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A SIMPLE METHOD USING TRACER GAS TO IDENTIFY THE MAIN  
AIR AND CONTAMINANT PATHS WITHIN A ROOM.

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## Synopsis

The main air- and contaminant flow paths or the spatial distribution of the age of air (or contaminant) in a room are of great interest to estimate the ventilation efficiency performance. A simple measurement method is presented, which consist to inject one or more tracer gases at locations of interest and to analyze the concentration at several other locations, carefully chosen for best accuracy.

Response functions can be fitted on these measurements, which are the age of the tracers or of the air or the concentration of the tracers in function of the location. As well the salient paths as the dead zones are determined from these functions.

The contribution presents the method and its application and validation on a well known and controlled room.

## List of symbols

$\underline{A}$	vector containing the coefficients $a, b_i, b_{ij} (i \neq j)$ and $b_{ii}$	
$a, b_i, b_{ij}$	coefficients in a model	
$C(\underline{r}, t)$	concentration of a tracer at location $\underline{r}$ and time $t$ .	[-]
$C_e(t)$	concentration measured in the exhaust duct	[-]
$C_o(t)$	concentration measured outdoors	[-]
$C_x(t)$	concentration measured at the location $x$	[-]
$f(\tau)$	probability density function of the age of air particles	[s <sup>-1</sup> ]
$F(\tau)$	probability function of the age of air particles	[-]
$\underline{M}$	effect matrix, resulting from an experimental design and a model	
$n$	air change rate	[s <sup>-1</sup> ] or [h <sup>-1</sup> ]
$\underline{r}$	vector locating a point in the space	[m]
$t$	time	[s]
$\underline{V}$	vector containing the measured quantities ( $v_1, v_2, \dots, v_n$ ) at $n$ locations	
$V$	volume of the room	[m <sup>3</sup> ]
$x_i$	coordinates of a measured point ( $i = 1$ to $3$ )	[m]
$\delta \underline{A}, \delta \underline{V}, \delta \underline{M}$	errors in the measurements of $\underline{A}, \underline{V}$ or $\underline{M}$ respectively	
$\ \underline{M}\ $	norm of the matrix $\underline{M}$	
$\ \underline{V}\ $	corresponding norm of the vector $\underline{V}$	
cond $\underline{M}$	condition number of the matrix $\underline{M}$	
$\Delta t$	time interval	[s]
$\varepsilon$	air change efficiency	[-]
$\varepsilon_c$	ventilation efficiency or pollutant removal effectiveness	[-]
$\mu_1$	largest eigenvalue of $\underline{M}^H \underline{M}$ , ( $\underline{M}^H$ being the hermitic conjugate of $\underline{M}$ )	
$\tau$	age of air	[s]
$\tau(\underline{r})$	mean age of air at location $\underline{r}$	[s]
$\tau_n$	nominal time constant of the room	[s]
$\langle \tau \rangle$	room mean age	[s]

## 1. Introduction

An efficient ventilation should bring fresh air to the inhabitants and quickly eliminate the indoor contaminants. The flow paths of the fresh and contaminated air are hence of great importance and a method useable to map either the age of air or the contaminant concentration is needed.

Demand controlled ventilation, in which the air flow rates are controlled to maintain the concentration of a characteristic contaminant under a target limit, needs gas sensors installed at locations which are representative of the inhabited volume of the room. These locations should be chosen according the pollutant sources, the characteristics of the ventilation system and the room itself to optimize the control of the air flows, hence to minimize the contaminant concentration. The

knowledge of the air flow pattern of the room may be of great help in the choice of these locations.

This pattern may be calculated using sophisticated codes<sup>[1]</sup> but can also be estimated in existing building by the measurement of the concentration pattern within a room resulting from a particular pollutant source or, if the pollutant source is not known, by the measurement of the age of the air in the room.

Various measurement methods, most of them using tracer gases, can be used to analyze the ventilation system characteristics and the spread of the contaminants in the room, and therefore to choose the best location of the contaminant probes.

More precisely, in a ventilated room, the scope of the measurements could be the followings:

To characterize the ventilation system:

- 1) determine the flow rate in the air ducts (pulsion, extraction, short-circuits);
- 2) measure the global air change rate in the room;
- 3) determine the ventilation efficiency of the system, that is the part of the total flow rate which comes from the ventilation system and the part coming from infiltration

To characterize the indoor air quality

- 4) determine the age of the air at different locations in the room;
- 5) get a map of the contaminant concentration resulting from several contaminant sources and a given flow pattern resulting of ventilation.

To characterize the complex room-ventilation system

- 6) determine the air exchange efficiency that is how the fresh air is distributed in the room;
- 7) determine the ventilation efficiency, that is the efficiency in extracting the contaminants generated in the room.

Each particular scope cited above in the introduction has its own measurement method(s) allowing to measure the necessary quantities. In this contribution, only the mapping experiment is described, together with an example measured in a well controlled test room.

