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SINGLE-ZONE STACK-DOMINATED INFILTRATION MODELING

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ABSTRACT

Simplified, physical models for calculating infiltration in a single zone, usually calculate the air flows from the natural driving forces separately and then combine them. For most purposes—especially minimum ventilation or energy considerations—the stack effect dominates and total ventilation can be calculated by treating other effects (i.e. wind and small fans) as perturbations, using superposition techniques. The stack effect is caused by differences in density between indoor and outdoor air, normally attributable to the indoor-outdoor temperature difference. This report derives an exact, but practical, expression for calculating the stack effect from the air densities and leakage distribution using the power law formulation of envelope leakage. The neutral height—the height at which there is no stack-related indoor-outdoor pressure difference—is a key intermediate in stack modeling. This report defines a computable parameter called ***stack height***, which contains all of the leakage distribution information necessary for estimating stack flows, thus freeing the model from specific assumptions (e.g. that the leakage is separable into evenly distributed floor, wall, and ceiling components). Example calculations including comparisons with other models, as well as validations using measured data from dwellings, are also presented. The dimensionless ***neutral level***, which is related to the neutral height, is often used as an indicator of leakage distribution and in superposition. Its definition and role in these regards are discussed in detail. The more exact formulation is then used to analyze the simple box cases normally assumed in infiltration modeling and other approximations. Measured ventilation data will be used to infer leakage distributions and neutral levels as well as for example calculations.

Keywords: Infiltration, Ventilation, Stack Effect, Validation, Dwellings, Indoor Air Quality, Single-zone, Modeling, Superposition.

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NOMENCLATURE

β_s	Neutral level [-]
ELA	Effective leakage area [m^2]
f_{β_s}	Neutral level factor [-]
g	Acceleration of gravity [m/s^2]
h	Height [m]
H	Box height [m]
h_n	Neutral height (stack only) [m]
H_s	Stack height [m]
K	Leakage coefficient
$k[h]$	Leakage coefficient per meter at height h
$\dot{m}$	Stack-induced mass flow rate of air [kg/s]
n	Leakage exponent [-]
P	(Air) pressure [Pa]
ΔP	Representative pressure drop across the envelope [Pa]
R	Box ratio [-]
ρ	Density (of air) kg/m^3
Q	Stack-induced air flow [m^3/hr]
X	Vertical leakage asymmetry[-]

Subscripts indicate values associated with:

$+$	*in*filtration
$-$	*ex*filtration
$\uparrow$	lighter air (i.e. leakage through upper part of building)
$\downarrow$	denser air (i.e. leakage through lower part of building)
o	reference
i	individual leak

INTRODUCTION

The calculation of infiltration-dominated ventilation usually requires the combination of wind-induced, temperature-induced, and mechanically-induced air flows. Complex models solve the problem by finding the pressure at each point on the envelope and then solving for the flow—modifying the internal pressure in order to satisfy the continuity equation.[1] Such an approach is very powerful, but may require inputs and computational requirements that may make it impractical. For many applications simpler models are desirable, even if less accurate. Each of these three mechanisms induces pressures across the envelope to drive the flow, but the spatial distribution of the pressure is different for each one of them. The focus of this report will be to develop appropriate expressions for the descriptions of the *stack effect* for use in single-zone modeling.

Stack Effect

The stack effect is the flow resulting from hydrostatic pressure differences caused by density differences in two fluid columns. For buildings, the fluid is air and the density difference is caused by bulk temperature differences. Although humidity and other variations in constituents of air can cause density differences, they are usually minor compared with normal temperature differences and will be ignored.

The physics of the stack effect is straightforward and has been understood for a long time. In 1926 Emswiler[2] defined the *neutral level* as the height at which there was no pressure difference and thus no flow, and related the flow through large openings in the building to the square-root of the vertical distance from the neutral height.

When there is internal resistance (i.e. airtight partitions or floors) the neutral level may not be unique and the pressure gradient complex. Such has been demonstrated for multistory buildings.[3] By definition a single-zone building has no significant internal resistances and so we will ignore such complexities. The stack effect, however, can cause large pressure differences even in single zones which can adversely affect mechanical ventilation systems.[4]

In tall structures or when the density difference is small, density gradients can play a significant role. We will, however, assume in this report that they are not important.

MOTIVATION

Many infiltration models are currently in use;[5] but the most widely used single-zone model is the LBL infiltration model,[6,7] which is included in the ASHRAE Handbook of Fundamentals.[8] A recent model, AIM-2, by Walker and Wilson[9] builds upon the LBL model and makes some generalizations. All of the single-zone models share the characteristic of treating the zone as a rectangular box, having a fixed floor and ceiling height. Thus all of the *vertical* leakage is concentrated at two heights. As the assumption is usually made that leakage in the walls is evenly distributed, the leakage distribution can be described by three parameters. In the case of the LBL model (and AIM-2), these three parameters are the total effective leakage area, *ELA*, the fraction of leakage in the vertical surfaces, R, and the fractional difference in the ceiling-floor leakage, X[‡].

