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VENTILATION AND COOLING

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Macroscopic Formulation and Solution of Ventilation Design Problems

no CFD,
but modes

STATIC → no heat capacity !

James W. Axley

Alles bekeken vanuit de
vraag: "Wat is de oppvl.
van de ramen" ?

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Synopsis

This paper will present a general approach that may be used to solve natural ventilation design problems typically addressed at the preliminary design stage – How wide should windows be opened in a given building for wind-driven cross ventilation on a moderate summer day? How should a ventilating monitor be configured to mitigate internal and solar gains on the same summer day? Established macroscopic equations governing airflow and heat transfer in multizone building systems will be reformulated to place airflow component characteristics – e.g., size of window opening, speed of a ventilating fan, or height of a monitor window – as key design parameters to be adjusted, selected, or in the special case of optimization, to be optimized. The resulting equations will establish constraints that must be placed on these design variables to guarantee the satisfaction of mass, momentum, and energy conservation. To these constraints, constraints relating to thermal comfort will be formulated in terms of the design parameters. It will be shown that together the conservation and comfort constraints serve to unambiguously define combinations of design parameters that are technically feasible – from these an optimal combination may be sought. The general approach will be applied to simple cases of cross ventilation and stack ventilation to demonstrate the utility of the approach.

List of Symbols

Variables

A area of window opening (m^2)

$[A]$ airflow coefficient matrix

$\{B\}$ vector of buoyancy effects

c_p heat capacity of air, constant pressure ($J \cdot kg^{-1} \cdot {}^\circ K^{-1}$)

C_i concentration of species "i"

C_d orifice opening discharge coefficient; typically ≈ 0.60

C_p wind pressure coefficient

$[C]$ thermal capacitance matrix

$\{E\}$ thermal excitation vector

$f(u,v\ ...)$ function of $u, v, \ldots$)

g the acceleration of gravity ($m \cdot s^{-2}$)

$[K]$ thermal conductance matrix

$\dot{m}$ air mass flow rate ($kg \cdot s^{-1}$)

M (effective) molecular weight of dry air $\approx 28.97\ g \cdot mol^{-1}$

p pressure (Pa)

q_{gain} total cooling load (W)

R universal gas constant ($J \cdot {}^\circ K^{-1} \cdot mol^{-1}$)

RH relative humidity

t time (s)

T temperature (°C)

v air velocity vector ($m \cdot s^{-1}$)

x, y, z spatial coordinates (m)

Δz stack height (m)

ϕ design parameter

ρ air density ($kg \cdot m^{-3}$)

$\hat{\rho}$ spatial average air density (1.2 $kg \cdot m^{-3}$ is sufficiently accurate here)

ΣUA the sum of building conductances ($m^2 \cdot {}^\circ K \cdot W^{-1}$)

Subscripts & Marks

i associated with zone "i"; indoor zone

j associated with a specific point

k associated with a region

l associated with an airflow path

n associated with a surface

o associated with outdoor conditions

db dry bulb temperature

mrt mean radiant temperature

eff effective

res CIBSE dry resultant temperature

$\hat{u}$ spatial average of u

1. Introduction

Advances made in methods to predict and measure building airflows have truly revolutionized the fields of building ventilation and air quality research and practice in the past two decades. Tracer gas techniques have been extended and refined to allow more accurate, better characterized, and more complete multizone measurements of airflows within buildings. A variety of rigorously defined ventilation effectiveness metrics have grown out of these advances and have placed ventilation system evaluation on a solid objective basis. Building pressurization techniques have been improved and extended to allow field measurement of building envelope, zone and component leakage air flow characteristics providing a more complete understanding of infiltration airflow paths in buildings and mathematical models to simulate them. Macroscopic methods of airflow analysis have been generalized to allow integrated modeling of wind-driven, buoyancy-driven, and mechanically-forced airflow in multizone building systems of arbitrary complexity. The global predictive capability of macroscopic simulation methods have been complimented by a constellation of microscopic methods of analysis, together placed under the more familiar rubric of Computational Fluid Dynamics, that allow investigation of the details of airflow around buildings and within single and, at this point, simply and well-connected collections of two or three rooms.

Consequently, we presently find ourselves armed with a veritable arsenal of tools to evaluate the thermal comfort, air quality and energy conservation efficacy of existing and proposed building ventilation systems that should, we hope, lead to improvements in building design, renovation, and operation. Yet, ironically, we have yet to develop tools to directly answer simple design questions relating to building ventilation: How wide should windows be opened in a given building for wind-driven cross ventilation on a moderate summer day? How should a ventilating monitor and building windows be configured to mitigate internal and solar gains on the same summer day? What size fan is needed to assist stack-driven air flow through the monitor on a more extreme summer day?

This paper seeks to address this problem.

2. Approach

A building system may be considered to be a three-dimensional continuum within which the *state variables* of temperature T, pressure p, air velocity v, and concentration C_i vary in space, x, y, z, and in time, t.

State Variables $\quad T(x,y,z,t),\ p(x,y,z,t),\ v(x,y,z,t),\ C_i(x,y,z,t)$

The variation of these state variables is governed, fortunately, by fundamental mass, momentum, and energy conservation principles – bound by environmental and thermal/mechanical/chemical boundary conditions – that allow prediction of the spatial and temporal variation of these state variables. Broadly speaking, two numerical approaches are commonly used for this prediction:

- *Microscopic analysis*, based typically on finite difference or finite element techniques, approximates the continuously defined state variables by a finite set of spatially discrete but temporally continuous state variables defined at or associated with discrete (mesh) points "j" within the continuum:

  Microscopic Discretization $\quad T(x_j,y_j,z_j,t),\ p(x_j,y_j,z_j,t),\ v(x_j,y_j,z_j,t),\ C_i(x_j,y_j,z_j,t)$

  Room airflow analysis using computational fluid dynamics techniques and conduction heat transfer and moisture transfer analysis using finite element or finite difference methods are the most familiar examples of microscopic analysis used today for building performance evaluation.

- *Macroscopic analysis,* based on idealizing the building system as a collection of one or more control volumes linked by discrete heat or mass transport paths, also approximates the continuously defined state variables by a finite set of spatially discrete but temporally continuous state variables but now the discrete state variables are associated with either the control volumes or discrete transport paths – i.e., discrete regions rather than geometric points within the continuum:

Macroscopic Discretization $T_k(t),\ p_k(t),\ \dot{m}_k(t),\ C_{i,k}(t)$

Here the subscript k is used to identify the discrete region – the control volume or discrete transport path – as, for example, well-mixed zones, 1D heat transfer paths through walls, windows, and floors, and airflow through doors, windows, cracks, etc. Macroscopic analysis applies the conservation principles to the discrete regions idealized as control volumes within the building system and thereby forfeits the ability to predict the spatial detail within these control volumes. Importantly, air velocity is simply replaced by predictions of total air mass flow rate through discrete air flow paths – air velocity is not directly predicted.