## 2. Planning of mapping experiments

Mapping the contaminant concentration of the age of air in a room can be of great advantage in studying the contaminant or air flow pattern and their effects on the occupants. Such maps were already calculated using computer codes<sup>[1]</sup> and some qualitative representations were drawn from measurements<sup>[2]</sup>.

The purpose of this paper is to propose a systematic way to obtain a coarse map of contaminant concentration or of the age of air from measurements. Such map allows to locate the dangerous locations or the dead zones within the measured room, and in the conditions prevailing during the experiments.

### 2.1. Minimum number of measurements

A map of any scalar variable  $v$  in a three-dimensional room is basically obtained by measuring the variable at each node of a network and interpolating between these nodes. Such measurement are however very expensive if not unfeasible: if only 5 values are taken on each axis, there are not less than 125 measurements. Therefore, it makes sense to look for methods needing a minimum number of measurements points.

The minimum number of measurements depends on the scope of the mapping experiment, or more precisely on the empirical model which is chosen to represent the map of the variable  $v$ . If a linear model is adopted, such as:

$$v = a + \sum_i b_i x_i \quad (1)$$

where  $x_i$  are the three coordinates of the measured point, only 4 measurements are needed to obtain a set of coefficients  $\{a, b_j\}$ . If more measurements are made, the coefficients may be obtained by a least square fit procedure, provided there is no (or negligible) uncertainty on the coordinates. If their coordinates differ for the other points, these supplementary measurement points give information on the validity of the used model.

If the linear model does not appear to be valid, more sophisticated models may be used. For example, a quadratic model:

$$v = a + \sum_i b_i x_i + \sum_{i \neq j} b_{ij} x_i x_j + \sum_i b_{ii} x_i^2 \quad (2)$$

which contains 10 coefficients can be chosen. Such model may already fit many practical situations and present minimal and maximal value(s). For that model, measurements at 10 locations is the minimum.

## 2.2. Location of the measurement points

The next problem is: where should we locate the measurement points? There are numerous possible experimental designs, but they do not give the expected results with the same accuracy. For example, it is obvious that, to fit a linear model of one dimension only ( $y = ax + b$ ), the location of the two measurement points (the minimum number) which gives the best accuracy on  $a$  and  $b$  is at the ends of the experimental domain. If the model is more sophisticated or for a larger number of dimensions, the choice is not so obvious. However, several tools exist for planning such experiments, which are found in the literature<sup>[3], [4]</sup> and are applied below.

Let us take a coordinate system in a rectangular room using as unit, for each direction, the half-length of the room in that direction. Any point in the room is then located by three numbers included in the interval  $[-1, +1]$ .

The experimental design can be represented by a rectangular matrix with 3 lines (one for each coordinate) and as much columns as measured points. For example the design:

Point No	1	2	3	4
x =	1	-1	1	-1
y	1	-1	-1	1
z	1	-1	1	-1

is very well suited to obtain the coefficients of a linear model. This design may be expanded the following way up to 10 locations for a quadratic model:

Point Nr	1	2	3	4	5	6	7	8	9	10
x	1	1	1	1	1	0	-1	-1	-1	-1
y	1	1	-1	-1	-1	0	1	1	-1	-1
z	1	-1	1	0	-1	0	1	-1	1	-1

which represent the 8 corners of the room, its center and the center of a vertical angle. This design looks pretty, but cannot provide the coefficients, because the resulting system of equations is singular. To elaborate the most efficient design, it is necessary to look at the method used to obtain these coefficients.

## 2.3. Criteria for location of the measurement points

For each point, the model (equation (1) or (2) or any other model) is applied, replacing the  $x_i$  by their values given by the experimental design. A system of equations (one equation for each location) is obtained this way, which can be written in a matrix notation:

$$\underline{V} = \underline{M} \underline{A} \quad (3)$$

where:

$\underline{V}$  ( $v_1, v_2, \dots, v_n$ ) is a vector containing the measured quantities at the  $n$  locations,

$\underline{M}$  is a matrix, each line of which corresponding to one location. Its first column is filled with ones and correspond to a constant in the model. The next 3 columns may contain the coordinates of the locations if the model contains linear terms. The next 3 columns may contain the products of these coordinates two by two (e.g.  $x_1 x_2, x_1 x_3, x_2 x_3$ ) in case of interaction terms and, for a quadratic model, the next three columns contain the squares of the coordinates. Other models will produce other matrices.

$\underline{A}$  is a vector containing the coefficients (e.g.  $a, b_i, b_{ij} (i \neq j)$  and  $b_{ii}$ ) of the model.

