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Title: **Reducing Cooling Loads With Under Roof Air Cavities**

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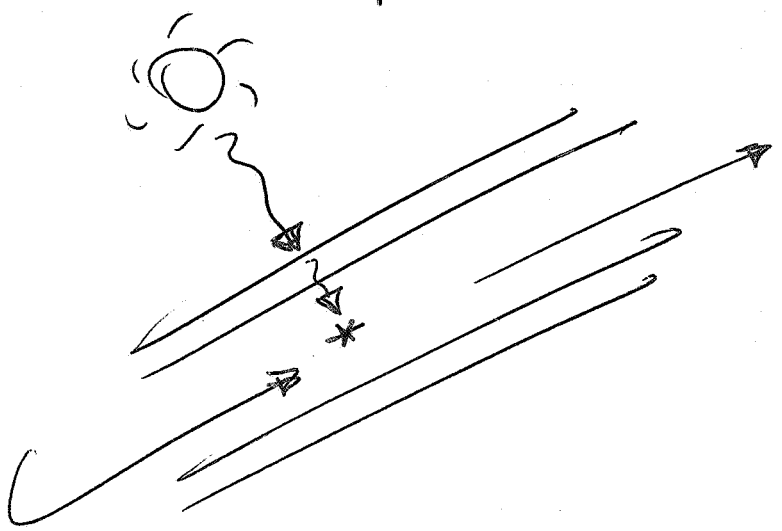

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Synopsis

In the present paper a model for steady-state thermal analysis of ventilated and unventilated light roofs is proposed. The aim of the work is to study the influence of thermo-physical and geometric parameters of the roof and boundary conditions (solar radiation) on the entering heat flow and the temperature distribution within the roof structure. Knowledge of this relationship, in fact, is important to optimise the roof "thermal" design in order to reach acceptable indoor conditions with low energy costs.
In the first part of the paper a calculation procedure for the forecast of internal temperatures and induced air flow rates is presented. The model can be used to analyse both closed and ventilated air cavities for different boundary conditions. From the mathematical point of view it is composed by a system of non-linear equation solved numerically.
The model output are: air temperature distributions, surface temperatures, heat fluxes and air flow rates (in the case of ventilated cavities).
The second part of the paper deals with the development of two simplified models, directly providing the entering heat flows. A non-dimensional analysis based on the Π-theorem provided the structure of the monomial expression, and the free parameters were determined through a parametric analysis performed by the complete model described in the first part.
The resulting model predicts the performance of light roofs with good accuracy.

Introduction

The under roof cavities represent a system characterised by the presence of two solid layers with an air space in between. These kinds of envelopes are specially used in the case of light roofs and allow an effective increase of the thermal insulation of the building, without a great increase of the structure weight. For these reasons these types of surfaces are widely employed in the roofing of large spaces such as atria, sporting halls and auditoria.
The correct design of these roofs requires a suitable calculation procedure producing information about the thermal fluxes going into the space, needed for thermal load calculation and HVAC plant design, and the temperature field inside the layers, which may give relevant information about the thermal stresses induced in the structure. Moreover, the availability of an automatic procedure, able to predict the behaviour of the system with little computational resources, allows the design optimisation of the roof thermal structure, aiming at achieving acceptable indoor conditions at lower energy costs.
As the aim is to provide an easy to use and time saving tool, the calculation procedure described in this paper has been implemented into a well known spreadsheet (Excel®), widely used and available to a great number of designers. More detailed and sophisticated calculation procedure are available, like CFD codes. However, these computer programs, that supply three-dimensional flow and thermal fields of the system, require a large amount of computational time and resources, and may not be used in practice to develop sensitivity analyses or design optimisation.
The calculation procedure takes into account two different kinds of roof cavities:
Closed air cavities: in which the air space is confined inside a closed volume by non-permeable surfaces.
Ventilated air cavities: in which the lower and upper part of the air cavity are open and the air is free to flow by means of natural or forced draught through the air space. Forced ventilation cavities are usually less employed and, besides, easier to analyse, because the air flow is imposed, while the air flow in natural ventilated cavities is an output itself. For these reasons in the following paragraphs attention will be devoted only to the last type of cavity roofs.
Since these two kinds of roof air cavities have different physical behaviour, two distinct predictive models are required.

