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POSTER 6

SCALING OF AIR FLOW PATTERNS IN ROOM VENTILATION

(Annex 20, Subtask 1, Research Item 1.23)

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SYNOPSIS

Is it possible to translate a computed flow field to a design case with different physical dimension? - This and related questions must be answered when the results of the "air flow pattern atlas", as proposed in the IEA Annex 20, should be applied to actual ventilation systems.

Looking up a case in the atlas and transforming results to an actual application is like interpolating in a table. If geometries are similar, scaling laws may be applied. The interpolation problem also arises when numerical or experimental data from literature must be translated to a case at hand. Scaling rules show whether and how measurements on scale models may be translated to full-scale.

The poster identifies dominant physical parameters of jet- and buoyancy-driven air flows in rooms. It lists non-dimensional parameters that are important for the air flow and those that are not. The difficulty of running scale-model tests for natural convection in large spaces is analyzed.

LIST OF SYMBOLS

Most symbols are explained in the text. The physical constants used in calculations are given below for air at 20 °C:

Gravitational constant, $g = 10$ m/s^2

Density of air, $\rho = 1.2$ kg/m^3

Coefficient of thermal expansion; for perfect gas equal to 1/T , where T is a mean absolute temperature,

$\beta = 1/300$ $°K^{-1}$

Specific heat at constant pressure,

$c_p = 1000$ $J/kg°K$

Kinematic viscosity of air, $\nu = 1.5E-5$ m^2/s

Thermal conductivity. $k = 0.025$ $W/m°K$

1. Introduction

The goal of the Research Item 1.23, a subtask-1 project of Annex 20, was to develop a concept for a design tool that allows the engineer to assess air flow pattern, comfort, and indoor air quality without running an expensive flow field simulation code. This was accomplished by pre-calculating many flow patterns in typical rooms for selected values of the relevant parameters to cover a wide range of conceivable design applications. The air flow patterns are stored in a data base and documented in a catalogue called the "Atlas", [1]-[4].

Of course, it would be nice if the pre-calculated cases could be "stretched" to an actual design case with the same geometric proportions. The usefulness of the "Atlas" would greatly be enhanced. This technique is successfully employed in many fields of engineering, such as aero- and thermodynamics [5].

The task of making laboratory experiments of air flows in large spaces also requires knowledge of scaling laws. The relevant parameters and problems related to scaling of air movement in atria and other large enclosures was discussed by Whittle [6].

In this paper, some principles of dimensional analysis and theory of modelling are reviewed and applications to room air flow are demonstrated. Studies by Heikkinen [7] and Chen et al. [8] suggest that the supply air jet of Annex-20 benchmark tests is characterized mainly by its mass flow and momentum. Therefore, these quantities have been used in the definitions of non-dimensional parameters [9], [10]. It is concluded that exact scaling with mixed and free convection is possible only to a limited extent.

2. Characteristic parameters of room air flow

The result of a numerical flow field computation carried out for a specific room may not only be applied to the original case but can be converted to other cases that are *similar* to the original. This technique is routinely used in calculations of the flow field around airfoils, for instance. The computation, that was done once, provides drag and lift coefficients. The actual lift force in physical units may be obtained for any size airfoil by simple multiplications, provided the non-dimensional parameters, such as Reynolds and Mach numbers are the same and the two profiles are geometrically similar.

For scaling to be valid, the original room geometry and air flow must be physically similar to the target situation. But this requires:

(1) Geometric similarity (e.g., rooms have the same height-to-length ratio, etc.), and

(2) the relevant characteristic dimensionless parameters must have the same numerical values (e.g., the same Archimedes number).

Before discussing scaling, some technical terms will be defined [2].