In the general case,  $\underline{M}$  is rectangular and the least square fit procedures is used:

$$\underline{A} = (\underline{M}'\underline{M})^{-1} \underline{M}' \underline{V} \quad (4)$$

where  $\underline{\underline{M}}^t$  is the transposed matrix of  $\underline{\underline{M}}$ . This equation is also valid if  $\underline{\underline{M}}$  is a square matrix, but reduces to the simpler equation:

$$\underline{\underline{A}} = \underline{\underline{M}}^{-1} \underline{\underline{V}} \quad (5)$$

Anyway, a matrix shall be inverted and the determinant of this matrix should not be zero, as it is the case for the 10 points design shown above! Since this determinant can be calculated before making the experiments, it is a first criterion for the choice of the experimental design.

If there are experimental errors  $\delta\underline{\underline{V}}$  in  $\underline{\underline{V}}$  or  $\delta\underline{\underline{M}}$  in  $\underline{\underline{M}}$ , these errors will propagate through the equations and errors  $\delta\underline{\underline{A}}$  will result in the coefficients in  $\underline{\underline{A}}$ . In case of complete experiments, that is if equation (5) is used, it is shown<sup>[5]</sup> that these errors are smaller than:

$$\frac{\|\delta\underline{\underline{A}}\|}{\|\underline{\underline{A}}\|} \leq \frac{\|\underline{\underline{M}}\| \cdot \|\underline{\underline{M}}^{-1}\|}{1 - \|\delta\underline{\underline{M}}\| \cdot \|\underline{\underline{M}}^{-1}\|} \cdot \left( \frac{\|\delta\underline{\underline{V}}\|}{\|\underline{\underline{V}}\|} + \frac{\|\delta\underline{\underline{M}}\|}{\|\underline{\underline{M}}\|} \right) \quad (6)$$

where  $\|\underline{\underline{M}}\|$  is a norm of the matrix  $\underline{\underline{M}}$  and  $\|\underline{\underline{V}}\|$  the corresponding norm of the vector  $\underline{\underline{V}}$ . Any norm can be used but it may be advantageous to use the smallest ones, which is the euclidian norm of the vector  $\underline{\underline{V}}$ :

$$\|\underline{\underline{V}}\|_2 = \sqrt{(\sum_i v_i^2)} \quad (7)$$

and the corresponding spectral norm for the matrix  $\underline{\underline{M}}$ :

$$\|\underline{\underline{M}}\|_2 = \sqrt{\mu_1} \quad (8)$$

where  $\mu_1$  is the largest eigenvalue of  $\underline{\underline{M}}^H \underline{\underline{M}}$ , ( $\underline{\underline{M}}^H$  being the hermitic conjugate of  $\underline{\underline{M}}$ ).

However, the following norms, which leads to faster calculations are often used:

$$\|\underline{\underline{V}}\|_1 = \sum_i |v_i| \quad (9)$$

and the corresponding norm for the matrix  $\underline{\underline{M}}$ :

$$\|\underline{\underline{M}}\|_1 = \max(\|\underline{\underline{M}}_j\|_1) \quad (10)$$

where  $\underline{\underline{M}}_j$  are the column vectors of  $\underline{\underline{M}}$ .

The number

$$\text{cond } \underline{\underline{M}} = \|\underline{\underline{M}}\| \cdot \|\underline{\underline{M}}^{-1}\| \quad (11)$$

appearing in (6) is of first importance here, since  $\|\delta\underline{\underline{M}}\| \cdot \|\underline{\underline{M}}^{-1}\|$  is generally much smaller than 1. It is the **condition number** of the matrix  $\underline{\underline{M}}$ , which multiplies the experimental errors and transmit these errors into the result  $\underline{\underline{A}}$ . This number depends only on the experimental design and on the chosen model. It can hence be calculated before doing any experiment and constitutes a second (and even better) criterion for the choice of the experimental design.

### 2.3. Examples of experimental designs

Several experimental designs were examined with the aim to map a 3-D cube with 10 measurements. The tested models were those of equations (1) and (2) plus an interaction model which is (2) without the square terms.

Several of these designs were found to be unusable (singular effect matrix or too large condition number for the quadratic model). The remaining designs are the followings:

C3 has 6 points in the center of the faces and 4 points at opposite corners. It was obtained by selecting 10 points out of the 27 points of a three level factorial<sup>†</sup> design in such a way that the matrix  $\underline{\underline{M}}$  gets the lowest condition number (figure 1). The same result is obtained starting from the 125 points of a 5-level factorial design.

C4 has 6 points near the center of the faces and 4 points at opposite corners. It was obtained by selecting 10 points out of the 65 points of a four level factorial design which present the lowest

<sup>†</sup> A k-dimensional, n-level factorial design is obtained by dividing the experimental domain (e.g. the interval [-1, 1]) on each axis into n equidistant levels. The complete factorial design contains all the points obtained by the  $n^k$  combination of the n possible values of the coordinates.

condition number for the matrix  $\underline{M}$ .