Real buildings, of course, are not usually simple boxes; there are split level, multiple story, and partially bermed buildings having vertical stacks and uneven surfaces. Palmiter and Brown[10] were the first to quantify the size of the errors caused by this assumption. They compared the definition of lowest leak to highest leak discussed in the ASHRAE Standard on air leakage[11] with an area weighted column of air and found that the former produced an average value 32% higher for their example houses. The two definitions should produce the same result for a simple box.

It is not necessary to determine which of the two definitions is superior to realize that the simple box model may be inappropriate for a large class of real buildings. The size of the error suggested by Palmiter and Brown provides a motivation to develop a stack model which does not rely on the box assumption and then to develop a more general derivation based on a simple box.

‡ Many box models use the parameters, R, X, β_s, etc. The definitions used in this report may be somewhat different, but will, in general, be similar in nature.

ENVELOPE LEAKAGE

The stack effect is buoyancy-caused, pressure-driven air flow through the envelope of the building. It is, therefore, important to understand the leakage properties of the envelope in order to understand the stack effect. Envelope leakage is conventionally treated as a power law.[12] The measurement of leakage is usually performed with a technique called fan pressurization[13] wherein the fan flow induces a shift in the internal pressure:

$$Q = K\Delta P^n \tag{1}$$

where the exponent $\frac{1}{2} \leq n \leq 1$ depending on the hydrodynamics of the leaks.

In addition to being measured from a fan pressurization test, the leakage parameters can be found from more advanced techniques[14,15]

The exponent is a particularly important characteristic of the flow for both understanding the behavior and modeling it. If the exponent were unity, the modeling would be linear and relatively simple. For most buildings, however, the exponent is in the range $0.55 \leq n \leq 0.75$ with $n=2/3$ being a typical value.[16]

The whole-building leakage is clearly made up of many parallel leakage paths. Although these leakage paths will not, in general, have the same exponent, it is commonly assumed that all macroscopic areas of the building can be treated as having the same exponent. Similarly, it is assumed that K is temperature independent; reference 12 demonstrates this independence only for the specific value of $n=2/3$

For the purposes of this report we will make these same assumptions. Thus, we will treat all leakage sites as though they are described by a single exponent and a temperature-independent leakage coefficient. It is clear, however, that a better understanding of the leakage process is needed.

Rather than describing leakage with a coefficient of mixed dimensions, leakage is often discussed in terms of an effective leakage area, **ELA** defined by

$$ELA \equiv K \sqrt{\frac{\rho_o}{2}} P_o^{n-\frac{1}{2}} \tag{2}$$

where the reference pressure is usually taken to be 4 Pa. Many of the models also use *ELA* directly, rather than K.

STACK EFFECT

The stack effect is caused by the hydrostatic pressure difference of two columns of fluid at different density. To be specific since indoor air and outdoor air are at different temperatures their densities are different and there is a stack effect. The pressure difference will be a function of the density difference and height:

$$\left|\Delta P\right| = \Delta\rho g \left|h - h_n\right| \tag{3}$$

where

$$\Delta\rho \equiv \rho_\downarrow - \rho_\uparrow \tag{4}$$

is the density difference between the two columns of air and the neutral height, h_n, is the height at which the pressure in the two columns is equal[↓]. Since we are ignoring density gradients, the density inside and out can each be represented by a single (average) value.

To calculate the air flow it is necessary to apply eq. 1 by integrating the leakage at each height.

$$Q_{\downarrow} = \int_{-\infty}^{h_n} k[h] \left(\Delta\rho g(h_n - h)\right)^n dh \tag{5.1}$$

$$Q_{\uparrow} = \int_{h_n}^{\infty} k[h] \left(\Delta\rho g(h - h_n)\right)^n dh \tag{5.2}$$

Note that we have separated the infiltration (positive pressures) from the exfiltration (negative pressures).

The neutral height is fixed by the requirement that the *mass* balance between infiltration and exfiltration be maintained.

$$\rho_{\downarrow} Q_{\downarrow} = \rho_{\uparrow} Q_{\uparrow} = \dot{m} \tag{6.1}$$

$$\int_0^{\infty} \left(\rho_{\uparrow} k[h_n+h] - \rho_{\downarrow} k[h_n-h]\right) h^n dh = 0 \tag{6.2}$$

It is conventional to use volumetric rather than mass flows to describe infiltration so we will seek a form for the stack flow as follows:

$$\rho_o\, Q = \dot{m} \tag{7}$$

which means we must use a reference density. Since eq. 1 is a volumetric flow equation, we can select the density in such a way as to have the stack flow be analogously defined:

$$Q = \mathrm{K} \int_{-\infty}^{\infty} \frac{k[h]}{2\mathrm{K}} \left|\Delta\rho g(h - h_n)\right|^n dh \tag{8}$$