Multizone building energy simulation, multizone indoor air quality analysis, and component-based approaches to HVAC system simulation are the familiar examples of macroscopic analysis used today for building performance evaluation.

In principle, these same state variables also determine the "state" of thermal comfort and air quality within the habitable portions of the building system – i.e., given the metabolic activity, clothing level, chemical sensitivity, etc. of occupants – however, the exact relation between these state variables and the state of thermal comfort and air quality is not presently understood with theoretical certainty. Nevertheless, a variety of semi-empirical thermal comfort indices have been developed over the years (see [1] for an overview) that relate thermal comfort to the primary comfort variables of dry bulb air temperature T_{db}, mean radiant temperature T_{mrt}, air velocity v, and relative humidity RH within rooms:

Primary Comfort Variables $T_{db}(x,y,z,t),\ T_{mrt}(x,y,z,t),\ v(x,y,z,t),\ RH(x,y,z,t)$

Microscopic methods of analysis provide the means to predict these comfort variables and, importantly, their spatial variation within rooms – air dry bulb temperature and velocity are directly predicted while mean radiant temperature and RH distributions may be easily computed at each of the room air mesh points from computed surface temperatures and vapor-phase water concentrations respectively. As a result, microscopic analytical evaluation of comfort in rooms has become one of the primary applications of computational fluid dynamics (see, for example, [2, 3]). In spite of the direct utility of the microscopic approach to thermal comfort prediction, several limitations must be noted:

- microscopic analysis is presently limited to single or simply connected multiple rooms of relatively simple geometry; whole building system analysis is beyond the current capability of microscopic analysis,
- microscopic analysis is expensive due to the special expertise needed to implement it, the time necessary to formulate, solve, and evaluate the results from microscopic analysis, and the fact that it is computationally demanding; for this reason it remains a research tool in North America and is very seldom applied in practice, and
- microscopic analysis presently supports only a trial and error approach to building design.

How about macroscopic analysis? Macroscopic methods can provide an economic and accessible means to predict simple measures of thermal comfort within rooms (e.g., spatially averaged room air dry bulb temperature, mean radiant temperature, air velocity, and relative humidity) although, in North America, these methods are most commonly used to predict annual and peak energy demands of heating and cooling systems in buildings instead. While they can not provide the spatial detail offered by microscopic analysis, macroscopic methods can be readily applied to whole building systems and configured to allow an integrated consideration of interacting building systems – e.g. heat transfer in the building fabric and envelope, HVAC systems, lighting systems, and natural ventilation systems. As in the microscopic case, however, these methods have been formulated to support only a trial and error approach to building design, although in the most current examples this design approach has been expedited via automated comparative analysis and augmented with design rules of thumb [4].

Here we will take a more direct approach to design. Macroscopic conservation equations governing airflow and heat transfer in multizone building systems will be used to establish the relation between system response and key design parameters associated with discrete airflow paths – e.g., size of window opening, speed of a ventilating fan, or height of a monitor window. Using one of a number of comfort metrics, a second relation will be generated to establish the link between the comfort index and the design parameters. Finally, a comfort criteria will be applied to identify combinations the design parameters that are feasible from a comfort point of view. In some instances it may also prove useful to add an *objective function* to narrow the selection of feasible design parameters to an "optimal" choice.

1.1. Mass & Energy Conservation

To begin, consider a building system idealized as a collection of well-mixed zones – not a necessary limitation of the approach, simply a convenience here – linked by discrete air flow paths and conductive heat transfer paths:

General case

difficult

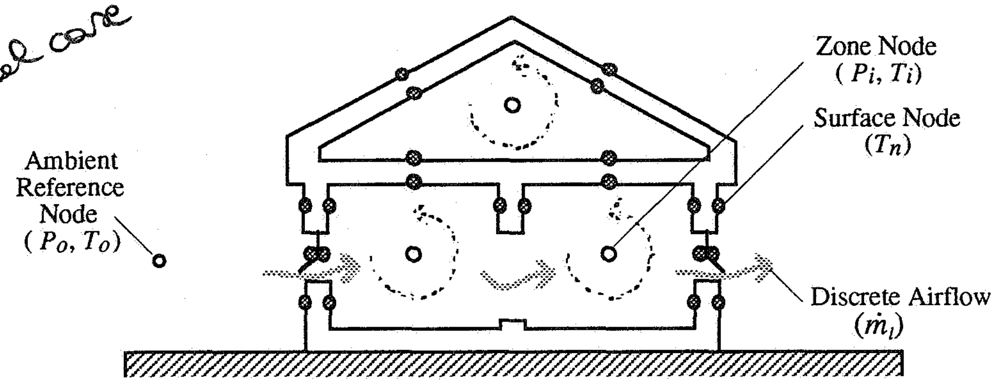

Fig. 1 Simplified representation of a multizone building idealization.

Macroscopic discrete state variables of pressure and temperature, p_i and T_i, will be associated with each of the zones – i.e., the pressure associated with a specific elevation within the zone identified above by "zone nodes" and the temperature associated with the spatial mean air temperature within the zone. Similarly, an outdoor ambient reference node will be associated with ambient pressure and temperature, p_o and T_o. Surface temperature variables, T_n, will be associated with the surfaces of each of the several conductive heat transfer paths within the building system and, finally, the mass flow rate of air, $\dot{m}_l$, through each of the several discrete airflow paths will be identified.

With these variables defined, one may apply mass and energy conservation principles to form systems of equations governing heat transfer and airflow in the building system (refer to [5-7] for details):

Heat Transfer: $$[K]\{T\} + [C]\frac{d\{T\}}{dt} = \{E\} \qquad (1)$$

Airflow: $$[A]\{p\} = \{B\} \qquad (2)$$

where:

- $\{T\}$ is a vector of zone and surface temperatures
- $\{E\}$ is the system "excitation vector" that accounts for thermal energy gains at each of the system nodes,
- $[K]$ is the system "conductance matrix" that is assembled from component or element conductances (i.e., UA terms for conductive transfer paths) and from airflow

contributions (i.e., the product of the air mass flow rate and heat capacity, $\dot{m}\hat{c}_p$, for discrete airflow paths),

$[C]$ is the system "capacitance matrix" that is assembled from component or element contributions to the thermal capacity associated with each temperature node,

$\{\boldsymbol{p}\}$ is a vector of zone pressures,

$[A]$ is the system airflow coefficient matrix assembled from the pressure-flow characteristics of the discrete airflow paths, and

$\{\boldsymbol{B}\}$ is a vector associated with buoyancy effects.