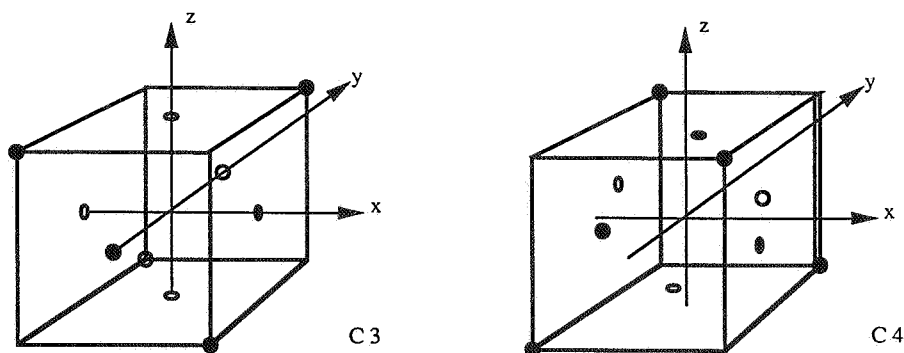


Figure 1: Experimental designs C3 (left) and C4 (right)

The hybrid design has two points on the z-axis ( $z = -0.136$  and  $z = 1.2906$ ), four factorial points on the plane  $z = 0.6386$  and a star-design in the plane  $z = -0.9273$ . Since there are points at locations larger than 1, all the coordinates should be divided by 1.2906. (figure 2).

The 3  $\Delta$  design is made with points at the summits of 3 triangles placed at levels  $z = -1, 0$  and  $1$ , the middle one being in opposite direction. The 10th point is at the center of the room.

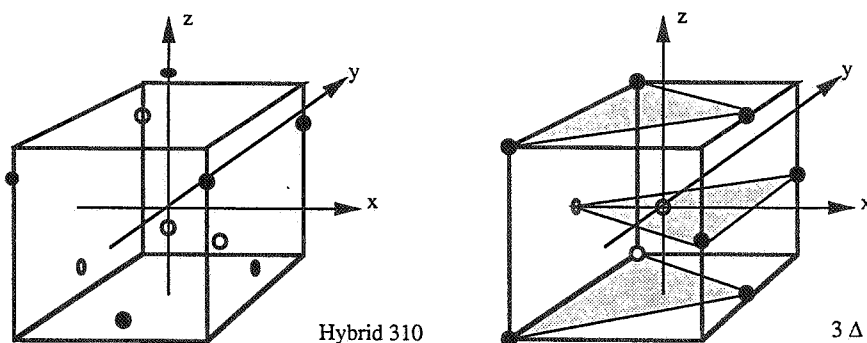


Figure 2: Hybrid (left) and 3  $\Delta$  (right) experimental designs

The Hoke D2 and the Rechtschaffner design have both 7 points at the summits of the cube and 3 points at the center of faces, but the position of these last three points with respect to the other differ strongly (figure 3). These designs were found in the literature<sup>[3]</sup> and chosen among others for having 10 measurement points.

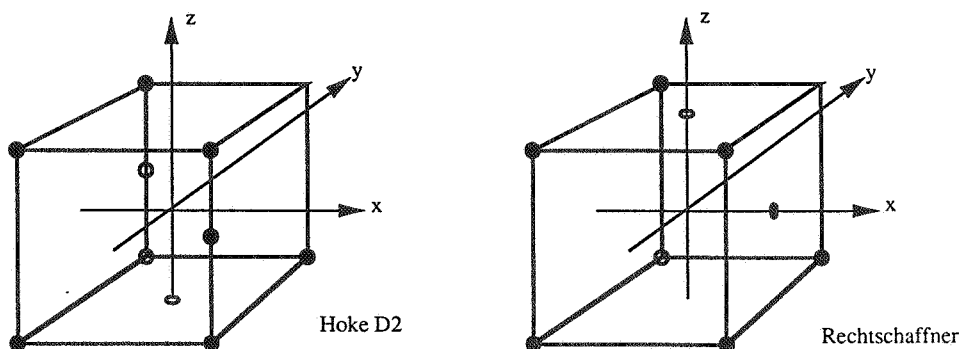


Figure 3: Hoke D2 (left) and Rechtschaffner (right) experimental designs

The condition number of  $\underline{M}'\underline{M}$  calculated according the equation (10) and (11) for these designs and three models are given in table 1.

Experimental design	Quadratic model	Interactions model	Linear model
C 3	<b>4.33</b>	<b>3.15</b>	<b>1.00</b>
C 4	6.67	2.62	1.15
Hybrid 310	6.70	<b>1.00</b>	<b>1.00</b>
3 Δ	8.88	1.414	1.11
Hoke D2	7.29	<i>3.19</i>	<i>1.66</i>
Rechtschaffner	9.24	2.03	1.23

Table 1: Condition number of  $\underline{M'M}$  for some experimental designs and three models. For each model, the best values are in bold characters and the worse in italics.