The density must then be as follows:

$$\rho_o \equiv \frac{\int_0^{\infty} \left(\rho_{\uparrow} k[h_n+h] + \rho_{\downarrow} k[h_n-h]\right) h^n\, dh}{\int_{-\infty}^{\infty} k[h] \left|h_n - h\right|^n dh} \tag{9.1}$$

↓ Note that the definition for neutral height in this report is specifically for the case in which only the stack effect operates. The actual height at which the indoor and outdoor pressures are equal can be affected by other factors (e.g. fans or wind).

Equivalent Stack

To gain some insight into these formidable integrals, we consider the case of a simple stack in which there is a leak, $K_\downarrow$, at the bottom ($h=H_\downarrow$) and another, $K_\uparrow$, at the top ($h=H_\uparrow$). This situation is exactly analogous to the stack effect in a building in which there are leaks only in the floor and ceiling.

Using the relationships from the previous section, the neutral height is

$$h_n - H_\downarrow = \frac{H_\uparrow - H_\downarrow}{1 + \left(\dfrac{\rho_\downarrow K_\downarrow}{\rho_\uparrow K_\uparrow}\right)^{1/n}} \tag{10}$$

and the mass flow through the stack is

$$\dot{m} = K \frac{\rho_\downarrow \rho_\uparrow \, \beta_s^n (1-\beta_s)^n}{\rho_\downarrow \beta_s^n + \rho_\uparrow (1-\beta_s)^n} \left(\Delta\rho\, g(H_\uparrow - H_\downarrow)\right)^n \tag{11}$$

where the neutral level is defined as

$$\beta_s \equiv \frac{h_n - H_\downarrow}{H_\uparrow - H_\downarrow} \tag{12.1}$$

$$= \frac{1}{1 + \left(\dfrac{\rho_\downarrow K_\downarrow}{\rho_\uparrow K_\uparrow}\right)^{1/n}} \tag{12.2}$$

and K is the total leakage

$$K = K_\downarrow + K_\uparrow = K_+ + K_- \tag{13}$$

We can rewrite the mass flow as follows:

$$Q = \frac{K}{2} f_{\beta_s} \left(P_s\right)^n \tag{14}$$

where the stack pressure, P_s, is the effective pressure drop across the leakage sites:

$$P_s = \Delta\rho\, g \frac{H_\uparrow - H_\downarrow}{2} \tag{15}$$

and the neutral level factor, f_{β_s}, is defined as follows:

$$f_{\beta_s} \equiv \left\{\frac{2^{1+n} \beta_s^n (1-\beta_s)^n}{\beta_s^n + (1-\beta_s)^n}\right\} \tag{16}$$

The reference density reduces to

$$\rho_o = \frac{\rho_\downarrow(\beta_s^n + (1-\beta_s)^n)\rho_\uparrow}{\rho_\downarrow \beta_s^n + (1-\beta_s)^n \rho_\uparrow} \tag{17}$$

We define a parameter, X, to describe the asymmetry in the leakage:

$$X \equiv \frac{\rho_\uparrow K_\uparrow - \rho_\downarrow K_\downarrow}{\rho_\uparrow K_\uparrow + \rho_\downarrow K_\downarrow} \approx \frac{K_\uparrow - K_\downarrow}{K} \tag{18}$$

or, equivalently,

$$X = \frac{\beta_s^n - (1-\beta_s)^n}{\beta_s^n + (1-\beta_s)^n} \approx n(2\beta_s - 1) \tag{19}$$

If we do this we can rewrite some of the previous expressions:

$$\beta_s = \frac{1}{1+\left(\frac{1-X}{1+X}\right)^{1/n}} \approx \frac{1}{2}(1+X)^{1/n} \tag{20.1}$$

$$f_{\beta_s} = 2^n \frac{1-X^2}{\left((1-X)^{1/n}+(1+X)^{1/n}\right)^n} \approx (1-X^2)^{(1+n)/2n} \tag{20.2}$$

$$\rho_o = \frac{\rho_\downarrow \rho_\uparrow}{\rho_\downarrow + \rho_\uparrow + X\Delta\rho} \approx \frac{\rho_\downarrow \rho_\uparrow}{\rho_\downarrow + \rho_\uparrow} \tag{20.3}$$

The approximations in eqs. 18 and 20.3 are true if the the density difference is small ($\Delta\rho << \rho$); the approximations in eqs. 19 and 20 are true if the leakage asymmetry is small ($X << 1$); and the approximations in eqs. 19, 20.1, 20.2 are also true if the exponent approaches unity ($n \rightarrow 1$). Although the exponent is normally closer to 2/3, the other two conditions are usually true.