Calculation models

Basic hypotheses common to closed and ventilated cavity roofs

The simplifying hypotheses, common for open and closed cavities, are:

- steady-state thermal and flow fields,
- solar radiance and optical properties of surfaces uniform along the roof surface and along the direction perpendicular to the air flow pattern,
- thermal gradients negligible along the cross direction of the roof.

For what concerns convective heat transfer the larger between the two following correlations has been adopted:

for natural convection (ASHRAE, 1977) $$h_c = 1.52 \cdot |\Delta T|^{\frac{1}{3}} \quad (1)$$

for mixed convection (ASHRAE, 1977) $$h_c = 5.62 + 3.9 \cdot v \quad (2)$$

where ΔT is the temperature difference between air and surface and v is the air velocity.
The steady-state behaviour assumption is justified by two factors: the first is that these types of roofs are usually made with light structures, with quite low thermal inertia, and hence the time constant of the system is low and heat flows almost instantaneously. The second is that the computer code is intended for design purpose, where the steady-state assumption is usually made.
For the calculation of atmospheric radiation the Cole's relation has been adopted (Fracastoro, 1985):

$$G_a = 222 + 4.94 \cdot T_e + 65 \cdot cc + 1.39 \cdot cc \cdot T_e \quad (3)$$

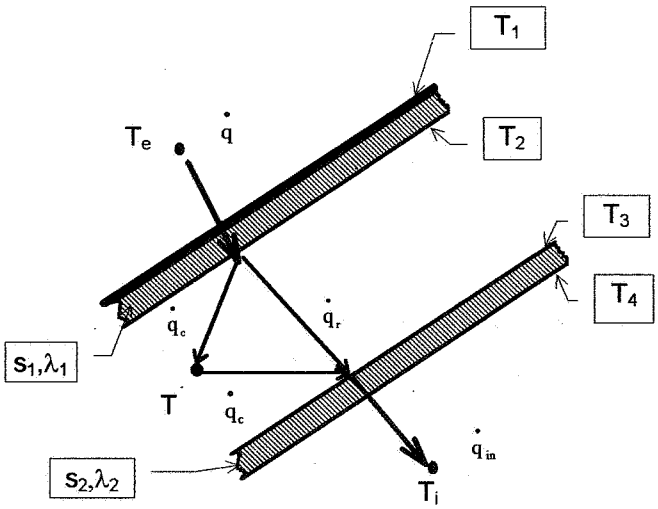
where T_e is the outdoor air temperature, measured in °C and cc, is the cloud cover factor, expressed as a fraction of unity.

Closed cavities

A roof with a closed air cavity is usually configured as:

- an upper layer made of a thin metal plate, often coupled with a thermal insulating panel,
- an air cavity, of quite low thickness (compared with the longitudinal roof length)
- a lower layer made of one or more panel with suitable thermo-acoustic properties.

Neglecting the thermal resistance of the metal plate, the calculation model for the closed cavities may be reduced to the scheme of figure 1.
For the inside surfaces of the air cavity the convective heat transfer coefficient has been calculated by means of the Raithby-Hollands (1985) correlation for closed, tilted cavities.
The total solar radiation I and infrared atmospheric radiation G_a impinging on the outdoor surface of the roof, inclined of an angle θ with respect to the horizontal plane, are absorbed with two different absorption coefficients, α_s and α_l, whose values depend on the optical properties of the outside roof surface. As a consequence, the equilibrium temperature of the upper layer is determined by the incident solar radiation, the re-radiation of the hot surface and the convective heat flow. Neglecting the longitudinal thermal gradients, the model will become one-dimensional, and the specific thermal flow rate,



$$\dot{q} = \alpha_s \cdot I + \alpha_1 \cdot G_a - \varepsilon_1 \cdot \sigma \cdot T_1^4 - h_{ce} \cdot (T_1 - T_e)$$

$$\dot{q}_r = F_\varepsilon \cdot \sigma \cdot (T_2^4 - T_3^4)$$

$$\dot{q}_c = h_c \cdot (T_2 - T_3)$$

$$\dot{q}_{in} = \left(\frac{s}{\lambda}\right)^{-1} \cdot (T_3 - T_4)$$

$$\dot{q}_{in} = h_i \cdot (T_4 - T_i) + F_\varepsilon \cdot \sigma \cdot (T_4^4 - T_i^4)$$

Figure 1 - Calculation model for closed air cavity.