The three latter designs lead to a large condition number for a quadratic model, that is to large errors in the estimates of the coefficients. C 3 seems to be the best if the model is quadratic or linear, when the hybrid and 3 triangles are better for interactions and linear models. However, the hybrid model does not map the whole domain and looks very asymmetric.

Finally recommended designs are C 3 if the quadratic or linear model is used and 3Δ for interaction or linear model.

It may also be interesting to draw the map of a contaminant (e.g. the CO<sub>2</sub>) in the flat volume where the heads of the inhabitants are located. In this case, a two-dimensional design may be used. Using a limited number of sampling points, the following designs may be recommended:

A 2-D full factorial design with 3 levels:

$$\begin{array}{l} X = \quad -1 \quad 0 \quad 1 \quad -1 \quad 0 \quad 1 \quad -1 \quad 0 \quad 1 \\ Y = \quad -1 \quad -1 \quad -1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \end{array}$$

which gives a condition number of 4.39 for a quadratic model. If 10 points are wanted, adding (-0.5, 0) and (0.5, 0) lowers the condition number to 3.48. The minimum number of experiments is 6 for a quadratic model. The design:

$$\begin{array}{l} X = \quad -1 \quad 1 \quad -1 \quad 0 \quad 1 \quad -1 \\ Y = \quad -1 \quad -1 \quad 0 \quad 0 \quad 0 \quad 1 \end{array}$$

has a condition number of 6.

For an interaction model, a recommended design is:

$$\begin{array}{l} X = \quad -1 \quad 1 \quad 0 \quad 1 \quad -1 \\ Y = \quad -1 \quad -1 \quad 0 \quad 1 \quad 1 \end{array}$$

which is also best for a linear model, even when the point (0,0) is removed. These designs have a condition number of 1, which is an absolute minimum.

### 3. Mapping the concentration of a contaminant

If the characteristics of the "natural" contaminant source(s) is known, they can be simulated either by the contaminant itself if it is easily analyzed for non-toxic concentrations (like CO<sub>2</sub> or H<sub>2</sub>O) or by the injection of tracers at appropriate flow rates at the same locations.

The air is sampled at regular intervals at the locations corresponding to the experimental design and analyzed. For each sampling time, or if the air- and contaminant flow pattern are in a steady state, a model is fitted on these points to estimate its parameters.

Once the parameters estimated, the model can be used to find the probable location of a minimum or maximum concentration, to estimate the average concentration or the total amount of contaminant within the room.

The ventilation efficiency, also called the pollutant removal effectiveness is generally defined as the ratio of two net concentrations:

$$\epsilon_c = \frac{C_e - C_o}{C_x - C_o} \quad (12)$$



where:

$C_e$  is the contaminant concentration in the exhaust air

$C_o$  is the contaminant concentration in the outdoor air

$C_x$  is the contaminant concentration of interest.

The concentration of interest may be the concentration at a particular location in the room, the maximum concentration or the average of the whole room. Depending on that definition, different efficiencies may be defined and measured.

It is obvious that from the map of the concentration  $C_x$  and the measurement of the exhaust concentration  $C_e$ , a map of the ventilation efficiencies can be obtained, as well as any other value as maximum, average or minimum efficiency. That assumes that the air leaves the room through a single exhaust duct.

When a first map is obtained, a "zoom" experiment may follow to get more details in an interesting part of the room. This experiment may use the same design but for a smaller volume.

#### 4. Age of the air

The particles of air coming from outside or from the ventilation system arrive at a given location in a room after a time  $\tau$  which vary from one particle to the other.  $\tau$  is called the age of the particle, as if it were born when entering the room.

Since there is a large number of air particles, a probability density  $f(\tau)$  that the age of particles arriving at a given location is between  $\tau$  and  $\tau+d\tau$  and a probability  $F(\tau)$  that this age is smaller than  $\tau$  can be defined. It is shown that these probability functions can be measured by registering the concentration  $C(t)$  of a tracer versus time, this tracer being either at uniform concentration at the beginning of the experiment, or injected at constant rate or as a short pulse at a single air inlet<sup>[6], [7]</sup>.

The mean age of air at a given location in the room can be deduced from these distributions. The convenient formulas to be used are found in the literature<sup>[6], [7]</sup>. If, as it was done in our experiments, the age is deduced from the decay in concentration of a tracer initially distributed in the room, the local mean age is calculated from:

$$\tau(r) = \frac{\int_0^{\infty} C(r,t) dt}{C(0)} \quad (13)$$

The average of the local mean age of air over the whole room is the room mean age  $\langle\tau\rangle$ . If the air leaves the room through a unique opening or duct, this room mean age can be also measured with tracer gas, the methods of injection being the same as before but the concentration  $C_e(t)$  being measured in the exhaust duct and using equation (15) below.

