0\_North} + a_{North}Z \\ u_{Bottom\_frontier}(X) = u_{0\_Bottom} + a_{Bottom}X \\ u_{Top\_frontier}(X) = u_{0\_Top} + a_{Top}X \end{cases} \qquad \begin{cases} w_{South\_frontier}(Z) = w_{0\_South} + b_{South}X \\ w_{North\_frontier}(Z) = w_{0\_North} + b_{North}X \\ w_{Bottom\_frontier}(X) = w_{0\_Bottom} + b_{Bottom}X \\ w_{Top\_frontier}(X) = w_{0\_Top} + b_{Top}X \end{cases}.$$

In the interfaces, the air density is considered homogeneous and the pressure hydrostatic :

$$\begin{cases} -\rho_{South}\left(u_{0\_South}^2 + \frac{a_{South}^2 h^2}{12}\right) + \rho_{North}\left(u_{0\_North}^2 + \frac{a_{North}^2 h^2}{12}\right) - \rho_{Bottom}\left(w_{0\_Bottom}u_{0\_Bottom} + \frac{b_{Bottom}a_{Bottom}l^2}{12}\right) \\ + \rho_{Top}\left(w_{0\_Top}u_{0\_Top} + \frac{b_{Top}a_{Top}l^2}{12}\right) = -P_0\left(-\frac{l}{2}\right) + P_0\left(\frac{l}{2}\right) \\ -\rho_{Bottom}\left(u_{0\_Bottom}^2 + \frac{a_{Bottom}^2 l^2}{12}\right) + \rho_{Top}\left(u_{0\_Top}^2 + \frac{a_{Top}^2 l^2}{12}\right) - \rho_{South}\left(w_{0\_South}u_{0\_South} + \frac{b_{South}a_{South}h^2}{12}\right) \\ + \rho_{North}\left(w_{0\_North}u_{0\_North} + \frac{b_{North}a_{North}h^2}{12}\right) = -P_0\left(-\frac{h}{2}\right) + P_0\left(\frac{h}{2}\right) + \rho_i g V \end{cases}$$

Four unknowns per interface have been introduced while only two equations per cell have been obtained. Continuity equations between interfaces are needed.

3.3. CONCLUSION

This method leads to the introduction of many supplementary unknowns, (4 per interface in 2-D cases and 9 per interface in 3-D cases). The resolution will be all the slower. Furthermore, writing continuity equation between interface involves the addition in the model of new macro-objects linking the interfaces. These are the reasons why, for the moment, this method has been put aside.

4. EMPIRICAL MODEL

4.1. DESCRIPTION OF ZONAL MODEL

In the zonal model studied by E. WURTZ [5], mass exchanges between cells are calculated from the equation of flows across large enclosures K. LIMAM et al.[2] :

$q_m = \int_S C\rho(\Delta P)^n dS$ where c and n are empirical coefficients.

E. WURTZ [5] has shown that with C equal to 0.83 and n equal to 0.5 where the flow is turbulent (usual case) and equal to 1 where the flow is laminar (for example when crossing a permeable wall), the zonal model gives results similar to those obtained with FLUENT.

4.2. ADVANTAGES AND LIMITS

The aforementioned method yielded good results for certain typical physical configurations (openings, cases for which flow is easily predictable) but falls short in particular in the case of natural convection. It is thus quite hard to demonstrate a thermally stratified problem, and impossible to represent properly a decelerated flow.
Furthermore, it is sometimes difficult to obtain the convergence of the simulation. This must be due to the coefficient n=0.5 that makes mass exchange equation non-linear.

4.3. REDUCTION OF THE MODEL

Pressure differences that occur in rooms are very small. As shown in figure 4.1, the function $f(\Delta P)=0.83*\Delta P^{1/2}$, separated in three intervals can be approximated by linear functions :

- $f_1(\Delta P)=0.2*\Delta P-0.75$ if $\Delta P\in[-10,-0.75]$,
- $f_2(\Delta P)=1.2*\Delta P$ if $\Delta P\in[-0.75,0.75]$,
- $f_3(\Delta P)=0.2*\Delta P+0.75$ if $\Delta P\in[0.75,10]$.