Geometry	is the complete geometric description of the solid envelope around the room air, expressed in physical units. Normally this is given as a large number, N, of points that describe the room inner surface including obstructions and furniture, $x_i \, , \, y_i \, , \, z_i \, , \qquad i = 1, \ldots N$
Size, H	is a typical dimension of the room, for instance the room height, in physical units.
Shape, S	is the non-dimensional geometry of the room, it is defined by the points $X_i = x_i/H \, , \quad Y_i = y_i/H \, , \quad Z_i = z_i/H, \qquad i = 1, \ldots N$
Location	is a particular position within the flow field, e.g., the specific point where a velocity was measured. It is given in physical units or dimensionless: $\mathrm{loc} = (x \, , y \, , z)$ or $\mathrm{LOC} = (X \, , Y \, , Z) = (x/H \, , y/H \, , z/H)$
Characteristic constant	is a parameter in physical units which has only one (constant) value throughout the experiment and is known before calculations. It is typical of the relevant physics of the problem. Examples: Inlet air velocity, v_o , viscosity, ν , also *size*, H.
Parameter P	is a non-dimensional group of characteristic constants, and hence an input quantity. A particular experiment may depend on several parameters, P_j, $j = 1, \ldots J$. Examples: $\mathrm{Re} = v_o H/\nu$, Ra, Ar, etc.

Variable Q is a non-dimensional group of output variables:

$$Q_k = Q_k(LOC), \qquad k = 1, \ldots K$$

$$= \text{function}(S, P_1, P_2, \ldots P_J, LOC)$$

Examples: Nu, v/v_o at a given location, etc.

A computation will probably still be done in physical units to reduce the risk of making input errors. The results may be converted to dimensionless form and stored in arrays of the form

$$Q_k(S, P_1, P_2, \ldots P_J, LOC), \quad k = 1, \ldots K$$

where LOC refers to the grid points of the computational mesh.

3. Air flow pattern with a supply air jet and a heat source

Some of the cases used within Annex 20, Subtask 1, to evaluate numerical methods, and many of the "Atlas" sample office rooms [3], dealt with air flow patterns that were driven by air jets from a supply diffuser and some had internal heat sources such as radiators or personal computers. In this section, the relevant characteristic constants are listed, and a corresponding set of non-dimensional parameters is proposed.

The numerical exercises with a supply air diffuser "HESCO", that blows the fresh air through many small directional nozzles, have shown that the inlet should be characterized by the total mass flow and momentum of the combined air jet [7], [8]. With these complex inlet devices, it is difficult to measure the effective inlet area. Therefore, the inlet cross-sectional area does not appear as characteristic constant.

The supply mass flow, m, and jet momentum, f, are:

$$m = \int \rho \; v \; dA \tag{1}$$

$$f = \int \rho \; v^2 \; dA \tag{2}$$

Where v is the local velocity component normal to dA, and the integral is taken over the cross section of the diffuser. With these two characteristic constants, the nominal inlet velocity and effective cross section are defined by

$$v_o = f / m \tag{3}$$

$$A = m / \rho \, v_o \tag{4}$$

The list of characteristic constants may include the following:

m	mass flow of air supply, eq.(1),
f	momentum (or thrust) of air supply jet, eq.(2),
H	*Size* of geometry, i.e., height of room,
ΔT	temperature difference between supply and exhaust air, in steady state, This temperature increase characterizes the combined effect of all internal heat sources that release energy into the room air. No separate parameter for heat input is required.
g	gravitational constant,
ρ	density,
β	coefficient of thermal expansion; for perfect gas equal to $1/T$, where T is a mean absolute temperature,
c_p	specific heat at constant pressure,
ν	kinematic viscosity of air,
k	thermal conductivity.

Do these 10 characteristic constants, together with the *shape* of the room, completely define the case? It is the experience and judgement of the engineer to know which parameters influence the physics of an air flow situation and which are not so important. So far, nothing has been said about radiation, which mainly transfers energy between surfaces but may also feed the air itself through infra-red absorption by the gas mixture (mainly by water-vapor or CO_2). Another factor not mentioned is the turbulence intensity of the supply air.

Assuming that the dominant parameters have been identified, dimensional analysis tells us that the number of independent *parameters* is = (number of physical constants) - (number of basic units). With *length, time, mass,* and *temperature* as basic units, we should find 10 - 4 = 6 non-dimensional parameters.

There are many ways to define a set of parameters, however certain groups are in standard use. Six parameters are proposed below [10]. Some may look unfamiliar, but they are still required to define the situation completely. Of course, other combination may be constructed depending on user preferences.

