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Flow Paths in a Swedish Single Family House

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Synopsis

The traditional description of a flow system with a multicell model, $V\dot{c}(t)=Qc(t)+p(t)$, may sometimes be to restrictive. The multicell theory require measurements, or reconstruction, of the tracer concentrations in a number of perfect and immediate mixed cells. Often, using of mixing fans and closing doors between cells are necessary to comply with the theory.

Another, very useful and more general description of a flow system is through the *weighting function.* Unlike the multicell model this is not an internal model, but only an input- output model and it is thus less informative. Still, knowing the weighting function is sufficient to determine the flows rates, the active volumes, the mean ages of exhaust air and of room air and the air exchange efficiencies.

Typically, these parameters are determined for *each flow path* from an input, often a supply air, to an output, often an exhaust air.

Contrary to common air exchange efficiency measurements, resulting in one ore a few figures, the determination of the weighting function results in a complete model which is also suitable for simulations.

In this paper, the theory is only surveyed and no derivations are given. A single family house with exhaust ventilation is used to demonstrate the method.

List of Symbols

$c(t)$	tracer gas concentration
$\mathrm{diag}(x)$	a diagonal matrix with the column or row vector x on the diagonal
e	the 1-column vector
e_e	the 1-column vector with n_e elements
$h(t)$	weighting function matrix
n	number of cells
n_e	number of exhaust air flows
n_s	number of supply air flows
$p(t)$	tracer gas injection
q	flow
t	time
t_n , T_n	nominal time constant
t_r , T_r	room mean age
E^i	i:th moment of h(t)
F	flow path matrix, n_e by n_s
Q	flow matrix, n by n
V	volume matrix, n by n

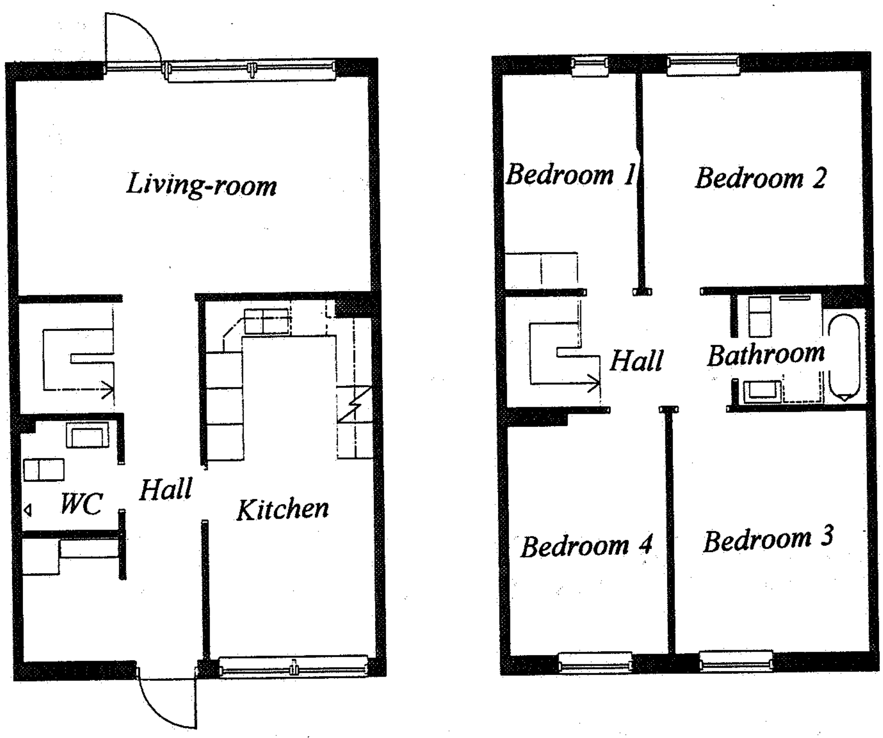
1 Introduction

The ventilation of a single family house is studied by means of tracergas measurements.

The house has a living-room, kitchen, toilet, hall with staircase on the first floor and four bedrooms, hall and bathroom on the second floor, see figure below. It has exhaust ventilation with outlets in kitchen, toilet, bathroom an one bedroom. Outdoor air inlets are through small openings at the windows in the living-room, kitchen and the bedrooms - and through infiltration.