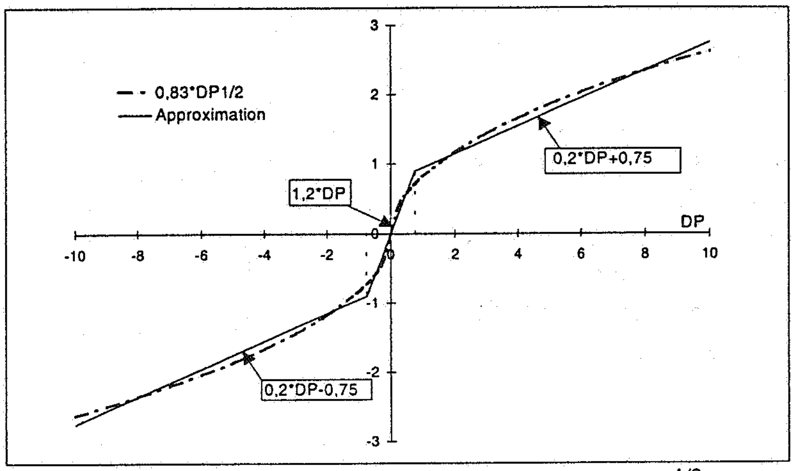

Figure 4.1 : Approximation of $f(\Delta P)=0.83*\Delta P^{1/2}$.

5. RESULTS
5.1. PRESENTATION OF SPARK

The Simulation Problem Analysis and Research Kernel (SPARK) is a modular environment that automates writing code for systems of non-linear equations. It was developed for building science but is applicable to other fields. First written for steady state problems, (J.L. ANDERSON [1]), it has been extended to handle transient problem by the addition of time integrator objects (E.F. SOWELL and al. [4]).
As TRNSYS, CLIM2000 and Allan Simulation, SPARK allows the user to build complex simulations by connecting smaller elements that can be objects (single equations) or macro-objects (equations subsystems).
Objects are automatically generated from equations expressed symbolically (J.M. NATAF and F. WINKELMANN [3])
SPARK use the graph-theoretic techniques to reduce the size of the equations system so that SPARK's Newton-Raphson solver words on the reduced equations set and, after convergence, the remaining unknowns are solved for.
The output is a C program that is automatically compiled and executed.

To build zonal models in SARK environment, two main object classes are created, they correspond to the cells and the interfaces between cells.

The cell class consists of the balance equations for the cell, the pressure drop equation and the perfect gas law while the interface class consists of the mass and energy flow calculations. These classes are used as many time as necessary to define the simulation and linked in the connection file.

5.2. COMPARISON ZONAL MODEL - SIMPLIFIED MODEL

Empirical zonal model and simplified one are studied from the simple configuration shown in figure 5.1 (ventilated room with heating floor (301K) and exterior wall (288K). The cells are 0,8m side squares. The results (flow rates, temperature density and pressure are represented in figure 5.2 and figure 5.3.

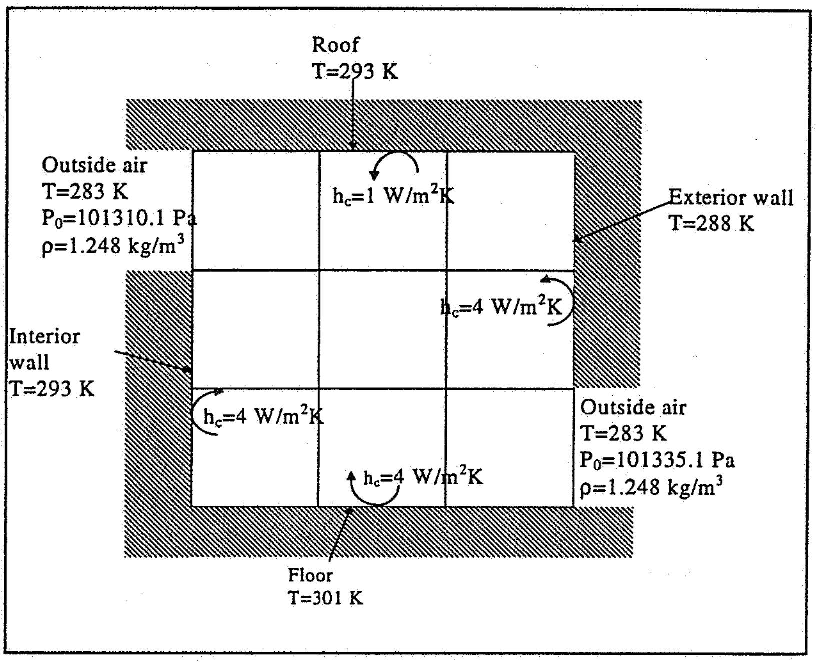

Figure 5.1 : Studied configuration

5.3. RESULTS AND CONCLUSION

The results given in figure 5.2 and figure 5.3 have been obtained with the simplified model and empirical model, they are presented as follows :

- *italic* : vertical and horizontal mass flow rates(kg/s),
- dotted outlined, from top to bottom : temperature (K), density (kg/m^3), pressure (Pa).