Solution of Equation 1 directly yields predictions of system temperatures $\{\boldsymbol{T}\}$. The determination of air mass flow rates is less direct – one first solves Equation 2 for the zonal pressures $\{\boldsymbol{p}\}$ then uses these solutions along with the individual discrete airflow path pressure flow relations to determine specific air mass flow rates, $\dot{m}_l$.

As written, these equations appear to be two systems of uncoupled linear equations but, in the usual case, they are in fact rather complex coupled systems of nonlinear equations. Specifically, the system airflow coefficient matrix is most often nonlinearly dependent on the state of pressures in the system, $[A] = [A(\{\boldsymbol{p}\}]$; the buoyancy effects vector is dependent on zone temperatures, $\{\boldsymbol{B}\} = \{\boldsymbol{B}(\{\boldsymbol{T}\})$; and the system conductance matrix and excitation vector are both dependent on the air mass flow rates $\dot{m}_l$ which, in turn, are dependent on both the system temperature and pressure vectors via the dependencies just noted for $[A]$ and $\{\boldsymbol{B}\}$.

The individual pressure-flow relations for the discrete flow paths are generally nonlinear but, nevertheless, depend on key design parameters, ϕ_l, of the flow path (e.g., size of window opening, speed of a ventilating fan, or height of a monitor window). That is to say, the mass flow rate of air through a given "l" discrete flow path is related to the zonal pressures the path links, say p_i, p_j and the design parameters:

$$\dot{m}_l = f(p_i, p_j, \phi_l) \tag{3}$$

Combining Equations 1, 2 and 3, one may, in principle, establish a relation between the system response (i.e., system temperatures and airflow rates) and the design parameters. In most practical situations, however, it will not be possible to establish this relationship formally as the combined system of equations will be hopelessly complex. Consequently, it will be necessary to establish the relation numerically by systematically varying key design parameters over a range of reasonable values and solving for the system response – i.e., for a given building and ambient and operating conditions. Whether formally or numerically derived, one may establish the relation between system response and the key design parameters for a given design problem:

$$\{\boldsymbol{T}\} = \{f_T(\phi_l)\} \tag{4}$$

$$\{\dot{\boldsymbol{m}}\} = \{f_{\dot{m}}(\phi_l)\} \tag{5}$$

1.2 Comfort Metrics and Criteria

A number of comfort indices could be considered. Here we'll limit consideration to two – room dry bulb temperature T_{db} and the CIBSE dry resultant temperature T_{res} – see [1] for a complete discussion of these and other comfort indices.

As noted above, macroscopic analysis forfeits spatial detail for numerical economy so we'll accept predictions of zone air temperature as a spatial average estimate of room dry bulb temperature for a given zone "i":

Metric 1:
$$\hat{T}_{db} \approx T_i \tag{6}$$

The CIBSE dry resultant temperature is defined in terms of the room dry bulb temperature, mean radiant temperature T_{mrt}, and air velocity v. Again we'll accept spatial average approximations. An area weighted average of computed surface temperatures will be taken as the spatial average of the mean radiant temperature in a given zone:

$$\hat{T}_{mrt} \approx \frac{\sum_{\text{zone i}} A_n T_n}{\sum_{\text{zone i}} A_n} \tag{7}$$

For room air velocity, we may estimate the spatial average velocity $\hat{v}_l$ at a discrete flow path "l" as:

$$\hat{v}_l \approx \frac{\dot{m}_l}{\hat{\rho} A_l} \tag{8}$$

where:

- $\hat{\rho}$ is, technically, the spatial average of the air density along the flow path but accuracy here is not important,
- A_l is the cross sectional area of the flow path

Here, we must be careful in the application of this approximation. At or in the "throw" of a ventilation opening, Equation 8 should provide a reasonable approximation of air velocities; away from the opening ventilation-driven velocities may diminish rapidly.

With these approximation in hand, the CIBSE dry resultant temperature may be defined as follows:

Metric 2:

$$\hat{T}_{res} \approx \begin{cases} 0.5\left(\hat{T}_{mrt} + \hat{T}_{db}\right) & ; \ \hat{v} \leq 0.1 \ ^{m}/_{s} \ \text{"still air"} \\ \left(\hat{T}_{mrt} + \hat{T}_{db}\sqrt{10\hat{v}}\right) \Big/ 1 + \sqrt{10\hat{v}} & ; \ \hat{v} > 0.1 \ ^{m}/_{s} \end{cases} \tag{9}$$

We'll place simple limits on the first measure of comfort to establish a comfort criteria:

Criteria 1:

$$20\,°C \leq \hat{T}_{db} \leq 26°C \tag{10}$$

For the dry resultant temperature metric, CIBSE recommends T_{res} = 20 °C for "still air" conditions in offices and provides a corrective increase in temperatures for air speeds greater than 0.1 m/s that may be approximated by $2.74\sqrt{\hat{v} - 0.1}$ °C. Thus the CIBSE comfort criteria for offices may be represented as:

Criteria 2a:

$$20\,°C \leq \hat{T}_{res} \leq 20\,°C + 2.74\sqrt{\hat{v} - 0.1} \tag{11}$$

In addition CIBSE recommends limiting air velocities to 0.3 m/s except in summers. Based on recent work of Arens and his colleagues [8] we'll add to the CIBSE criteria, Equation 11, an upper (summer) limit on air velocity of 1.5 m/s:

Criteria 2b:

$$\hat{v} \leq 1.5\ m/s \tag{12}$$

Note, we have not included RH humidity in our considerations. The CIBSE comfort criteria apply for relative humidities between 40 and 70%, consequently the applications to follow should be practically useful within this range. By adding system equations that allow a prediction of room RH to the above heat transfer and airflow equations – Equations 1 and 2 – and including additional comfort criteria for RH, however, the basic approach taken may be easily extended to include this important aspect of comfort.

1.3 Feasible & Optimal Design Configurations

By combing the system response results developed in terms of the key design parameters, indicated functionally by Equations 4 and 5, with one of the comfort metrics, Equation 6 or 9, we may establish the relation between the comfort index and the design parameters. It is useful to think of this relation geometrically as a comfort index surface defined relative to the key design parameters – the "design space". The comfort criteria presented above also define surfaces in the "design space." The intersection of the comfort index surface and comfort criteria surface defines, rather unambiguously, combinations of the key design parameters that are technically feasible. If then we can define some objective function in terms of the key design parameters, we can search through the feasible combinations of design parameters to find an optimal configuration. The examples presented below should made this distinction between feasible and optimal clear and reveal the procedures used to evaluate them.