This description is complete for a system with exactly two, localized leaks, but it cannot be used in the general case without further refinement. First we must interpret the two leakage sites to be the total leakage area below and above the neutral level respectively:

$$\mathrm{K}_\downarrow \equiv \int_{-\infty}^{h_n} k[h]\, dh \approx \mathrm{K}\,\frac{1-X}{2} \tag{21.1}$$

$$\mathrm{K}_\uparrow \equiv \int_{h_n}^{\infty} k[h]\, dh \approx \mathrm{K}\,\frac{1+X}{2} \tag{21.2}$$

We define the equivalent stack bottom ($H_\downarrow$) and stack top ($H_\uparrow$) so that the mass flow is correct:

$$\mathrm{K}_\downarrow \rho_\downarrow \left(\Delta\rho\, g(h_n - H_\downarrow)\right)^n = \dot{\mathrm{m}} = \mathrm{K}_\uparrow \rho_\uparrow \left(\Delta\rho\, g(H_\uparrow - h_n)\right)^n \tag{22}$$

$$H_\uparrow \equiv h_n + \left(\frac{\int_{h_n}^{\infty} k[h](h-h_n)^n dh}{\mathrm{K}_\uparrow}\right)^{1/n} \tag{23.1}$$

$$H_\downarrow \equiv h_n - \left(\frac{\int_{-\infty}^{h_n} k[h](h_n-h)^n dh}{\mathrm{K}_\downarrow}\right)^{1/n} \tag{23.2}$$

Thus, as can be directly verified, eqs. 11,12,15 can be used in the general case with these definitions.

Stack Height

We now seek to compress all of the leakage distribution information into a single parameter so as to express the infiltration as follows:

$$Q = \frac{K}{2}\left[\Delta\rho\, g \frac{H_s}{2}\right]^n \tag{24}$$

where H_s is called the ***stack height.***

The general relationship for the stack height can be found by comparing this to eqn 8:

$$H_s = 2\left[\frac{\int_{-\infty}^{\infty} k[h]\left|h - h_n\right|^n dh}{K}\right]^{1/n} \tag{25.1}$$

Equivalently, the stack height can be related to the equivalent stack top and bottom as follows:

$$H_s = f\beta_s^{1/n}\,(H_\uparrow - H_\downarrow) \tag{25.2}$$

Thus all of the information about the leakage distribution (relevant to the stack effect) can be contained in a single parameter, the stack height. Other formulations, such as simple box models, can be converted into this form.

LEAKAGE DISTRIBUTION EXAMPLES

The following examples illustrate the procedure for three houses of differing construction. The procedure calculates the stack height and leakage distribution parameters; specific temperature conditions could be used to calculate the flows. The leakages are quoted in effective leakage area, which is linearly related to K by eq. 2 and can be used in its stead.

For the examples below we will estimate the neutral height by ignoring the density differences, assuming that any unaccounted for leakage is evenly distributed in height, and defining the floor level as the zero in height. Operationally, one can use an exponent of unity in eq. 6.2 to estimate the neutral height, in which case

$$h_n \approx \frac{\sum_i K_i h_i + (K - \sum_i K_i)H/2}{K} \tag{26.1}$$

or, equivalently

$$h_n \approx \frac{H}{2} + \sum_i \frac{K_i}{K}(h_i - \frac{H}{2}) \tag{26.2}$$

where H is the height of the house and h_i is the height at which leak K_i is located.

Having determined the neutral height, X, β_s, and H_s can then be calculated from eqs. 12,18, and 25.

Default House

Consider a two-story (5m) tall house with a total leakage area of 700cm^2 (with an exponent of 0.65) and 200m^2 of floor area (10m x 10m footprint). If we know nothing else about the leakage distribution, we might reasonably assume that the envelope is uniformly porous in which case there is 175cm^2 of floor leakage, there is 175cm^2 of ceiling leakage, and there is 350cm^2 in the walls. Using the approximate expression for the neutral height,

$$h_n \approx \frac{5}{2} + \frac{175}{700}(5-2.5) + \frac{175}{700}(-2.5) = 2.5m \tag{27}$$

(This result is, in fact, that from the exact neutral height expression, eq. 6.2)

$$\left(\frac{H_s}{2}\right)^{.65} = \frac{\frac{350}{2.5*1.65}\,|-2.5|^{1.65} + \frac{350}{2.5*1.65}\,|2.5|^{1.65} + 175\,|-2.5|^{.65} + 175\,|2.5|^{.65}}{700} \tag{28}$$