$q = \dot{Q}/S$, will be uniform over the whole roof surface. Furthermore, the energy balance of the roof yields:

$$\begin{cases} \dot{q} = \dot{q}_c + \dot{q}_r \\ \dot{q}_c + \dot{q}_r = \dot{q}_{in} \end{cases} \tag{4}$$

More details on the mathematical models of the closed cavity may be found in (Fracastoro et al., 1997).

Ventilated cavities

When the air cavities are tilted respect to the horizontal the air flows upwards due to natural draught ("chimney effect") and exchanges heat with the adjacent surfaces, progressively increasing its enthalpy and temperature. Therefore, there will be thermal gradients along the air flow direction and the heat flows crossing the roof will increase in the air flow direction. The system is now strongly two-dimensional, and the mathematical model that describes the thermal and fluodynamic behaviour of the roof will be made of a set of non-linear differential equations. This system has been solved numerically, discretizing the real system in a number of elements adopting an up-wind scheme. The resulting algebraic system is described in figure 2 for the generic jth-element.

- S_j is the area of the j-th element, with $S = \sum_j S_j$,
- s_1 ed s_2 are the thickness of the two (upper and lower) insulating panels.

The energy balance of the roof yields:

$$\begin{cases} \dot{Q}_1 = \dot{Q}_r + \dot{Q}_2 \\ \dot{Q}_4 = \dot{Q}_r + \dot{Q}_3 \end{cases} \tag{5}$$

In order to complete the mathematical model the enthalpy balance has to be written for the air flow inside the cavity:

$$\dot{Q}_{aria,j} = \dot{m} \cdot c_p \cdot (T_{j+1} - T_j) = \dot{Q}_{3,j} + \dot{Q}_{4,j} \tag{6}$$