If, as it is often the case, the air leaves the room through multiple openings, leaks and outlets, the technique mentioned above cannot be used. However, the local age of air  $\tau(r)$  can be measured and mapped as well as a concentration, using the same experimental design. The average of the local mean age of air over the whole room is the room mean age of air  $\langle\tau\rangle$ .

#### 5. Measurements

##### 5.1. The experimental chamber

In order to evaluate the feasibility of the method presented above, measurements were done in a well controlled room called CHEOPS for "CHambre Expérimentale d'Observation de Phénomènes de Stratification" (figure 4). This room has airtight, lightweight walls, floor and ceiling. Its internal dimensions are  $3.5 \times 4.0$  m with a height of 2.7 m, its volume is hence  $37.8 \text{ m}^3$ . It is completely contained in a second enclosure measuring  $5 \times 6 \times 4$  m, which is made of panes plated with 10 cm polystyrene foam, in order to control the temperature and temperature gradients within the internal room.

For that particular experiment, an inlet and an outlet openings were managed in two walls, as shown in figure 4. The airtightness of the room with closed openings was measured and found to be so

high that air flows through parasitic leaks can be neglected when compared to the main air flow.

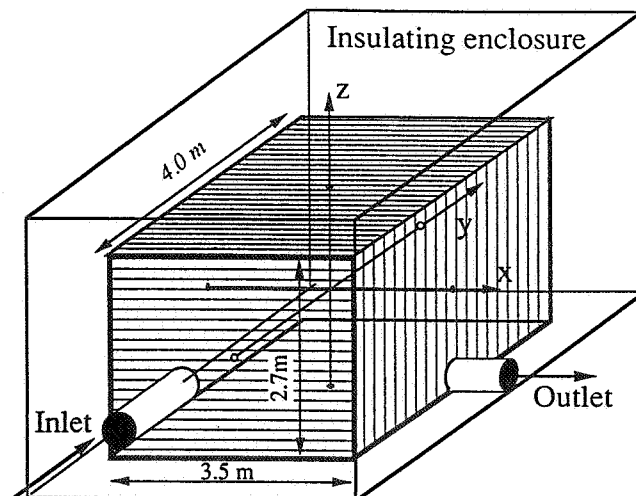


Figure 4: The experimental chamber CHEOPS, with the inlet and outlets used for the present experiment.

## 5.2. The experiments

Before the experiments, the ends of 10 sampling tubes are fixed at 10 different locations according to the two selected experimental designs, namely C3 and 3A. The other ends of these tubes are connected to a sampling scanner allowing to analyze in sequence the air samples coming from these 10 locations. The analyzer was a Binoss from Leybold Heraeus and the tracer was  $N_2O$ .

During the experiments, an air flow rate of  $18.9 \text{ m}^3/\text{h}$ , (which corresponds to an air change of  $0.5 /\text{h}$  or a nominal time constant of 2 hours) was blown at the inlet. The air flow rate was controlled by both an air flow meter and tracer gas (constant concentration) techniques.

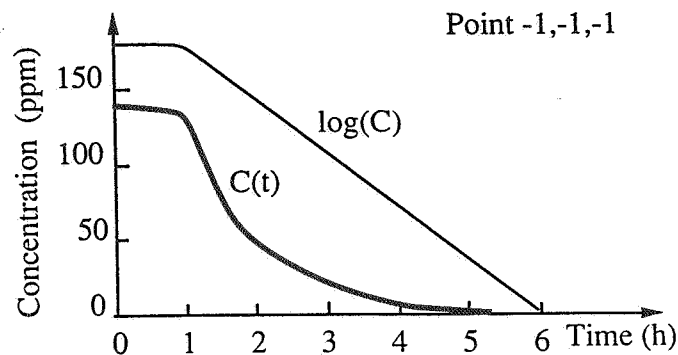


Figure 5: Record of the concentration versus time (full line) at one location in the room, after cutting off the tracer injection. The dotted line shows the log of the concentration on an arbitrary scale. Note that the decay starts nearly one hour after the tracer injection stops.

After stabilization of the air flow and of the concentration, that is after about 4 hours, the tracer injection was stopped and the decay of the concentration was measured at 10 different locations according to the two selected experimental plans (figure 5). The scanning interval between two channels was 42 seconds, so the time period between two measurements at the same location was 420 s or 7 minutes.