The values of the parameters are

$$H_s = 3.57 \qquad X = 0 \qquad \beta_s = 0.50 \tag{29}$$

Slab-on-Grade House

Consider a single-story (2.5m), slab-on-grade house of 700cm^2 of leakage area (with n=0.65) in which the only leakage is in the walls and at ceiling level. There is 140cm^2 of leakage at the level of the ceiling (2.5m), the remainder is assumed to be spread evenly in the walls. Using the approximate expression for the neutral height we get

$$h_n \approx \frac{1}{700}\left(140*2.5 + \frac{560*2.5}{2}\right) = 1.5m \tag{30}$$

The stack height can now be calculated as follows:

$$\left(\frac{H_s}{2}\right)^{.65} = \frac{\frac{560}{2.5*1.65}*|-1.5|^{1.65} + \frac{560}{2.5*1.65}*|1|^{1.65} + 140*|1|^{.65}}{700} \tag{31.1}$$

$$H_s \approx 1.3 \tag{31.2}$$

More exact values of the parameters are

$$h_n = 1.52m \quad H_s = 1.34m \quad X = 0.03 \quad \beta_s = 0.52 \tag{32}$$

The stack height is significantly less than the floor-to-ceiling height because of the concentration of leakage at ceiling height without any compensating leakage at floor level, even though the net asymmetry is small.

Crawlspace House

Consider a single-story, crawlspace house of 700cm^2 of leakage area (with n=0.65). 210cm^2 of leakage is at floor level, there is a 50cm^2 dryer vent 1m above the floor, 140cm^2 of leakage at the level of the ceiling (2.5m), and a total of 100cm^2 of leakage at the 4m level due to (insulated) flues, vents, and chimneys; the remainder is assumed to be spread evenly in the walls. Using the approximate expression for the neutral height we get

$$h_n \approx \frac{1}{700}\left(50 + \frac{200*2.5}{2} + 140*2.5 + 100*4\right) = 1.5m \tag{33}$$

The stack height can now be calculated as follows:

$$\left(\frac{H_s}{2}\right)^{.65} = \frac{210*1.5^{.65} + \frac{200}{2.5*1.65}*1.5^{1.65} + \frac{200}{2.5*1.65} + 50*.5^{.65} + 140 + 100*2.5^{.65}}{700} \tag{34.1}$$

$$H_s \approx 2.3 \tag{34.2}$$

More exact values of the parameters are

$$h_n = 1.44m \qquad H_s = 2.30 \qquad X = -0.07 \qquad \beta_s = 0.45 \tag{35}$$

This house has the same total leakage as the first example, but a significantly larger stack height (and therefore, stack effect), because the leakage sites are vertically separated. The stack flow for this example will be about 70% larger than the first one because of the different leakage distributions. Thus, the stack height is similar to the floor-to-ceiling height.

Daylight Basement

Consider a two-story house in which the bottom level is partially bermed and half of it is taken up by a garage. Again the total leakage is 700cm^2 (n=0.65). There is floor level leakage of 150cm^2; there are 100 cm^2 of leakage between the two stories (2.5m); there are 100 cm^2 of leakage at the ceiling level (5m) ; there are 50 cm^2 of leakage at 7m above the floor from a chimney. The remainder of the leakage is assumed to be spread evenly in the walls. Thus, the neutral height is as follows:

$$h_n \approx \frac{1}{700}\left(\frac{300*5}{2} + 100*2.5 + 100*5 + 50*7\right) = 2.6m \tag{36}$$

From which we can calculate the stack height:

$$\left(\frac{H_s}{2}\right)^{.65} = \frac{150*2.6^{.65} + \frac{300}{5*1.65}2.6^{1.65} + 100*.1^{.65} + \frac{300}{5*1.65}2^{1.65} + 100*2.4^{.65} + 50*4.4^{.65}}{700} \tag{37.1}$$

$$H_s \approx 3.2 \tag{37.2}$$

More exact values of the parameters are

$$h_n = 2.57m \qquad H_s = 3.12m \qquad X = -0.15 \qquad \beta_s = 0.38 \tag{38}$$

Even though the building is twice as tall as in the second example, the stack height is only somewhat larger, because of the concentration of leakage near the middle of the building. Note that there is significant asymmetry (and an accordingly small neutral level) because there is a significant leak just below the neutral height.

BOX MODELS

As discussed in the introduction many models treat the building as a box. There is leakage in the floor, the ceiling, and the walls (of height H), but in order to define it as a simple box we must make an assumption about how the leakage is distributed in the walls. In the LBL model, it is assumed that all wall leakage is evenly distributed. We assume here that there can be different amounts of leakage in the walls above and below the neutral height, but that the distribution in leakage in the walls mirrors that of the floor and ceiling:

$$\frac{K_{floor}}{K_{\downarrow}} = \frac{K_{ceiling}}{K_{\uparrow}} = R = \frac{K_{ceiling} + K_{floor}}{K} \tag{39}$$

Thus the box ratio, R, represents the fraction of the leakage lumped at the floor and ceiling.