Some tracer gas experiments have been done in the house. N_20 was used as a tracer and was injected according to a certain pattern at nine locations, the six known air inlets, the toilet and the halls at first and second floor

In the second section the experiments are analysed with a traditional multicell model. Some shortages are pointed out and the need of a more general model is established. In the next two sections the weighting function and its usage is briefly introduced first in the single inflow - single outflow case then in the multiple inflow - multiple outflow case. In the sixth section the method is used to analyse the experiments of the single family house. The paper ends with a summing up which also includes some words about some difficulties encountered



2 Multicell Modelling

Multicell modelling refers to a family of similar methods to determine the flow matrix Q and the diagonal volume matrix V of the model: $V\dot{c}(t)=Qc(t)+p(t)$, see Chap.3 of Roulet and

Vandaele (1991). Here, an improved version of the method described in Hedin(1989) has been used. This is a single gas, iterative method with a derivative or integral approximation based on the so far found model and a constrained least square parameter estimation.

Two experiments will be presented, one with and one without mixing fans. The used equipment permits nine cells and a reference concentration measurements. The upstairs hall and bathroom was therefor treated as one cell and this door was open. All other doors were closed. Note that the two halls the staircase and the living-room are not separated by doors. This volume is neither well treated as one single cell nor as three separate cells as being tried here.

The two graphs below shows the modelled tracer concentrations, $c_{mod}(t)$, as a function of time. The small rings corresponds to the measurements The fit looks to be acceptable at least in the first case but the errors are in fact larger than in similar experiments in other houses. Without mixing fans the errors increase about four times.

The identified flow rates and volumes for the first case are shown in the tables below.

Air Flow Rates [l/s]

from → *to ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*hall downstr.*	*kitchen*	*living room*	*toilet*	*bath../hall*	*infiltra-tion*	*total*
bedroom1	-									8.8	8.8
bedroom2		-								8.2	8.2
bedroom3			-							9.2	9.2
bedroom4				-						8.8	8.8
hall downstr	0.4			2.8	-	0.6	50.8	3.1	15.9	8.2	81.7
kitchen					4.7	-				4.2	8.8
living-room	0.3	0.7	1.5	0.7	42.1		-		8.0	10.5	64.8
toilet					14.8					.0	14.8
bathroom	1.0	5.7	7.2	5.3	7.1		13.0		-	4.5	43.8
exfiltration	7.2	1.8	0.5		13.1	8.2		11.7	20.0		62.5

Volumes [m^3]

bed-room1	*bed-room2*	*bed-room3*	*bed-room4*	*hall downstr*	*kitchen*	*living-room*	*toilet*	*bath../hall*	*total*
19.7	23.5	32.6	26.6	27.5	40.8	56.9	6.6	31.6	265.6

Even if the fit looks good there are some problems worth to discuss. It is not easily seen from the graph above, but a closer look on the data shows that then injecting tracer to the downstairs hall the concentration became higher at the toilet then in (at least the measuring point) of the hall itself. This is due to an imperfect mixing in the hall (in relation to the large flow to the toilet and the location of the tracer injection). Such a situation cannot be explained by a multicell model, unless the number of cells is increased. Another, more easily seen, shortcoming is the bad modelling of the bedroom 2 concentration. Also this is probably due to incomplete mixing. With better mixing the peak had been lowered. The multicell model cannot fit both this to high peak and the correct decay.

3 The Weighting Function

The weighting function, also called the impulse response function, is a basic input- output relation of a linear, time invariant system. It is defined, see e.g. chap.1 of Kailath(1980), as

h(t) = the response of a linear system at time t to a unit impulse at time 0

For a system with n_s inputs and n_e outputs its straightforward to define a weighting function matrix (n_e by n_s), here also denoted by h(t), where the elements $h_{ij}(t)$ are defined as

$h_{ij}(t)$ = the response at output No. i of a linear system at time t to a unit impulse to input No. j at time 0

In- and outputs can be selected in many ways. Here it is natural to choose the supply and exhaust so that h_{ij} is the relation from a release of tracer in the supply air No. j to the resulting concentration in the exhaust air No. i.

A main problem is, of course, how to actually find the weighting function. In fact, the discrete time - not the continuously time - weighting function is determined and the necessary corrections are performed, but all these issues are elaborated elsewhere (Hedin(1993), Jensen(1988)).