From these 10 decay curves, the local mean age of air were deduced using equation (13). These results are reported on table 2, for two experiments with two different designs.

x	y	z	3Δ	C3
-1	-1	-1	1.43	1.86
0	0	-1		1.83
1	0	-1	2.16	
-1	1	-1	1.41	
1	1	-1		1.56
0	-1	0		1.05
1	-1	0	1.02	
-1	0	0	1.09	1.06
1	0	0		0.99
0	0	0	1.06	
0	1	0		1.08
1	1	0	1.02	
-1	-1	1	0.94	
1	-1	1		0.98
0	0	1		0.99
1	0	1	1.00	
-1	1	1	0.96	0.96
Average of the ages (eq. 16)			1.209	1.236

Table 2: Ages of air in the CHEOPS chamber for two experiments. The coordinates are given in units corresponding to the half dimensions of the chamber. The air outlet is at 1, 0, -1 and the inlet at -0.5, -1, 0

First a quadratic model was adjusted on these data for both experiments. Since the coefficients of the y, xy and yz terms were very small, a limited model was also tested, using:

$$\tau(\mathbf{r}) = a + b_x x + b_z z + b_{xz} x z + b_{xx} x^2 + b_{yy} y^2 + b_{zz} z^2 \quad (14)$$

The table 3 gives the coefficients of these two models and the room mean ages obtained by different methods.

One of the measurement point in design 3Δ is located at the air outlet. The room mean age of air can hence also be measured using:

$$\langle \tau \rangle = \frac{\int_0^{\infty} t C_e(t) dt}{\int_0^{\infty} C_e(t) dt} \quad (15)$$

where  $C_e(t)$  is the tracer concentration in the only air outlet of the room. This value can be compared with the other values obtained either by the direct average of the measurements of the local mean ages:

$$\langle \tau \rangle = \frac{1}{N} \sum_i \tau_i(\mathbf{r}_i) \quad (16)$$

or by the average of the obtained map: if a linear or interaction model is used, the room mean age of air is simply the constant term of the model. If a quadratic model is used, then:

$$\langle \tau \rangle = a + \frac{2}{3} \sum_i b_{ii} \quad (17)$$

where  $a$  is the constant term of the model and  $b_{ii}$  are the coefficients of the square terms.

Model:	Quadratic		Limited	
Coefficient for:	3Δ	C3	3Δ	C3
Constant	1.06	1.08	1.06	1.08
X	0.08125	-0.03	0.08125	-0.0583
Y	0	0.01		
Z	-0.4075	-0.42	-0.4075	-0.3867
X*Y	0	-0.05		
X*Z	-0.1725	0.09	-0.1725	0.08
Y*Z	0.01	0.03		
X <sup>2</sup>	0.1112	-0.055	0.1112	-0.055
Y <sup>2</sup>	-0.2325	-0.015	-0.2325	-0.015
Z <sup>2</sup>	0.3275	0.33	0.3275	0.33
Mean age (eq. 17)	1.197	1.253	1.197	1.253
Room mean age measured at the outlet (eq. 15, 3Δ design):				1.21 h

Table 3: Coefficients of two models calculated from measurements done with similar conditions but with two different experimental designs. The coefficients which are very small in the quadratic model are put to zero in the limited model.

A map of the local mean age of air can be drawn using the model, as it is shown in figure 10.

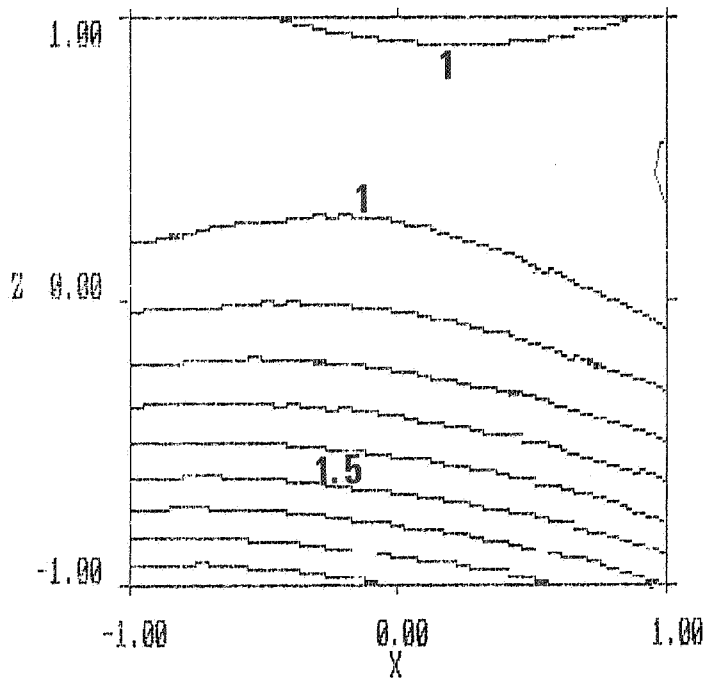


Figure 10: Lines of constant age of air in the plane  $y = 0$  drawn with the limited model adjusted on the measurements done with the C3 design. The younger air is on the top of the room even with the inlet at the mid-plane.

Finally, the air change efficiency, which expresses how quickly the air in the room is replaced, is calculated by:

$$\varepsilon = \frac{\tau_n}{2\langle\tau\rangle} = \frac{2}{2.4} = 0,83 \quad (18)$$

where  $\langle\tau\rangle$  is the room mean age of air and  $\tau_n$  is the nominal time constant of the room. This efficiency is 1 for piston ventilation and 0.5 for complete mixing.