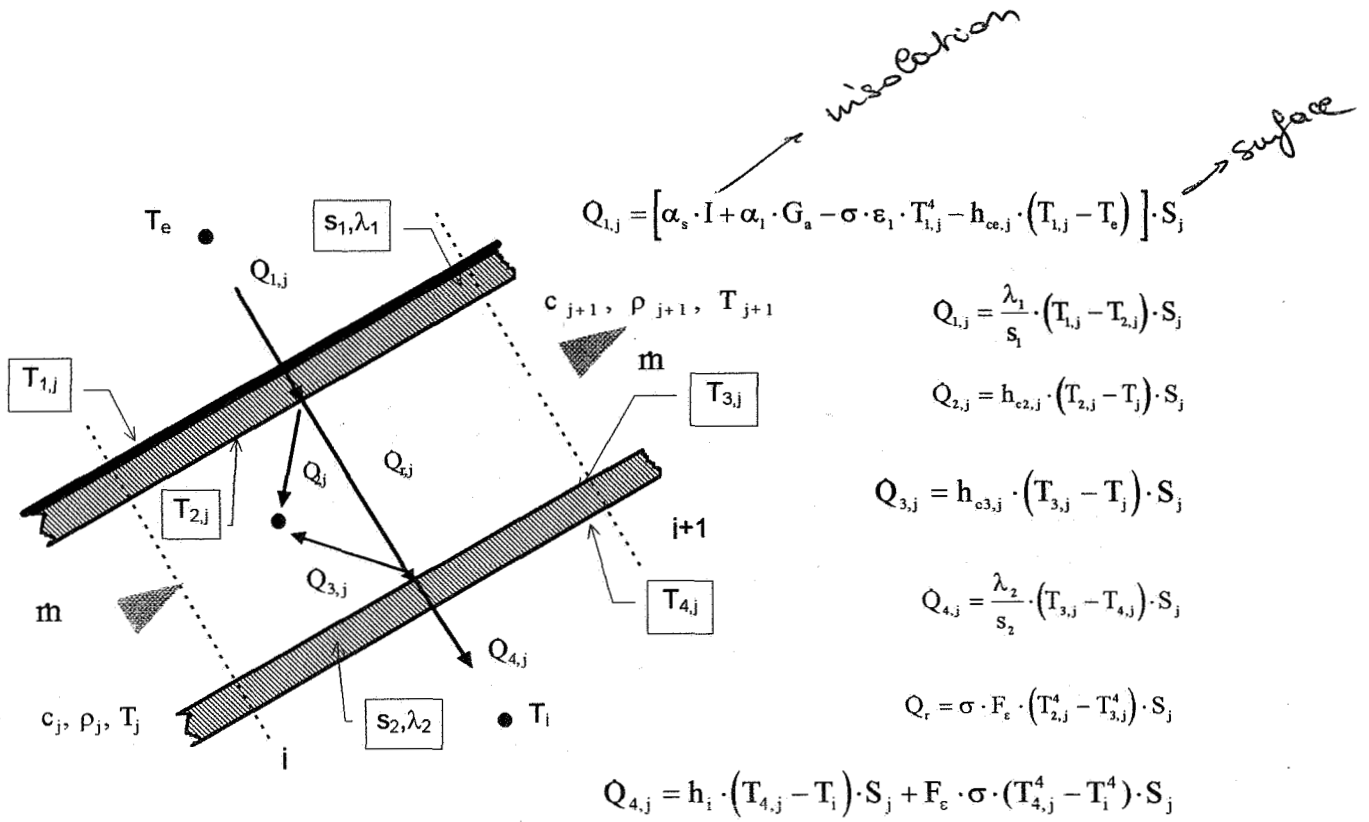


Figure 2 - Calculation model for open air cavity.

The air mass flow rate is also a function of the fluodynamic properties of the flow channel. In fact, the sum of all the pressure drops, due to distributed and concentrated losses in the various elements along the roof shall equal the total static pressure difference existing in the outdoor air between the entrance and the exit of the air cavity:

$$\sum_j \Delta p_j = \sum_j -\rho_j \cdot \left(\frac{c_{j+1}^2 - c_j^2}{2} + g \cdot \Delta z_j + l_{w,j} \right) = -\rho_{aria,est} \cdot g \cdot \Delta z_{tot} \qquad (7)$$

Numerical solution

In order to solve the non-linear system of algebraic equations obtained from the mathematical model of air cavities discussed in the previous paragraph, a numerical iterative procedure has been used. The two models have been implemented into an Excel® spreadsheet that, following the two schematic flow-charts shown in figure 3, allows a quick solution of the problem.

The computational time needed for the closed cavity analysis is negligible, and no problems of convergence have been experienced.

For what concerns the open cavities, the calculation procedure may require some minutes of CPU time (employing a PC Pentium pro 200 MHz with 32 Mb of system memory). Critical boundary conditions for convergence are: low solar radiation, high degree of thermal insulation and low tilt angle roof. A suitable guess of the outside surface temperature field and of the air mass flow rate is also needed for the first step calculation. A recommendable procedure is to use the results of the closed cavity as a first guess for the open case simulation.