Optimization is a tempting objective yet the definition of a reasonable objective function may not be obvious nor have the technical rigor associated with the approach presented above to determine feasible solutions. Knowledge of a range of feasible configurations is likely to be of the greatest value to the building designer anyway, as the designer is invariably searching for design configurations in a far more complex "design space" that given by the ventilation problem – e.g., for windows, in addition to thermal comfort, the designer must consider an array of functional considerations such as daylighting and view potential, economic and constructional constraints, as well as more "formal" issues relating to planning strategies, spatial character and composition and detail of the building's facade.

3. Applications

In this section we'll apply the approach presented above to two relatively simple cases – wind-driven cross-ventilation and stack-ventilation of a simple single-zone building under steady-state conditions of heat transfer.

3.1 Wind-Driven Cross Ventilation of a Single Zone Building

Consider a simple single-zone model of a building ventilated by wind-driven cross-ventilation:

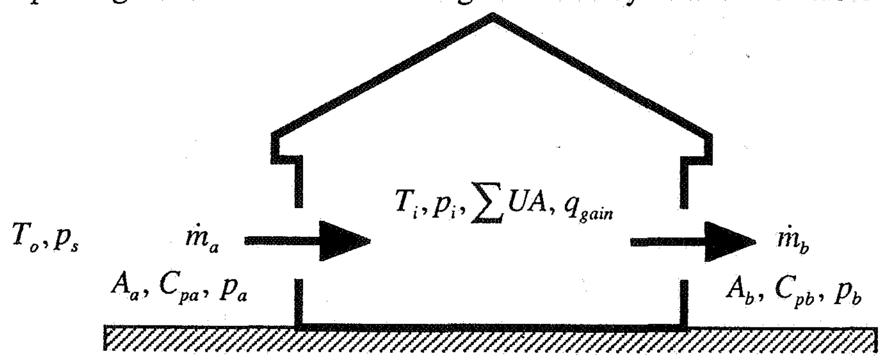

Fig. 2 Single-zone cross-ventilated building idealization and variables.

A steady wind, characterized by a stagnation pressure p_s, approaches the building from the left as air passes through a window "a" of cross sectional area A_a and out through window "b" of cross sectional area A_b. A_a and A_b will be taken as the key design parameters that will be adjusted to achieve thermal comfort. Wind pressure coefficients at the windows C_{pa} and C_{pb}; building conductances $\sum UA$; internal gains q_{gain}; and outdoor air temperature T_o are assumed known a priori. Two unknown state variables are associated with this simple idealization – the zone air temperature T_i and the zone pressure p_i defined relative to a specific elevation which, here, will be taken along the horizontal centerline through the two windows.

With the problem thus defined, we can form the heat transfer system equations by demanding conservation of thermal energy (as experienced analyst we recognize kinetic energy and pressure work terms associated with the flow will be negligible in comparison to the uncertainty in the thermal energy terms):

Heat Transfer: $$\sum UA\left(T_i - T_o\right) + \left(\hat{c}_p \dot{m}_b T_i - \hat{c}_p \dot{m}_a T_o\right) = q_{gain} \qquad (13a)$$

or in the form of Equation 1 above:

1kW
+ solar gains
+ . . .

$$KT_i = E \quad with \quad K = \left(\sum UA + \hat{c}_p \dot{m}_b\right) \quad and \quad E = q_{gain} + \left(\sum UA + \hat{c}_p \dot{m}_a\right) T_o \tag{13b}$$

We'll model the airflow through the windows using the familiar orifice equation as it has proven (and continues to be proven [9]) to be a reliable model:

$$m_a = C_d A_a \sqrt{2\hat{\rho}(p_a - p_i)} \quad and \quad m_b = C_d A_b \sqrt{2\hat{\rho}(p_i - p_b)} \tag{14}$$

and estimate the wind pressures acting at each of the windows using the approach wind stagnation pressures and appropriate wind pressure coefficients, C_{pa} and C_{pb}, although here more research is needed to provide proper estimates of these pressure coefficients as the size of the window openings increase the porosity of the building [10]:

$$p_a = C_{pa} p_s \quad and \quad p_b = C_{pb} p_s \tag{15}$$

With these relations in hand we may demand the conservation of air mass flow and form the system airflow equations for the problem:

Airflow:

$$\dot{m}_a - \dot{m}_b = 0 \tag{16a}$$

$$C_d A_a \sqrt{2\hat{\rho}(C_{pa} p_s - p_i)} - C_d A_b \sqrt{2\hat{\rho}(p_i - C_{pb} p_s)} = 0 \tag{16b}$$

In this case, the airflow equations are uncoupled from the heat transfer equations and thus we may directly solve for the airflow rates:

$$\dot{m}_a = \dot{m}_b = C_d \sqrt{\frac{A_a^2 A_b^2}{A_a^2 + A_b^2}} \sqrt{2\hat{\rho}\, p_s (C_{pa} - C_{pb})} \tag{17}$$

Substituting this solution into the system heat transfer equation, Equation 13, we may solve for the zone air temperature in terms of the design parameters A_a and A_b:

$$T_i = \frac{q_{gain}}{\hat{c}_p C_d \sqrt{\frac{A_a^2 A_b^2}{A_a^2 + A_b^2}} \sqrt{2\hat{\rho}\, p_s (C_{pa} - C_{pb})} + \sum UA} + T_o \tag{18}$$

Note the first square root term in the denominator defines, an equivalent single orifice opening A_{eff} that plays a key role in these equations. For a series of openings in cross-ventilation flow, we may generalize the result ~~defined by Equation 18~~ by replacing this term with this effective orifice opening defined as (after Flourentzou, 1996 #1314):

$$\left(\frac{1}{A_{eff}}\right)^2 = \sum \left(\frac{1}{A_i}\right)^2 \tag{19}$$

Zeer bel. vge!

Given the form of this expression, it becomes clear that the smallest opening in the series will tend to control the airflow.

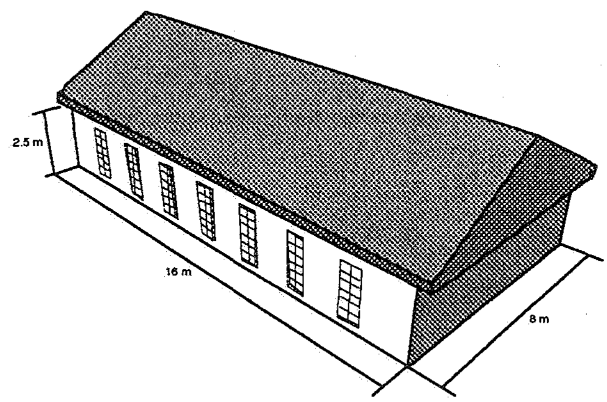

Fig. 3 Representative building for application of approach.