Now, assuming that we know the weighting functions, the following notation is useful to avoid writing a lot of integrals (in the matrix case the integration is done element-by-element)

$$E^0 = E^0(h(t)) = \int_0^\infty h(t)dt$$

$$E^1 = E^1(h(t)) = \int_0^\infty t\,h(t)dt$$

$$E^2 = E^2(h(t)) = \int_0^\infty t^2\,h(t)dt$$

It is known from Jensen(1988) and (partly from) Sutcliffe(1991) that the following relations hold in the case of one inflow and an equal outflow and no in- or exfiltration.

flow $$q = \frac{1}{E^0}$$

active volume $V_a = \dfrac{E^1}{E^0 E^0}$

nominal time constant (mean residence time) $t_n = \dfrac{E^1}{E^0}$

room mean age $t_r = \dfrac{E^2}{2E^1}$

air change efficiency $N_a = \dfrac{E^1 E^1}{E^2 E^0}$

This means that *all the parameters: flow, volume, mean age of flow and volume and the air change efficiency are easily computed from the weighting function, h(t)*. This is true also in the multiple inflow - multiple outflow case. In the next section the corresponding formulas will be given (but not derived) for the multiple case.

{Then the moments are calculated from the discrete time impulse response $(\hat{E}^i, i = 0,1,2)$ the following corrections must be done

$$E^0 = \hat{E}^0, \quad E^1 = \hat{E}^1 - T_s / 2\hat{E}^0 \quad and \quad E^2 = \hat{E}^2 - T_s \hat{E}^1 + T_s^2 / 6\hat{E}^0$$

Here, T_s is the sample interval. }

4 The Flow Path Matrix

In the multiple case, there are up to $n_e n_s$ flows to deal with. This is best done if the flows are organised in a matrix here called the flow path matrix. The flow path matrix, F, is an n_e by n_s matrix whose elements f_{ij} are the net flow rate from the supply airflow No. j to the exhaust airflow No. i. We write 'net' flow rate as, internally in the system, there may be large circulated flows, but f_{ij} is the resulting (purging) flow of supply air leaving at the specific exhaust.

The flow path matrix is calculated from the 0:th moment of the weighting function matrix as

$$F = diag(q_e) E^0 diag(q_s)$$

where

input (supply) flows $q_s = E^{0^T} \setminus e_s$

output (exhaust) flows $q_e = E^0 \setminus e_e$

The other parameters are calculated as

active volumes $V_a = F.* E^1 ./ E^0$

nominal time constants (residence times) $T_n = E^1 ./ E^0$

room mean ages $T_r = 0.5 E^2 ./ E^1$

air change efficiencies $N_a = E^1 .* E^1 ./ (E^0 .* E^2)$

Note that ./ and .* denotes simple element-by-element division and multiplication respectively, but, \ denotes matrix left division i.e. solving of a linear equation system, which may involve fairly heavy computations. In fact, it is a critical point how to solve these two equations. It is recommended - though not necessary - to collect data so the linear equation systems became over determined and to solve them with some least square method.If n_e=n_s=1 the equations above will reduce to those given in the last section for the scalar case.

5 Relations to the multicell model.

The model above tells different properties than the multicell model does. Naturally - as the multicell model is a complete internal model - it is possible to derive the same pieces of information from a known multicell model {V,Q}. To give two exempla, it hold that

$$\tilde{F} = -\, diag(-e^T Q)\; Q^{-1}\, diag(-Qe)$$

$$\tilde{T}_n = -\, diag(-e^T Q)\; Q^{-1}\, V\, Q^{-1}\, diag(-Qe)$$

The ~ sign is used to show that these two matrices are of dimension n by n.

Of course, the opposite is not true. It is not possible to extract the model {V,Q} from the weighting function or parameters derived from it.

6 The Case Study

Now we return to the experiments in the single family house described in the two first sections. Application of the theory above results in the parameters of the two following pages.

Air Flow Rates [l/s]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./hall*	*infiltra-tion*	*total supply + infiltr.*
bedroom1	2.9	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.7	5.0
kitchen	0.1	0.2	0.2	0.2	0.2	2.3	0.1	0.0	0.1	5.1	8.3
toilet	1.0	2.1	2.1	2.3	3.6	0.1	4.0	0.0	1.0	2.3	18.5
bathroom	1.0	2.8	2.8	2.4	1.0	0.0	1.8	0.0	1.5	6.9	20.3
total exhaust	5.0	5.4	5.2	5.0	4.8	2.4	5.9	0.0	2.6	15.9	52.1

Active Volumes [m^3]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./hall*	*total supply*
bedroom1	8	4	0	0	0	0	0	0	0	13
kitchen	1	2	3	2	1	9	1	0	1	20
toilet	11	28	20	17	8	1	18	0	4	108
bathroom	6	33	16	11	4	0	8	0	3	81
total exhaust	26	68	40	30	13	11	27	0	8	223