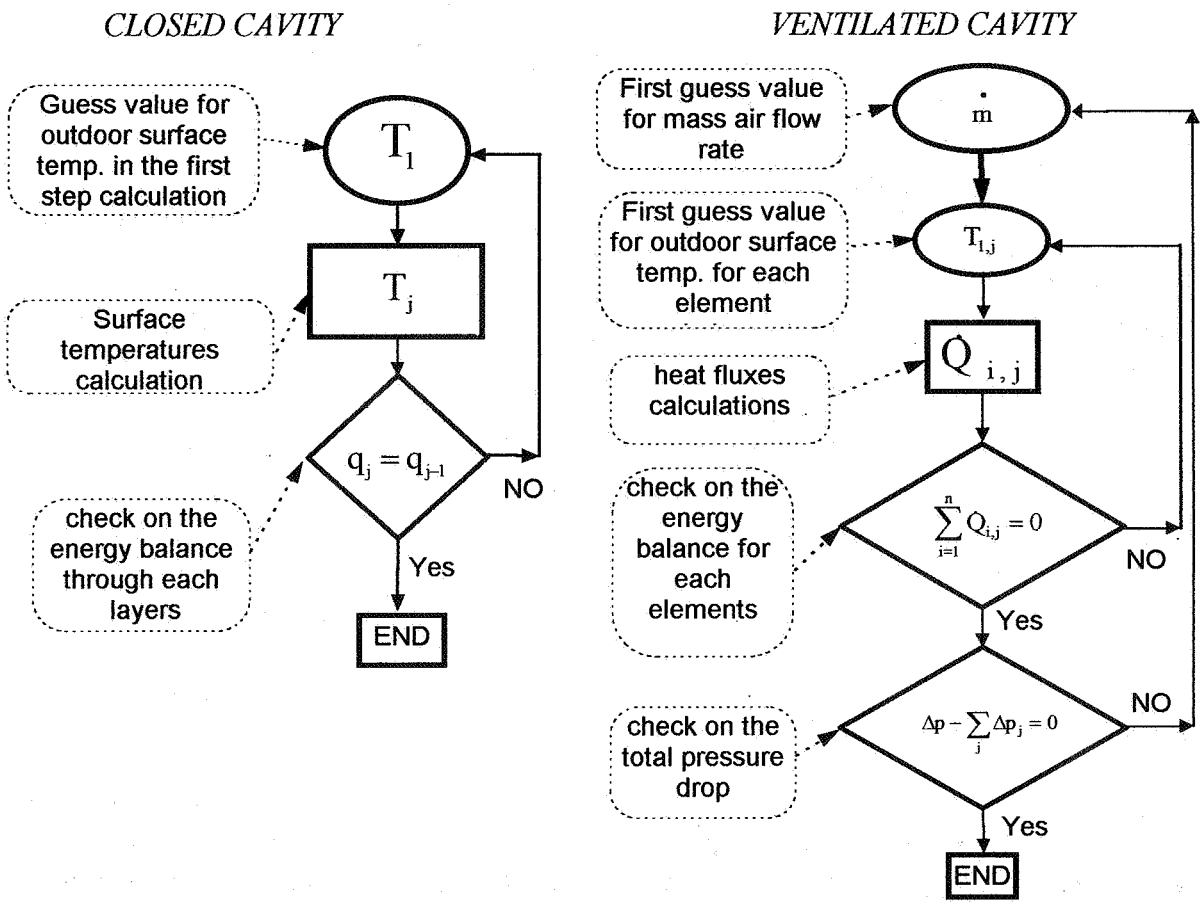


Figure 3 - Flow chart of the solution procedure for the close and open air cavity model.

Models application

Closed air cavity

Once the entering heat flow is normalized respect to the absorbed solar radiation, it turns out to be dependent only on the ratio of the outside film resistance to the total roof resistance, according to the classical expression:

$$\frac{Q}{\alpha \cdot I \cdot S} = \frac{R_{out}}{R_{tot}} \tag{8}$$

where

R_{out} is the value of the outside surface coefficient,

R_{tot} is the total thermal resistance of the roof, which may be calculated adding to the conductive thermal resistance of the insulation layers the outside and inside surface resistances and the thermal resistance of the air cavity.

The air cavity thermal resistance is, in practice, inversely proprtional to the gray surfaces view factor, F_ε, according to the expression:

$$R_{int} = \frac{0.12}{F_\varepsilon}.$$

Figure 4 compares the performance of ventilated and closed air cavities in reducing the solar heat load. This chart plots the non-dimensional parameters:

$$\xi = \frac{Q_{in,vent.}}{Q_{in,closed}} \quad \text{and} \quad \eta = \frac{Q_{in}}{I \cdot S},$$

versus the total conductive thermal resistance. The presence of the closed air cavity, even with very low total conductive resistance, allows to reduce the solar heat load to 15% of the solar radiance. This fraction may be further reduced to a value of 5%, when the total thermal resistance is higher than 1 m^2K/W.