To better understand the character and use of Equation 18, consider the representative single story building illustrated above. With R20 (3.5 m^2°K/W) wall insulation, R30 (5.3 m^2°K/W) roof insulation, and R4 (0.7 m^2°K/W) windows the total building conductance of the envelope is ΣUA = 71.1 W/°K. For combined solar and occupant generated internal gains of 4 W/ft^2 (43 W/m^2) the total internal gain is q_{gain} = 5,504 W. Finally, an approach wind stagnation pressure of p_s = 15 Pa (corresponding to an approach wind velocity of 5 m/s or 11 mph), an outdoor air temperature of T_o = 20 °C, and wind pressure coefficients of C_{pa} = 0.8 and C_{pb} = –0.7 will be assumed and reasonable values will be assigned to the discharge coefficient (C_d = 0.6) and the average air density and heat capacity ($\hat{\rho}$ = 1.2 kg/m^3 and $\hat{c}_p$ = 1,004 J/kg-°K).

Substituting these values in Equation 18, the response becomes specific and may be plotted as shown below:

$$T_i = \frac{5{,}504\ W}{4{,}409\sqrt{\frac{A_a^2 A_b^2}{A_a^2 + A_b^2}}\ W/°K + 71.1\ W/°K} + 20\ °C \tag{20}$$

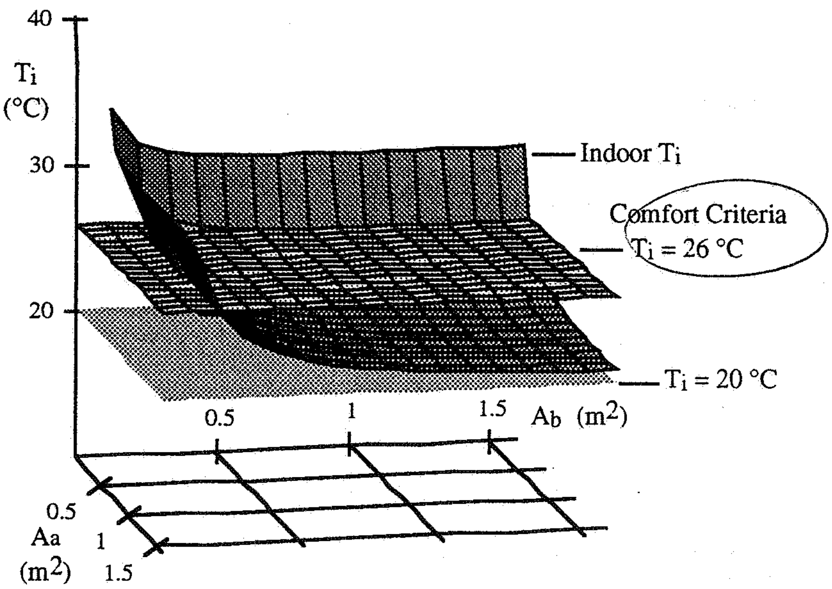

Fig. 4 Indoor air temperature response and comfort criteria for the representative building using cross ventilation with an approach wind velocity of 5 m/s.

As expected, the indoor air temperature response asymptotically approaches the outdoor air temperature for large window openings.

Comfort Metric and Criteria 1

The simple room air temperature comfort criteria defined by Equation 10 is also plotted on Figure 4 – simple horizontal planes corresponding to the upper and lower bounds of the comfort criteria, 26 °C and 20 °C. The intersection of the upper bound comfort criteria surface and the indoor temperature response - here, taken as the comfort index – then, establishes the minimum feasible combinations of window openings needed to achieve thermal comfort. This intersecting curve - the feasible solution curve for T_i = 26 °C - is plotted below as a solid line along with a more moderate feasible solution curve for T_i = 23 °C plotted as a dotted line:

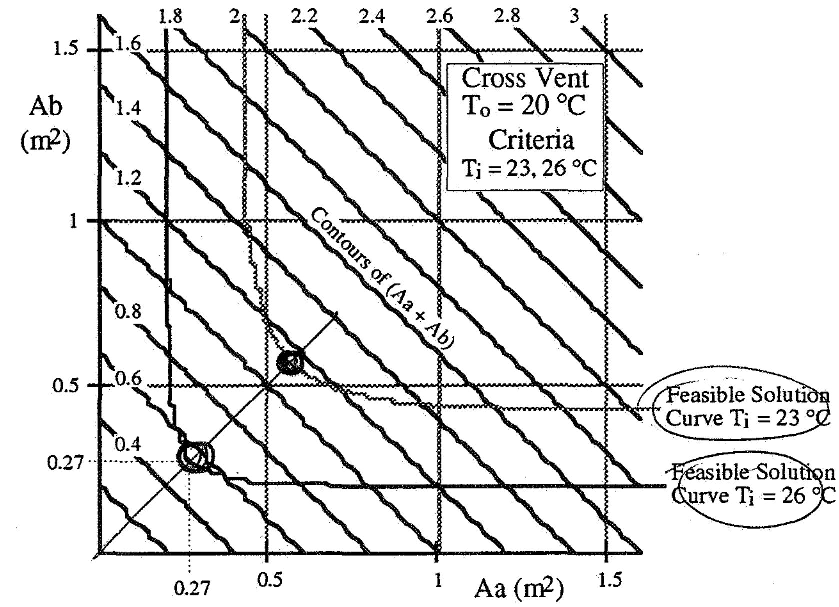

Fig. 5 The curve of feasible combinations of window openings for the cross-ventilation design problem for the simple air temperature comfort metric.

Using these curves, the building designer could select any number of window opening combinations to achieve a thermal comfort objective. For example, a combination of a 1 m^2 windward window opening, A_a, and a 0.20 m^2 leeward window opening, A_b, would achieve the 26 °C comfort objective while to achieve the 23 °C comfort objective with the same windward window opening, the leeward opening would have to approximately doubled to $A_b \approx 0.40$ m^2. Guided by a feasible solution curve, a building designer could attempt to meet other design objectives while maintaining a given thermal comfort objective without being constrained to a specific "optimal" solution.

Nevertheless, in some instances it may also be possible to establish an minimization objective that would allow the designer to narrow the search to a specific "solution." As a demonstration of this, consider the objective to minimize the sum of the window openings, $(A_a + A_b)$, possibly in an effort to minimize cost. Contours of equal values for this *objective function* are also plotted on Figure 5. Given the simplicity of this objective function and the fact that feasible solution curve, Equation 18, is symmetric about the line $A_a = A_b$, the minimum (or optimal) solution is obtained when $A_a = A_b$ which for the specific case at hand is $A_a = A_b = 0.27$ m^2. In the complex and rich world of architectural design, this particular objective function may or may not have much merit.