The neutral level can still be calculated from eq. 20, but it is conventional (in box models) to define the neutral level based on the height of the box (from the floor):

$$\beta_{box} \equiv \frac{h_n}{H} \tag{40}$$

As can be verified, this definition of neutral level is equivalent to the one used previously—***provided the box assumptions are valid*** (i.e. $\beta_{box}=\beta_s$ only if eq. 39 is true).

Following the development of the previous sections, the flow equation for the simple box is very similar to that for the simple stack case:

$$Q = \frac{K}{2}\,\frac{1+nR}{1+n}\,f_{\beta_s}\left(\Delta\rho\, g\frac{H}{2}\right)^n \tag{41}$$

As expected, this solution reduces to the simple stack case for $R=1$.

The stack height can be calculated from the box model parameters as follows:

$$H_s = \left(\frac{1+nR}{1+n}\,f_{\beta_s}\right)^{1/n} H \tag{42}$$

Equivalent Box Parameters

Few houses are really simple boxes and follow the assumptions above, but it is possible to find a set of parameters that is equivalent to the actual situation. Some of the parameters can be determined from direct measurement while others must be inferred. If the leakage distribution is known, then $X, h_n, \beta_s, and\, H_s$ can all be directly calculated using eqs. 18-25.

We can calculate values for H and R that must exist for the box assumptions to be true. The equivalent box height inferred from the neutral level and height:

$$H = \frac{h_n}{\beta_s} \tag{43}$$

The assumption that eqs. 38, 39 and 42 are true is the ***box assumption***. The box parameter can only be calculated by making these assumptions and is as follows:

$$R = \frac{\dfrac{1+n}{f_{\beta_s}}\left(\dfrac{H_s}{H}\right)^n - 1}{n} \tag{44}$$

Eqs. 43 and 44 define the equivalent box height and ratio for the case in which the leakage distribution can be calculated directly. If the box assumptions were valid, then the equivalent box ratio would lie between zero and unity, but in general R can have a larger range. If R is greater than unity it implies that there is significant leakage below the nominal floor or above the nominal ceiling. If R is less than zero, it implies a concentration of leakage near the neutral level.

The equivalent box parameters, H and R, can be calculated for our four examples and are summarized in the discussion.

INTERPRETATION OF FIELD MEASUREMENTS

The above approach assumes detailed knowledge of the leakage distribution. For parametric studies or design purposes it may be possible to define the leakage distribution exactly, but when measuring an existing structure it may not be possible to know where all of the leakage is. It is possible to measure the neutral height directly (from pressure measurements) and the stack height from the whole-building leakage, temperature difference and the infiltration rate (using eq. 24).

If a nominal building height is assumed then a set of *apparent* box parameters can be found. The neutral level can then be determined from eq. 40, the leakage asymmetry can be determined from eq. 19 and then the box ratio can be determined from eq. 44.

In order to demonstrate this stack formulation, it is necessary to have field measurements of both the envelope leakage and ventilation with only the stack effect in operation. Such a case study has recently been done by Palmiter and Bond.[17] The sites were in the Puget Sound area of the state of Washington and were relatively new construction. Information not explicitly contained in this report was required for the calculations below.[18]

In all sites continuous multizone air flow measurements were made using tracer gases[19] and while all mechanical systems as well as weather and surface pressures were monitored. The infiltration data for these examples will only include those periods in which the stack effect dominated and there were no effects from HVAC systems.

Site 1

This site was a two-story crawlspace home with attached garage, built in 1988 to the "Super Good Cents" program specifications; it had tight-fitting windows and electric baseboard heating. The Super Good Cents program requires the presence of mechanical ventilation, which consisted of a central exhaust fan in the attic with five ports in upstairs closets and a through-the-wall 5" diameter inlet port (with damper) located 1.6m above the first-story floor. In addition, there were two ceiling and three mid-level exhaust vents, all of which had back-draft dampers. There was also a dryer vent at floor level. The total leakage area was measured at $560 cm^2$ with an exponent of 0.63.

The measured neutral height for this site was 7.75 ft., which implies an apparent neutral level of 0.48 based on the 16.25 ft nominal height. The stack-induced air change rate was approximately $0.28\ h^{-1}$ which yields a stack height of 11.5ft from the average temperature difference of 11.3 ^{o}C. The apparent R is 0.50 and the apparent X is -.03.

Site 2

This site is also a two-story crawlspace house with attached garage. Built in 1979, it has been the subject of detailed measurement before[20] There are three ceiling vents, one mid-level vent, one ground level dryer vent, and one fireplace on each floor. While the ceiling in site 1 was flat, this site had a partial cathedral ceiling. The total measured leakage area is 1089 cm^2 with an exponent of 0.66.