Reynolds number $\quad Re = (f/m)\ H / \nu = v_o\ H / \nu$

Archimedes number $\quad Ar = \beta\ \Delta T\ g\ H\ m^2 / f^2$

$= \beta\ \Delta T\ g\ H / v_o^2$, with v_o of eq.(3)

Prandtl number $\quad Pr = \rho\ c_p\ \nu / k$

Size-of-inlet parameter

$P_4 = m^2 / (\rho\ f\ H^2) = A / H^2$ with A of eq.(4)

Temperature-ratio parameter

$P_5 = \beta\ \Delta T = \Delta T / T$

Thermal-to-potential-energy parameter

$P_6 = c_p\ \Delta T / g\ H$

These J = 6 non-dimensional parameters, together with a description of the *shape* (non-dimensional geometry) of the room, and its details, should uniquely characterize the boundary conditions of the flow under investigation. Other parameters may be constructed, but they will always be combinations of the six above, as for example

$$P_5^2 / Ar\ P_6 = v_o^2 / c_p\ T\ .$$

This last combination is proportional to the square of a Mach number.

It may surprise that the air change rate (*ach*) does not appear. It is not a characteristic parameter because it has dimension of reciprocal time; two air flow patterns may be similar even if the corresponding air change rates are different.

It turns out that only the first four parameters have any significance for room air flow. P_5 and P_6, and the Mach number as well, are unimportant. The complete set of six parameters has been listed for formal reasons.

The size-of-inlet parameter, P_4, would be given by the proportions of room and air diffuser geometries if the effective inlet cross section, A of eq.(4), is always proportional to the geometric inlet area. P_4 is needed to determine each of f and m and not only their ratio, $f/m = v_o$. The effective area, A, which is not in the set of characteristic constants, may be computed from f and m, eq.(4). It is assumed that P_4 will not change much between geometrically similar rooms.

In conclusion then, it can be stated that for practical purposes, two forced-ventilation non-isothermal air flow patterns are similar if Re, Ar, P_4, and Pr are the same. To satisfy these four definition equations, a total of 10 variables are available. But six of these are physical constants and are known as soon as the medium (air) and ambient conditions are given. That means that only the quantities m, f, H, and ΔT are free to solve the system of four equations. A closer look shows that none of these variables appears in the Prandtl number, i.e., Pr is already nailed down by the physical constant. And we end up with four variables for three equations.

In this example, one of the four may be arbitrarily chosen, - for instance the *size* (H) of the room, - and the others are then fixed by the similarity requirement.

4. Air flow pattern with free convection

The approach of analysis is the same here as in the previous section. But it is understood that for free or natural convection no air is blown into the room. Therefore, the parameters related to the air supply device disappear from the list of characteristic constants:

H	*Size* of geometry, i.e., height of room,
ΔT	temperature difference between typical hot and cold surfaces,
g	gravitational constant,
ρ	density,
β	coefficient of thermal expansion (= 1/T),
c_p	specific heat at constant pressure,
ν	kinematic viscosity of air,
k	thermal conductivity.

With these 8 characteristic constants, 4 independent parameters may be formed [10]:

Rayleigh number $Ra = \beta\, c_p\, \Delta T\, g\, H^3\, \rho \,/\, \nu\, k$

Prandtl number $Pr = \rho\, c_p\, \nu \,/\, k$

Temperature-ratio parameter

$$P_5 = \beta\, \Delta T = \Delta T \,/\, T$$

Thermal-to-potential-energy parameter

$$P_6 = c_p\, \Delta T \,/\, g\, H$$

Two free-convection air flow situations are now similar if the geometric configurations (*shape*) are similar and these J = 4 parameters have the same values. Experience tells us that only the first two are important in room air flow. Of the 8 variables appearing in the two definition equations, 6 express physical properties of the medium. Theoretically, the two free variables, H and ΔT, can now be adjusted to obtain the desired values of Ra and Pr. But where are the similar cases when all variables are already committed?

As with mixed convection, only physical constants appear in the Prandtl number. Hence, once the fluid and ambient conditions are selected, Pr is fixed, and may or may not have the desired value. That means, one of H and ΔT may now be arbitrarily chosen, and the other results from imposing the Rayleigh number. Under the assumptions made above, experiments 1 and 2 will be similar if

$$(\Delta T\, H^3)_2 = (\Delta T\, H^3)_1 \qquad (5)$$

5. Limits for scaling air flow patterns in rooms

The aim of this investigation is to find a "cheap" way to transfer information from a pre-calculated air flow pattern to a design situation that is of immediate interest. For instance, the computed air flow pattern with a Rayleigh number of 8.3×10^{10} may be applied to any geometrically similar room with the same Rayleigh number.