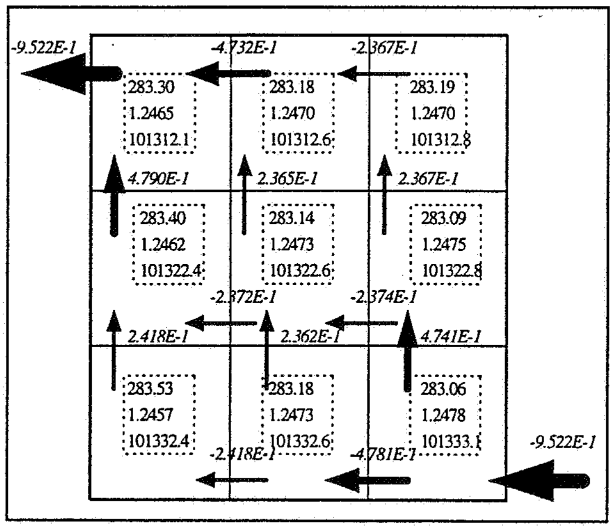

Figure 5.2 : Simplied model, results.

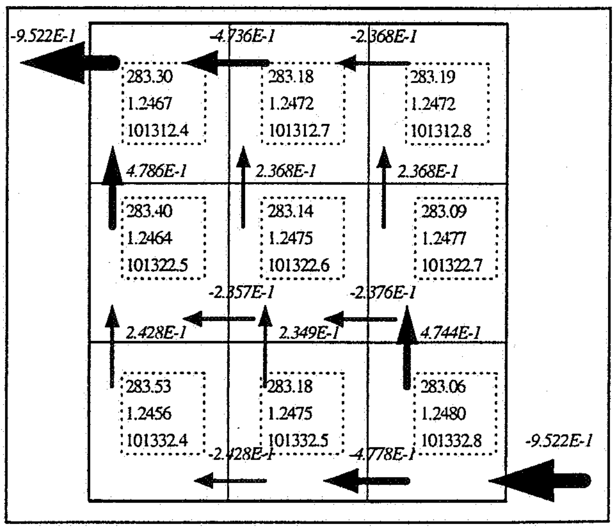

Figure 5.3 : Empirical model, results.

One can notice that the temperatures and mass flow rates obtained are very similar while the differences between pressures are more important. It does not really matter because pressure is not the most interesting result in thermal and mass transfers representation.

6. CONCLUSION

The best compromise between simplicity of the model, convergence of the simulation and calculation time seems to be the simplified model presented in chapter 3.3. This model gives as good results as the empirical one, thus in very short calculation times, so it will be possible to couple it with other models. The aim of the next studies will be the coupling of the simplified zonal model with wall models, comfort models, moisture and pollutants transport models. It is also projected to create new sorts of cells that represents plumes or jets.

ACKNOWLEDGEMENTS :

The authors wish to thanks Mr Francis ALLARD for his help and discussion during this work

REFERENCES :

1. ANDERSON J.L.
« A network definition and solution of simulation problems »
Technical Report 21522. Lawrence Berkeley Laboratory. September 1987.

2. LIMAM K., INARD C., ALLARD F.
« Etude expérimentale des transferts de masse et de chaleur à travers les grandes ouvertures verticales »
Ventilation et renouvellement d'air. Lyon. GEVRA, 1991.

3. NATAF J.M. WINKELMANN F.
« Symbolic modelling in building energy simulation »
LBL Report LBL-35439. Lawrence Berkeley Laboratory. April 1994.

4. SOWELL E.F., BULH W.F., NATAF J.M.
« Object-oriented programming, equation-based sub-models, and system reduction in spank. »
Proceedings of Building Simulation' 89, pages 141-146, Vancouver, British Colombia, Canada, June 1989.

5. WURTZ E.
« Modélisation tridimensionnelle des transferts thermiques et aérauliques en environnement orienté objet »
PhD Thesis. Ecole Nationale des Ponts et Chaussées. 1995.