Comfort Metric and Criteria 2 nieuwe comfort T°

With the general approach established by ~~this first example~~, we may move on to the far more complex case of utilizing the CIBSE dry resultant temperature comfort index, Equation 9, and the associated comfort criteria, Equation ~~11. We'll limit~~ consideration to conditions just inside the windward window where mean air velocities may be estimated, by Equation 8, as:

$$\hat{v} = \frac{m_a}{\hat{\rho} A_a} = C_d \sqrt{\frac{2\, p_s}{\rho} \frac{A_b^2 (C_{pa} - C_{pb})}{(A_a^2 + A_b^2)}} = 3.67 \sqrt{\frac{A_b^2}{(A_a^2 + A_b^2)}} \;\; m/s \qquad (21)$$

Using this result, the specific solution Equation 20, and the strategy outlined above to estimate the spatial average mean radiant temperature we may establish the relation between the dry resultant temperature and the design parameters – the window openings A_a and A_b. For even this simple case, however, the algebra becomes formidable. Using a computational symbolic math processor the result below was obtained:

$$\hat{T}_{res} = \begin{cases} 0.98\left[\dfrac{5500}{4409\,A_{eff}+71.1}+20\right]+0.27 \;\; ; \;\; 3.67\dfrac{A_{eff}}{A_a} - 0.1 < 0 \\[2ex] \dfrac{0.96\left[\dfrac{5500}{4409\,A_{eff}+71.1}+20\right]+\left[\dfrac{5500}{4409\,A_{eff}+71.1}+20\right]\sqrt{36.7\dfrac{A_{eff}}{A_a}}+0.53}{\sqrt{36.7\dfrac{A_{eff}}{A_a}+1}} \;\; ; \;\; 3.67\dfrac{A_{eff}}{A_a} - 0.1 \geq 0 \end{cases}$$

Plotting this result along with the CIBSE comfort criteria presented above we again obtain two intersecting surfaces that define the technically feasible design combinations that will achieve the thermal comfort objective. Two cases are plotted below – the first places no limit on indoor air velocity and the second places an upper limit of 1.5 m/s (i.e., as discussed above).

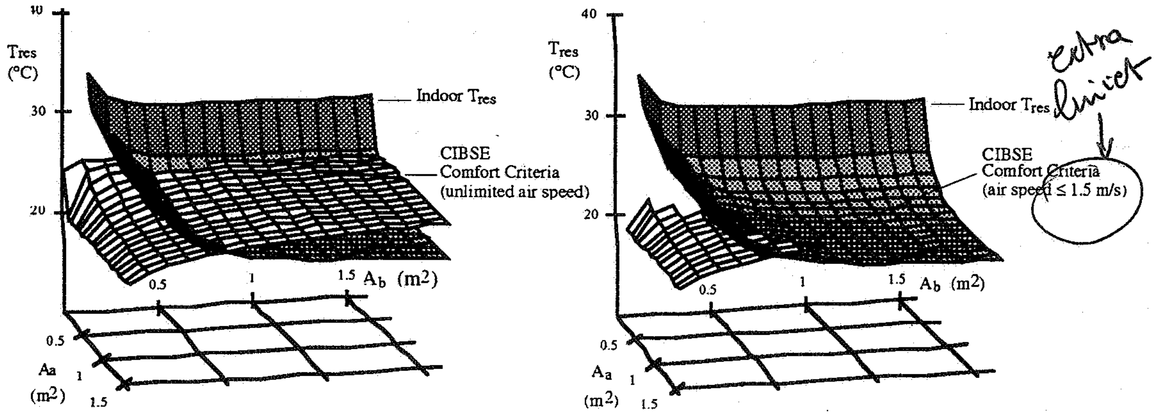

Figure 6. CIBSE dry resultant temperature response and comfort criteria for the representative building for cross ventilation with an approach wind velocity of 5 m/s.

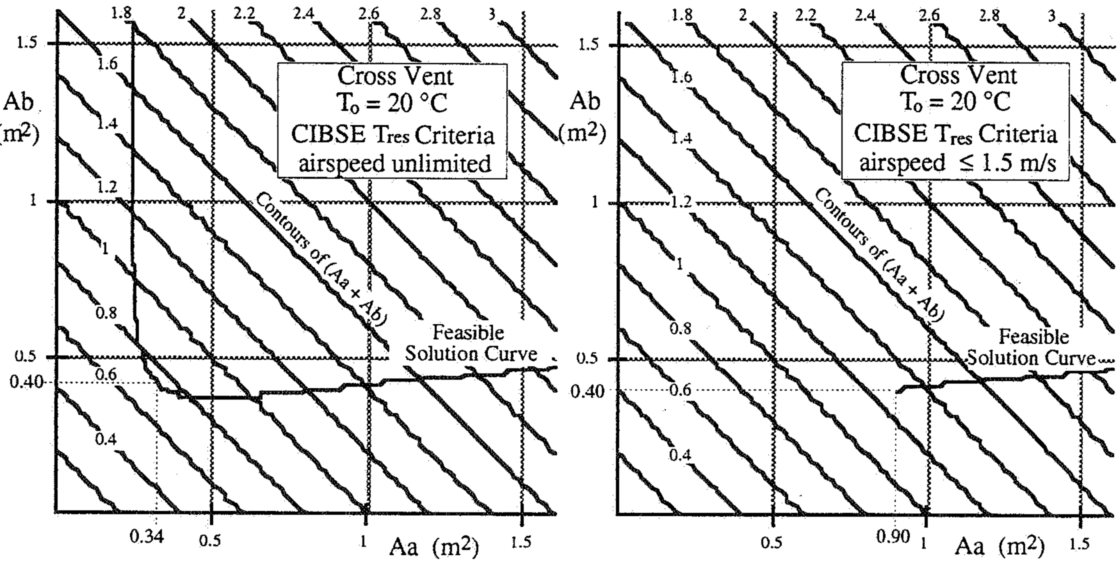

Fig. 7 Curves of feasible combinations of window openings for the cross-ventilation design problem for the CIBSE dry resultant temperature comfort metric.

With this more complete comfort metric the response and comfort criteria surfaces are geometrically more complex, but have the same general features found using the simpler comfort metric. The intersection of the response and comfort surfaces are, however, significantly different – the limit on indoor air speed quite literally limits the intersection of these curves. These intersections, again define the technically feasible combinations of window openings that may be used to achieve the comfort objective. They are plotted in Figure 7 for the two limits placed on indoor air speed.