The neutral height was measured at 8.45ft which implies an apparent neutral level of 0.52 based on the 16.25ft nominal height. Stack induced ventilation was approximately 0.41 h^{-1}, which yields a stack height of 13.1ft from the average temperature difference of 9.3 ^{o}C. The apparent R is 0.67 and the apparent X is -0.02.

Site 3

This site is a split-level home with an integral garage built in 1984 and has partial slab and partial crawlspace. There are two ceiling vents two mid-level vents, and one dryer vent and fireplace on the the lower floor. The total measured leakage area is 902cm^2 with an exponent of 0.70. (Note: this leakage is based on depressurization only.)

The neutral height was measured at 8.48ft which implies an apparent neutral level of 0.52 based on the 16.25ft nominal height. Stack induced ventilation was approximately 0.32 h^{-1}, which yields a stack height of 13.1ft (for depressurization) from the average temperature difference of 8.3 ^{o}C. The apparent R is 0.76 and the apparent X is 0.02.

Site 4

This site was a new manufactured home build under the BPA Residential Construction Demonstration Program. It was single story, but had a cathedral ceiling section. There was a make-up air system whose exhaust (with a damper near the mid-plane of the house) was sealed for the quoted data and accompanying slot inlets. There are three ceiling vents and one dryer vent; for The total measured leakage area is 286 cm^2 with an exponent of 0.64.

The neutral height was measured at 4.6ft which implies an apparent neutral level of 0.49 based on the 9.33ft nominal height. Stack induced ventilation was approximately 0.16 h^{-1} which yields a stack height of 4.8ft from the average temperature difference of 16.5 ^{o}C The apparent R is 0.11 and the apparent X is -0.01.

DISCUSSION

We have applied our stack formulation to four examples where the leakage distribution is known and to four measured instances where it was not. This dataset is summarized in Table 1 below:

TABLE 1: Summary of Examples and Measurements										
	ACTUAL				EQUIVALENT		APPARENT			
Case	h_n	H_s	X	β_s	H	R	H	R	X	β_{box}
Default	2.5	3.57	0	0.50	5.00	0.50	5.0	0.50	0	0.50
Slab	1.52	1.34	0.03	0.52	2.91	0.00	2.5	0.20	0.14	0.61
Crawlspace	1.44	2.30	-.07	0.45	3.23	0.51	2.5	0.90	0.10	0.58
Basement	2.57	3.12	-.15	0.38	6.72	0.05	5.0	0.33	0.02	0.52
Site 1	2.36	3.51	?	?	*4.7*	*0.57*	4.9	0.50	-.03	0.48
Site 2	2.55	3.99	?	?	*5.1*	*0.63*	4.9	0.67	-.02	0.52
Site 3	2.58	4.27	?	?	*5.2*	*0.72*	4.9	0.76	0.02	0.52
Site 4	1.40	1.46	?	?	*2.8*	*0.11*	2.8	0.11	-.01	0.49
italic values are undetermined from measurements, but calculated assuming X=0, β_s=½.										
APPARENT values are based on the nominal building height.										

The first four columns of data represent the true value of the parameters as determined from direct measurement. In the case of the four measured sites, the leakage distribution (i.e. asymmetry or true neutral level) was not determined. The next two columns represent the values of H and R that would be true for the equivalent box. Since the data were missing from the four measured sites these columns were calculated assuming leakage symmetry. The last four columns represent the apparent distribution parameters based on the nominal height of the structures.

One notes that the apparent neutral level, β_{box} is not always the same as the actual neutral level, β_s, and similarly for X. Although calculations based on the four apparent values will yield the correct stack effect, the superposition of the stack effect with other driving forces requires the actual value of neutral level.[21]

The apparent neutral levels of the four measured sites are all quite close to 1/2 (i.e. $X \approx 0$) and one might be tempted to generalize this result. These four sites, however, are all-electric homes and, therefore, do not have as many flues as fossil-fuel heated homes for which a neutral level in the range of 0.6 to 0.7 (i.e. $X \approx 0.2$) would be more appropriate.

In our formulation the neutral level is not a very sensitive factor for calculating the stack effect, because we take into account both infiltration and exfiltration. Because R indicates the relative distance of leaks from the neutral level (in box models), it is significantly more important. The eight cases show a wide range for the box ratio. Site 1 appears very similar to the Default case in that R=.5. Sites 2 and 3 have high values of R suggesting a large amount of ceiling and floor leaks (including the leaky ductwork noted in the report). Site 4, the manufactured home, has quite a low value of R due undoubtedly to the factory-tight construction of the floor and ceiling assemblies.

The data might suggest that a relatively high value of R is appropriate for stick-built homes, but the sample is too small to be conclusive. The importance of this parameter to the result implies that more field measurements should be made to categorize the value of R for different construction types.