Annex-20 test case **d2** has a radiator at 55 °C and a window surface at 5 °C. Taking ΔT equal to the difference of these two temperatures, and H equal to the room height (2.5 m), the Rayleigh number is 8.3×10^{10}, with a Prandtl number of 0.72. Scaling to a room height of $H = 3.0$ m would require a temperature difference of $\Delta T = 29$ °C (using eq.(5)). This kind of scaling is illustrated in fig. 1. Small changes in H require a large change of ΔT to maintain similarity.

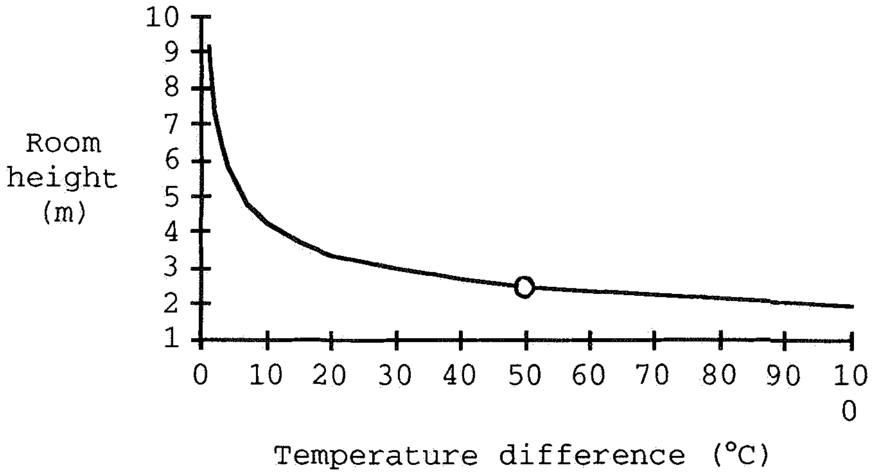

Fig. 1 Combinations of room height, H, and temperature difference, ΔT, that result in a Rayleigh number, $Ra = 8.3 \times 10^{10}$, assuming the remaining physical parameters are kept constant, eq.(5).

For air flow with forced or mixed convection, two non-dimensional parameters (Re and Ar) plus the *shape* of the geometry must be conserved to maintain similarity, assuming both experiments are conducted in atmospheric air (same Prandtl number) and have the same size-of-inlet parameter, P_4 (Section 3). The free variables are H, v_o, and ΔT. The third parameter, P_4, together with v_o is used to determine m and f.

The possible range of scaling will be illustrated by the Annex-20 test case **e2**, a summer-cooling situation with mixed convection. The following parameters are formed with the room height $H = 2.5$ m, $v_o = 4$ m/s, and $\Delta T = 6$ °C:

Test case **e2**:
$Re = 6.7 \times 10^5$
$Ar = 0.031$
$P_4 = 0.00128$

To see how the physical parameters may be varied, v_o is eliminated from Re and Ar to yield

$$Ar\ Re^2 = \beta\ \Delta T\ g\ H^3 / \nu^2 \qquad (6)$$

Again, the product $\Delta T\ H^3$ must be kept constant for fixed Ar and Re, eq.(5). When a pair of values is chosen, v_o follows from the definition of Re, and m and f from P_4.

The limitations to scaling of mixed-convection flows imposed by similarity rules become obvious when the variables H, v_o, and ΔT are combined in one graph, fig. 2.