In the unlimited indoor air velocity case, the feasible solution curve is seen to be similar to that obtained for the simpler comfort metric – falling, interestingly, between the feasible solution curves for T_i = 23 and 26 °C – but now it is no longer symmetric about the $A_a = A_b$ line. Consequently, if we apply the same objective function to minimize total window opening area (note the contours on these plots again) we'll obtain an "optimal" solution of $A_a = 0.34$ m^2 and $A_b = 0.40$ m^2 – a slight bias toward a larger leeward window opening, a position that has been presented as a cross ventilation design guideline [11]. On the other hand, if indoor air velocities are limited, all feasible solutions and the "optimal" least area solution ($A_a = 0.90$ and $A_b = 0.40$, as shown above) involve configurations where the windward opening is significantly greater than the leeward. That is to say, for this particular building case, inlet air velocities must be limited by increasing the resistance to flow at the leeward window(s) to satisfy the 1.5 m/s limit place on indoor air velocities.

3.2 Stack Ventilation of a Single Zone Building

Consider, now, a similar simple single-zone model utilizing a stack ventilation strategy:

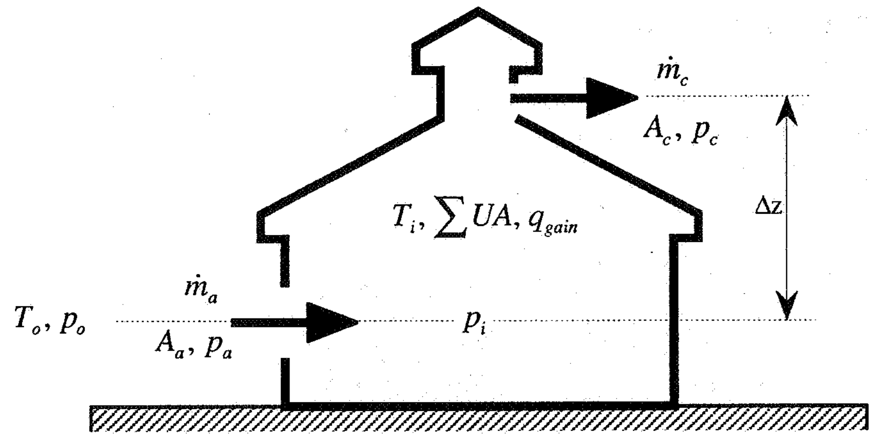

Fig. 8 Single-zone stack-ventilated building idealization and variables.

Again, air enters a window opening A_a, but now exits at a higher opening A_c located a distance Δz above the lower opening. These three variables – A_a, A_c, and Δz – will be taken as the key design parameters that will be adjusted to achieve thermal comfort in this case. All other variables will be defined as before. The ambient pressure p_o and the indoor pressure p_i are defined here relative to the same elevation.

Again, heat transfer system equations may be formed by demanding the conservation of thermal energy:

$$\textit{Heat Transfer:} \quad \sum UA\left(T_i - T_o\right) + \left(\hat{c}_p \dot{m}_c T_i - \hat{c}_p \dot{m}_a T_o\right) = q_{gain} \tag{22}$$

Again, airflow through the windows will be modeled using the orifice equation. In the absence of wind-driven pressures, both indoor and outdoor air pressures will be assumed to vary hydrostatically in proportion to the indoor and outdoor air densities, ρ_i and ρ_o respectively – the, now, conventional assumption of macroscopic airflow analysis [5-7]:

$$m_a = C_d A_a \sqrt{2\bar{\rho}(p_o - p_i)} \ \ and \ \ m_b = C_d A_b \sqrt{2\bar{\rho}\Big((p_i - \rho_i g \Delta z) - (p_o - \rho_o g \Delta z)\Big)} \tag{23}$$

The airflow system equations may be formed by demanding conservation of airflow:

Airflow:

$$m_a + m_b = 0 \tag{24a}$$

$$C_d A_a \sqrt{2\bar{\rho}(p_o - p_i)} + C_d A_b \sqrt{2\bar{\rho}\Big((p_i - \rho_i g \Delta z) - (p_o - \rho_o g \Delta z)\Big)} = 0 \tag{24b}$$

Finally, we'll use the ideal gas law to estimate air densities indoor and out:

$$\rho = \frac{p\,M_{air}}{RT_{^\circ K}} \ \ where \ \ \frac{p\,M_{air}}{R} \approx 352.6 \, {}^{kg-{}^\circ K}/_{m^3} \ \ and \ \ T_{^\circ K} = T + 273.15 \tag{25}$$

In this case, the heat transfer and airflow equations are coupled through both the airflow rate and buoyancy terms and the resulting nonlinearity is pathological. Consequently, it was not possible to <u>explicitly</u> solve the resulting equations for the indoor air temperature T_i in terms of the design variables, A_a and A_c. Nevertheless, these equations may be combined to form an equation that <u>implicitly</u> defines such a relation:

$$\frac{q_{gain}^2}{\Delta T^2} - 2\frac{\Sigma UAq_{gain}}{\Delta T} + \Sigma UA^2 = \frac{\left(2\frac{\Sigma UA\,q}{\Delta T} - \Sigma UA^2 - \frac{q_{gain}^2}{\Delta T^2}\right)A_b^2}{A_a^2} + 2\frac{\bar{\rho}\Delta z C_d^2 \bar{c}_p^2\, 352.6\, g \Delta T A_b^2}{(Ti + 273.15)(To + 273.15)} \ ; \ \Delta T = T_i - T_o \tag{26}$$

This implicit solution may then be plotted using numerical root-finding methods available in popular math processing programs.