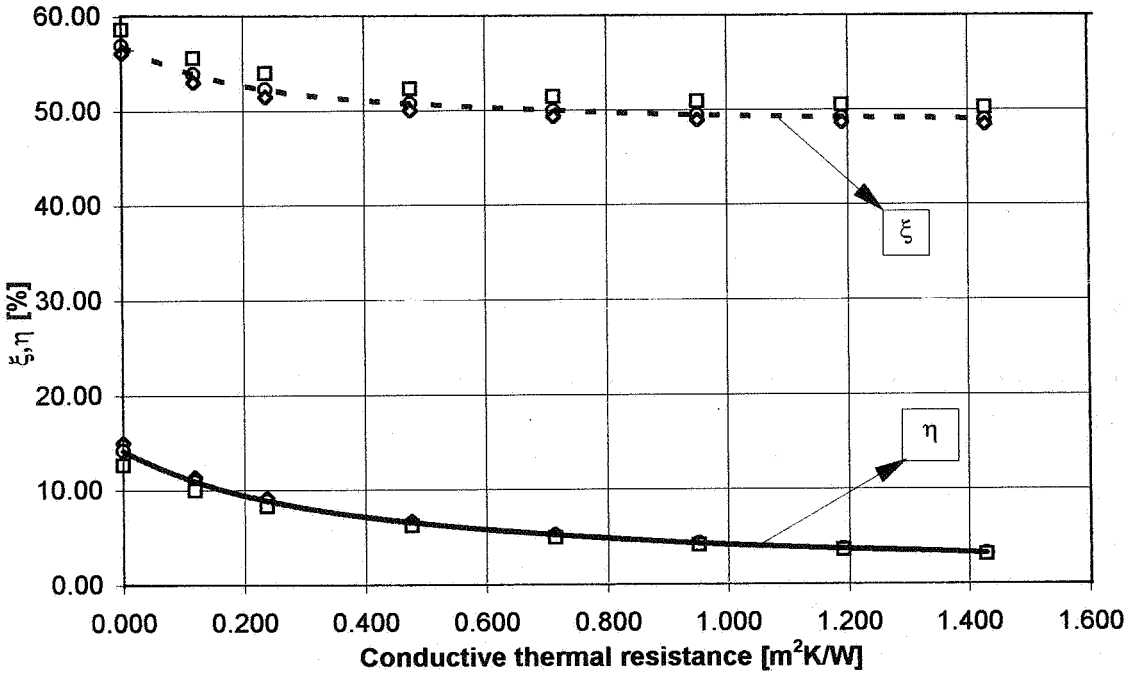


Figure 4 - Ventilated and closed air cavity performances.

These positive effects are amplified if the air cavity is ventilated, with entering heat fluxes lower than 60 % of the closed case, down to less than 50 % when R_{tot} becomes higher than 1 m^2K/W (fig. 4 has been plotted supposing the insulation concentrated in the lower panel).

Simplified model for ventilated air cavity

The numerical procedure obtained for the naturally ventilated configurations is a flexible calculation tool, that however requires a certain amount of computational resources and time. In order to provide a procedure for first attempt calculations, a simplified method, based on dimensionless analysis, has been developed.

The study has been split into two different parts:

1) investigate the behaviour of a non-insulated ("bare") ventilated roof considering only the effects of the boundary conditions (solar radiation, I) and geometrical parameters (roof length, L, duct roughness, e, and hydraulic diameter, D, roof angle referred to horizontal plane, θ) (i.e. the roof is made of an upper and lower thin metal slab having a negligible conductive resistance, and only a radiation shield function).
2) analyse the influence of:

- total conductive thermal resistance of the insulating panels,
- distribution of the thermal resistance between upper and lower slab.

Bare roof

For what concerns the "bare" roof, applying the Π-theorem to the ventilated air cavity model, it is possible to reduce the relations between the main physical influencing variables into a monomial formula containing five dimensionless groups:

$$\Pi_1 = A \cdot \Pi_2^m \cdot \Pi_3^n \cdot \Pi_4^o \cdot \Pi_5^p$$

where:

$$\Pi_1 = \frac{\frac{Q_{in}}{a}}{I \cdot L} \tag{9}$$

$$\Pi_2 = \frac{L \cdot \sin\theta}{D} \qquad \Pi_3 = \frac{R^* \cdot T_e}{g \cdot L \cdot \sin\theta} \qquad \Pi_4 = \frac{e}{L} \qquad \Pi_5 = \frac{\alpha \cdot I}{h_e \cdot T_e} = \frac{T_{s,a} - T_e}{T_e} \tag{9'}$$

being:

Q/a the total net heat flux entering the enclosure per unit of roof width, T_e air outdoor temperature, $T_{s,a}$ sol-air temperature, h_e surface heat transfer coefficient, α external roof surface absorption coefficient, R* air elasticity constant, g gravity acceleration.