Additional conclusions based on the mechanical systems performance of these sites have been made by Palmiter.[22]

Comparison to LBL Stack Model

The box version of this stack model can be compared with the LBL stack model (which is a box model). If we set $n=½$, eq. 41 becomes equivalent to the LBL stack model (ref 6,7). Doing so reveals that the two equations are similar in form, but slightly different in interpretation (i.e. the box assumptions used are not identical). Additionally, this model allows the correct calculation of mass flow by including a modified density.

In the LBL model the parameters R and X focused exclusively on the floor and ceiling leakages. In this new model the interpretation of these parameters has been expanded to include more general cases, which is quite useful in interpreting field measurements of infiltration.

Stack-Induced Pollutant Entry

Some pollutants (such as those in soil gas) may be driven into the building by stack-induced pressure differences. This pressure can be easily calculated for any height in the building (eq. 3). Since the competitive effects of pollutant entry and infiltration would be simultaneously affected, it is necessary to solve them simultaneously to find the concentration of the pollutant.

A detailed discussion of this is beyond the scope of this report, but as an example we can solve the problem for the special case in which a pollutant enters the structure through a small leak (of the same exponent as the house) driven by the inside-outside pressure at floor level. (Such a case might be reasonable for radon entry into a slab-on-grade house.) The steady-state concentration of this pollutant would thus be

$$C = C_\infty \frac{\mathrm{K}_{crack}}{\mathrm{K}} \frac{2}{1-\frac{1+nR}{1+n}X} \tag{45}$$

where C_∞ is the concentration of the gas entering through the K_{crack} leak.

This example serves to demonstrate how the exposure will be a function of the leakage distribution. Specifically, if X approaches unity (i.e. a lot of high leakage) the exposure could be quite large, but once X gets into a more normal range (i.e. below 0.7) the concentration is not a strong function of the distribution; even making X go highly negative cannot make the exposure arbitrarily small.

SUMMARY

The model developed herein can be summarized as follows. The whole-building leakage parameters K and n, combined with the temperature and density differences, interact with the leakage distribution to give the stack-induced ventilation:

$$Q = \frac{\mathrm{K}}{2}\left(\Delta\rho\, g \frac{H_s}{2}\right)^n \tag{46.1}$$

Because of the density differences between inside and outside air this flow is neither the volumetric infiltration nor exfiltration, but rather is at an intermediate density (given by eq. 17),

$$\rho_o \approx \frac{\rho_+ \rho_-}{\rho_+ + \rho_-} \tag{46.2}$$

which is quite close to the density at the average inside/outside temperature.

If the leakage distribution is assumed known then the stack height can be calculated:

$$H_s = 2 \left[\frac{\int_{-\infty}^{\infty} k[h] \left| h - h_n \right|^n dh}{\mathrm{K}} \right]^{1/n} \tag{47.1}$$

where the integral can be converted to a sum for localized leaks and the neutral height is calculated from eq. 6 or estimated from eq. 26.

The neutral level, β_s, is a useful parameter for quantifying the vertical distribution of the leakage and can be calculated from either the leakage distribution directly or equivalently from the vertical asymmetry parameter, X.

$$\beta_s = \frac{1}{1 + \left[\frac{\rho_\downarrow \mathrm{K}_\downarrow}{\rho_\uparrow \mathrm{K}_\uparrow}\right]^{1/n}} = \frac{1}{1 + \left[\frac{1-X}{1+X}\right]^{1/n}} \tag{47.2}$$

Although the neutral level is not strictly necessary for the calculation of the stack effect, it is necessary for other functions such as the superposition of other driving forces or for the estimation of the entry of some pollutants.

In a real building it may be difficult to know the entire leakage distribution and one can make some estimates by assuming the structure can be treated as a box. From eq. 42 the stack height can be estimated from the height of the box and the parameter, R:

$$H_s \approx \left[\frac{1+nR}{1+n}\right]^{1/n} H \tag{48}$$

The problem still remains to estimate R, which quantifies how well the leakage is spread out. (For example, if the leakage is evenly spread in the walls R is zero, if it is all concentrated at the floor and ceiling R is unity; lumped leakage near the neutral level will decrease R, while lumped leakage outside of the floor and ceiling level will increase it.) Some case studies which would yield R were presented, but sufficient data are lacking to provide guidelines on the estimation of this parameter.

Early field measurements indicated a need for improvements to stack models to handle different construction types and leakage distributions. The model developed in this report is more general and more robust than its predecessors. Current field measurements, combined with the model, have allowed us to infer useful information about the leakage distribution in some typical house styles and have demonstrated both similarities and differences from some of the conventional assumptions. The current dataset of measurements is too small to generalize leakage distribution conclusions, but expansion of this effort could led to useful guidelines in the future. Such guidelines would allow a better understanding of typical leakage distributions and, hence, of residential ventilation.

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