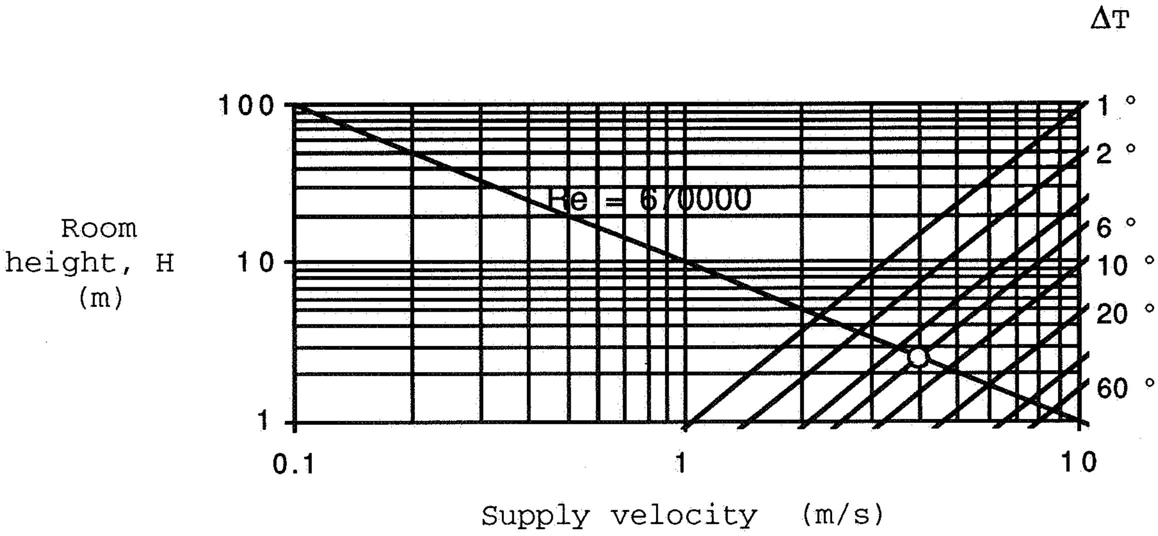

Fig. 2 Mixed convection: Variation of room height, H, supply velocity, v_o, and temperature difference, ΔT, for $Re = 6.7 \times 10^5$ and $Ar = 0.031$. The array of parallel lines represents constant Ar number at different ΔT.

The Annex-20 test case **e2** is at the intersection of the 6°-line and the Reynolds-number line. If the room height is increased to 4.5 m the supply velocity must be reduced to 2.2 m/s and the temperature difference to about 1 °C. Small relative changes of *size*, H, require a large adjustments of ΔT. This makes scaling difficult.

If the flow is known to be fully turbulent, the condition on Reynolds number may be relaxed, but the H-v_o-ΔT triple should still be on the array of Ar = 0.031 lines (fig.2). This last relationship is also expressed by the definition of the Archimedes number.

6. Scaling of measured or computed results

The results or dependent variables should be expressed as "*Variable* Q_k" of Section 2, i.e., in non-dimensional form. With the set of physical parameters, as introduced in Sections 3 and 4, the Q_k are transformed back into physical quantities.

As shown in the previous section, it is not always possible to reach similarity, even if geometries are similar. If the characteristic parameters P_j do not differ too much, it is suggested [2] to use the same Q_k in both situations. This is often done with heat transfer coefficients, - or with lift and drag coefficients of an airfoil section, - which are assumed to vary little between applications. This procedure is still much better than transferring physical quantities from one case to another.

7. Conclusions

This investigation leads to the following conclusions:

- To scale geometrically similar *free-convection* cases, the product $H^3\ \Delta T$ (height x temperature difference) must be kept constant to conserve Rayleigh number. Thus, scaling up the dimensions of a room is restricted to a narrow range because a slight increase of H requires a large reduction of ΔT.

- To scale geometrically similar *mixed-convection* cases, the product $H^3 \Delta T$ (height x temperature difference) and $v_o H$ (supply velocity x height) must be kept constant to conserve Rayleigh and Reynolds numbers. As in free convection, scaling is also restricted for the same reason.

Hypotheses based on discussions at Annex-20 meetings and on various Annex reports:

- If the flow of two geometrically similar *mixed-convection* cases is fully turbulent it may be admissible to relax Reynolds-number similarity within a small range. Then, the combination $\Delta T\ H\ /\ v_o^2$ must be kept constant to conserve Archimedes number. (Caution, if surface heat transfer is strongly influenced by location of laminar-to-turbulent transition in boundary layers).
- It is hypothesized that measured or computed results of one case may be transferred to a geometrically similar case, even if the non-dimensional parameters do not have exactly the same values. To do this, the *non-dimensional* variables (e.g., Nu, v/v_o, etc.) must be transferred, not the physical ones.
- The supply air jet is characterized by its *mass flow* and *momentum* (thrust). Nominal air velocity, v_o, and effective inlet cross section, A, are derived parameters. A method to measure jet momentum needs still to be developed.

These conclusions are based on using air. Substituting other gases or water for air as test medium may lead to more freedom of scaling.

8. ACKNOWLEDGMENTS

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