Applying the computer code described in the previous section to a set of more than 100 data[1], including different boundary conditions and air cavity configurations, it is then possible to derive a number of "working points" for the ventilated cavities, by means of which the unknown coefficients may be determined.

The best-fit of the calculated values has lead to the following formula:

$$\Pi_1 = 0.0033 \cdot \Pi_2^{0.2967} \cdot \Pi_3^{0.4245} \cdot \Pi_4^{0.0365} \cdot \Pi_5^{0.1545} \tag{10}$$

The reliability of this formula is acceptable, as it is shown in figure 5 where the exact dimensionless parameter Π_1 is plotted versus the value "predicted" by means of eqn. 10. For all the considered configurations the prediction error lies inside a range from -10 % to +7 %.

Effect of insulation

The effect of the total conductive resistance and of the distribution of this resistance between the upper and lower slabs may be taken into account introducing a suitable dimensionless parameters ~~defined as~~:

$$\varphi = \frac{Q_{in,insulation}}{Q_{in,R=0}} \tag{11}$$

[1] these calculations have been performed assuming: equal inside and outside air temperature (in order to take into account only the solar radiation shield effect), constant atmosphere radiation $G \approx 355$ W/m^2, equal long and shorrt-wave absorption coefficients.

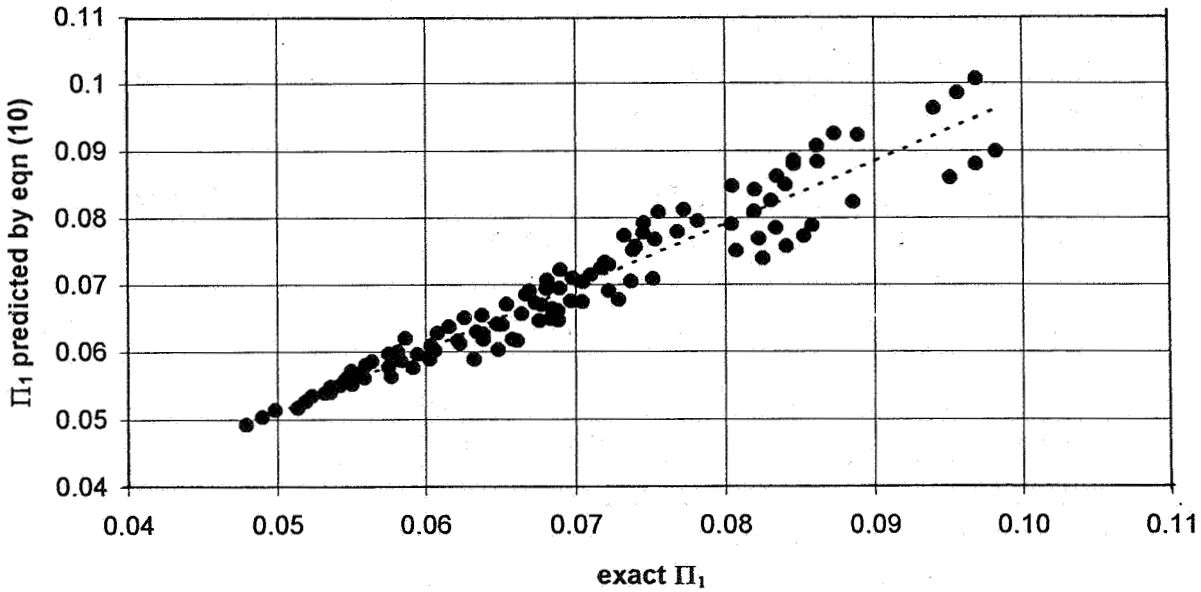


Figure 5 - Comparison between predicted and calculated Π_1 value.

It is possible to express φ only as a function of the total (upper plus lower slab) conductive thermal resistance and of the distribution of such resistance between the two slabs, having the other parameters, such as I, ϑ, L, D, e, practically no influence.
This is clearly shown in figure 6 where φ is plotted versus the fraction of thermal conductive resistance placed in the upper slab respect to the total.