Nominal Time Constant [min]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./hall*	*total supply*
bedroom1	48	216	157	-	-	-	-	-	-	65
kitchen	153	218	238	151	125	69	151	-	206	203
toilet	193	228	157	119	39	196	74	-	75	111
bathroom	95	195	95	80	59	-	74	-	34	101
total exhaust	87	210	127	101	46	75	75	0	54	103

Air Change Efficiencies [-]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./hall*	*total supply*
bedroom1	0.48	0.58	-	-	-	-	-	-	-	0.34
kitchen	0.78	0.63	0.68	0.68	0.58	0.53	0.66	-	0.62	0.47
toilet	0.55	0.58	0.64	0.64	0.35	0.73	0.53	-	0.51	0.44
bathroom	0.51	0.48	0.55	0.62	0.54	-	0.62	-	0.30	0.40
total exhaust	0.38	0.53	0.56	0.61	0.38	0.51	0.55	-	0.35	0.41

Air Flow Rates [l/s]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./ hall*	*infiltra-tion*	*total supply + infiltr.*
bedroom1	4.7	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.9	7.0
kitchen	0.0	0.1	0.4	0.4	0.8	2.6	0.7	0.0	0.1	3.3	8.5
toilet	0.3	1.8	2.0	2.8	3.5	0.0	3.6	0.0	0.9	5.0	19.9
bathroom	0.5	2.9	3.2	3.0	1.0	0.0	2.4	0.0	1.8	6.8	21.5
total exhaust	5.5	5.1	5.6	6.2	5.3	2.6	6.7	0.0	2.9	17.0	56.9

Active Volumes [m^3]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./ hall*	*total supply*
bedroom1	12	2	0	0	0	0	0	-	0	15
kitchen	0	1	7	7	6	12	7	-	1	42
toilet	2	23	19	21	8	0	13	-	4	89
bathroom	1	18	16	15	3	0	8	-	3	64
total exhaust	16	43	41	44	17	13	28	-	8	209

Nominal Time Constant [min]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall down*	*toilet*	*bath./ hall*	*total supply*
bedroom1	44	117	-	-	-	-	-	-	-	48
kitchen	-	164	290	284	116	80	150	-	163	133
toilet	102	203	154	125	38	-	61	-	73	100
bathroom	48	101	83	86	51	-	57	-	25	72
total exhaust	48	140	124	117	53	80	70	-	47	87

Air Change Efficiencies [-]

supply → *exhaust ↓*	*bed-room1*	*bed-room2*	*bed-room3*	*bed-room4*	*living room*	*kitchen*	*hall. down*	*toilet*	*bath./ hall*	*total supply*
bedroom1	0.47	0.68	0.00	0.67	0.00	0.00	0.00	..-	0.00	0.46
kitchen	-	0.78	0.61	0.62	0.59	0.49	0.65	..-	0.63	0.46
toilet	0.68	0.60	0.65	0.64	0.40	-	0.62	-	0.58	0.47
bathroom	0.63	0.54	0.56	0.55	0.61	..-	0.60	-	0.29	0.50
total exhaust	0.48	0.53	0.50	0.52	0.41	0.49	0.54	-	0.35	0.44

7 Summing Up

Describing a single or multicell flow system, the first choice is the traditional multicell model $V\dot{c} = Qc + p$. This model is easy to understand and easy to use . However, there are cases when this model fails. The description with a flow path matrix and the weighting function for each flow path may then be a good substitute. This model tells little about internal flows but it is sufficient to compute flow rates, active volumes, mean ages, and air efficiencies for each flow path.

Some problems and drawbacks associated with weighting function modelling should be mentioned: 1) It is a time-invariant model. That means the flows must be constant (at least during the experiment). 2) If there are many inputs, then the method is very time consuming. A remedy to this may be to use a multiple gas equipment, which would shorten experimental time considerably. 3) It is not hard to construct (theoretical) examples, especially with circulated flows, when the weighting function would be very hard to determine because a not negligible part of the response is coming after long time with low intensity thus giving a low signal-to-noise ratio. 4) Air infiltration and exfiltration, or to be more specific, air inflows that is not possible to homogeneously label with tracer and outflows where it is not possible to measure the mean tracer concentration will inherently cause problems and require additional presumptions about the system.

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