We'll apply this result to the representative building presented above – a reasonably conventional single story building with total cooling load q_{gain} = 43 W/m^2 (4 W/ft^2). The indoor air temperature response is plotted below, for this stack-ventilated case, along with the indoor air temperature comfort criteria of T_i = 26 °C:

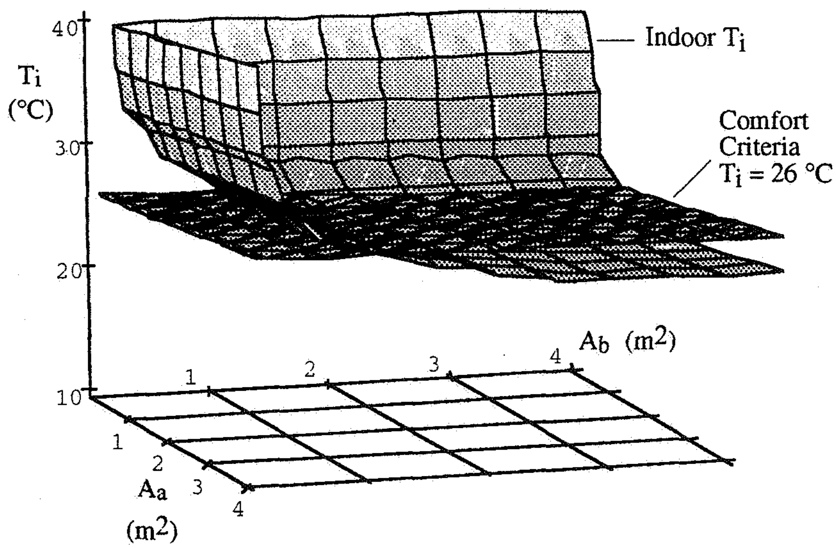

Fig. 9 Indoor air temperature and comfort criteria for the representative building using stack-ventilation with an outdoor air temperature of T_o = 20 °C.

As expected, the indoor air temperature approaches the outdoor air temperature for large window openings. The intersection of the comfort criteria surface and the indoor air temperature response surface establishes combinations of window openings that will achieve the 26 °C comfort objective – the feasible solution curve. Feasible solution curves were generated for a range of stack heights, the third design parameter, and are plotted below. In addition, the inlet air velocity is also plotted as it varies with inlet window opening, A_a.

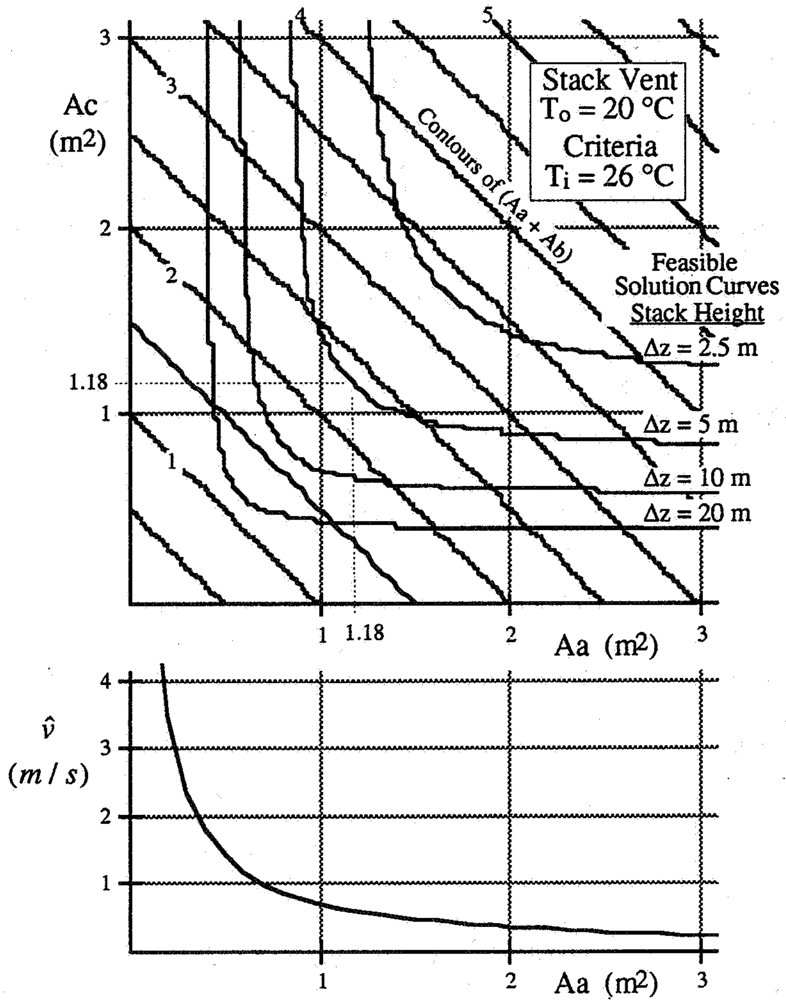

Fig. 10 Feasible solution curves for a range of stack heights, Δz, and inlet air velocity for the representative building using stack-ventilation with an outdoor air temperature of $T_o = 20$ °C.

A building designer could use these feasible solution curves in much the same way as before to guide design decisions while maintaining the comfort object (i.e., here $T_i = 26$ °C). For example, with a reasonable stack height of 5 m the designer could achieve the 26 °C comfort objective with $(A_a, A_c) \approx (3.0\ m^2, 0.8\ m^2)$, $(A_a, A_c) \approx (2.0\ m^2, 0.9\ m^2)$, $(A_a, A_c) \approx (1.0\ m^2, 1.4\ m^2)$, or choose the combination that minimizes the total window opening of $(A_a, A_c) \approx (1.18\ m^2, 1.18\ m^2)$. If the designer felt it was important to limit inlet air velocities to, say, 0.5 m/s then inlet window openings would have to greater than, approximately, 1.5 m^2 regardless of stack height (i.e., to simultaneously achieve the 26°C objective).

4. Conclusion

A general approach to solve ventilation design problems typically addressed at the preliminary design stage has been presented that is based on well-established macroscopic analysis theory. Equations, based on conservation of mass, momentum, and energy are formulated that relate room response to the key ventilation design parameters. These conservation equations may then be combined with comfort criteria also formulated in terms of these key design parameters to identify, unambiguously, feasible combinations of the design parameters that will achieve the thermal comfort objective. While it is argued that the description (i.e., curve, surface, or hypersurface) of these feasible combinations are likely to be of greatest use to the building designer, one may, in addition, add an objective function to allow the search for an "optimal" combination. It is likely, however, that the "optimal" solution will be sub-optimal from the point of view of a skilled designer in as much as such a designer will, in effect, be searching in a design space of far greater complexity than that defining the ventilation problem.

The application of the general approach to a simple single-zone building was demonstrated. Both wind-driven cross-ventilation and buoyancy-driven stack-ventilation schemes were considered along with thermal comfort assessed in terms of a) the indoor air temperature and b) the CIBSE dry resultant temperature with air velocity correction. Although simple, these cases are likely to be practically useful to designers of residential and small commercial and office buildings. From a general perspective, these simple cases demonstrate that the equations that result from the approach may be expected to be pathologically nonlinear and symbolically insoluble, thus, in general, numerical methods will have to be applied to evaluate both the relation between the chosen comfort index and the design parameters and the feasible combinations of these design parameters that will achieve the comfort objective. For simple problems this may readily be achieved with available math processing programs; for more complex cases it will be more reasonable to extend the capabilities of existing building energy simulation, airflow, and indoor air quality programs.

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