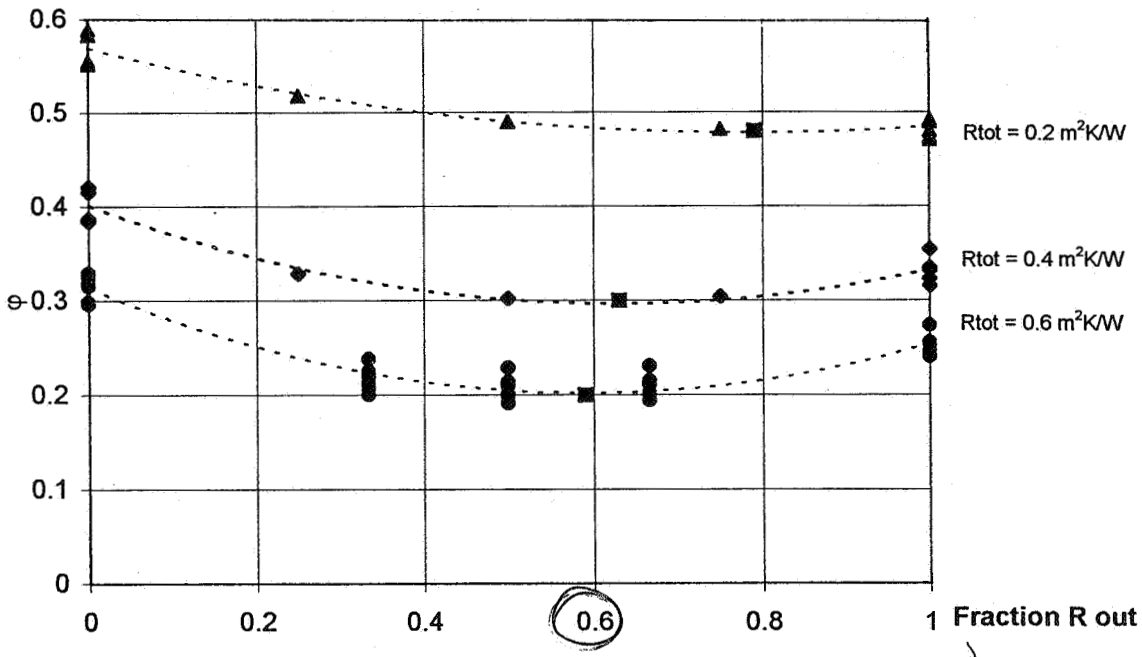


Figure 6 - Influence of thermal insulation.

optimale positie
± 60% van isolatie
in buitenste slab,
40% in binnenste

The points gather around a parabolic trend with quite low dispersion. The non-dimensional heat flux, φ, shows a minimum when the 60%-80% of the conductive thermal resistance is located in the upper panel. For these configurations the total heat flux entering the enclosure through the roof, is only 75 % of the heat flux that would be transmitted if all the insulation were located in the lower panel.

Finally, once the parameter Π_1 has been calculated by means of eqn. (10) and φ has been read by means of figure 6, Q_{in} may be determined as:

$$Q_{in} = \varphi \cdot \Pi_1 \cdot I \cdot L \cdot a \qquad (12)$$

The assumptions under which the simplified model has been developed are given in Table 2.

Table 2 - Ventilated cavity. Constant values adopted.

Parameter	Symbol	Value	Units
"long-wave" absorption coefficient	α_l	0.85	-
Cloud cover factor	cc	0.10	-
Outdoor air temperature	T_e	25	°C
Indoor air temperature	T_i	25	°C

Conclusions

The paper describes the models developed by the authors to simulate the thermal behaviour of roofs with closed and/or ventilated air cavities. The models have been implemented on a commercial spreadsheet and allow a quick and easy calculation of the relevant outputs, such as air flow rates, temperature distribution, and entering heat fluxes. Some examples are reported which provide some insight on the performance of these structures, and show the reduction of solar heat gains achievable both with closed and ventilated air cavities. In particular, ventilated roofs may lead to a reduction of 50 % and more respect to closed air cavity roofs. Furthermore, the model shows that there is an optimal positioning of insulation slabs in the case of ventilated air spaces, while positioning is unrelevant for closed ones.
A further step has led to the development of a procedure for "hand calculation" of the heat gain through a ventilated roof. This procedure yields results correct within ± 10 % respect to the computer model.
Future developments will include the experimental validation of the codes.

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