## 6. Discussion and conclusions

The comparison of the figures in table 2 show that:

- 1) as expected, there are important differences (up to a factor 2) in the age of air at different locations. It confirms that either a strong mixing or sampling at several places is necessary to measure the air change rate or the nominal time constant of the room with tracer gas techniques.
- 2) There are slight differences between the two experiments, as it can be seen by the values marked in bold characters, which are measured at the same locations. This effect is particularly high at the point -1, -1, -1. Even when the experiment is done in very similar conditions, tiny changes in air temperatures may affect the flow pattern.

As shown in table 3, the coefficients of the model obtained with two different designs does not differ much. Moreover, the room mean ages obtained by various ways from the measurements are very close together. This value is affected neither by small changes in air temperature nor by the experimental planning, as far as it is good enough.

Hence, if there is not only one air outlet, or a fortiori when the air outlets cannot be clearly identified, the room mean age of air may be obtained by averaging several measurements taken at convenient locations. It can be safer however to deduct it from the coefficients of a fitted model.

Finally, it is shown that, if the experiments are well designed, it is possible to draw a map either of the concentration of a contaminant and the related quantities as the ventilation efficiency or of the local mean age of air, with a reasonable amount of measurements. From these maps, as well the main fresh or contaminant flows as the dead or well ventilated zones can be deduced.

However, sharp details cannot be reproduced with a limited amount of measurement points. It is the case of the distribution of the age of air (or of the contaminant concentration) near the fresh air inlet (or the contaminant source). If the number of measurement points is limited, these few measurements can be used to calibrate a simulation with a sophisticated code (e.g. Phoenics) at these locations. The code is then used to interpolate correctly between the measurements and more confidence is gained in the description of the details.

## 7. Acknowledgements

This research was sponsored by the NEFF ("Nationaler Energie-Forschungs-Fonds" or National Energy Research Funds). The codes NEMROD from LPRAI, Marseille and XSTAT (John Wiley and sons) were used to design and analyze the experiments.

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Discussion

Paper 41

**S.Irving (Oscar Faber, UK)**

How much of this new information on measurement techniques will be included in the revised Measurement Techniques being done within Annex 20?

*C-A Roulet (LESO, Switzerland)*

*A text close to the present paper is already included in part IV of the Measurement Guide.*

**M.Sherman (LBL, USA)**

One cannot compare the accuracy of two models by comparing their condition numbers or regression coefficients. The linear model may have a better condition number than the others, but that does not imply it is better - a model with no parameters fits the data perfectly. When one is selecting different models the choice of model should be based on the underlying physics not the statistics.

*C-A Roulet (LESO, Switzerland)*

*I completely agree with you. As proposed in the paper, the condition number is useful to compare two experimental designs, once a model is chosen.*

**J.Axley (MIT, USA)**

If the form of variation of a scalar variable throughout the domain is known a priori (e.g. to be well approximated by linear interpolation, say) then the procedure described is appropriate. But for age of air we do not know the form of variation a priori, specifically we should expect significant gradients across boundary layers. Thus with this in mind it would be best not to measure as the actual physical limits of the domain (i.e. room) but rather somewhat within this physical boundary.

*C-A Roulet (LESO, Switzerland)*

*1. Your comment is completely right. In fact measurements points were located at the extremities of a volume 1m smaller than the room in each direction, that is the points were located at 50 cm of the walls. 2. If the model is not known a priori, a linear model (the simplest) may be chosen and an experimental design planned accordingly. Between the various possible designs, just choose the one having the more common points with a design which is good for 2nd degree model. This way, you can add a minimum number of experiments to the first measurements done if the linear model does not reproduce the realities well enough. And so on for more sophisticated models.*

**R.Grot (Lagus Applied Tech., USA)**

Did you consider using incomplete function basis's. This technique is used in finite element methods. You could also reverse your question; given N measurements, what is the best interpolation scheme?

*C-A Roulet (LESO, Switzerland)*

*There are techniques to answer the reverse question\*, but a criteria shall be first defined: what is best? (\*see references in the paper). However, I remain convinced that it is best to know what knowledge is needed, that is to define a model and adjust the experimental plan accordingly.*

**R.J.F.Van Gerwen (Cauberg-Huygen, Netherlands)**

The best condition number for the linear model is smaller than for the quadratic model. As the condition number gives more or less the ratio of errors in the results and errors in the measurements one would expect a linear model to be better than a quadratic model. On the other hand a quadratic model uses more measuring points and therefore is more accurate. This seems a contradiction to me.

*C-A Roulet (LESO, Switzerland)*

*It is not. A low condition number signifies that the parameters will be determined with a better accuracy, but does not mean that the model will better fit on the reality. Usually, a model with a large number of parameters fits the reality better than another with a low number of parameters. However, it is not recommended to multiply the number of parameters without limit.*