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Paper 11

COUPLED AIR FLOW AND HEAT CONDUCTION MODEL FOR MECHANICALLY VENTILATED FOUNDATIONS

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1. SYNOPSIS

Rising moisture from the ground has caused quite a lot of damage on foundations of Swedish buildings. It is in some constructions possible to prevent this by mechanical ventilation below the floor or below the concrete slab.

This paper will present a model for coupled air flow and heat conduction for mechanically ventilated foundations.

The presented model uses analytical expressions for the air flow in an air-permeable layer below a rectangular building. Analytical double-periodic functions of elliptic type are used.

The ground temperature is simulated by using a time-dependent finite difference method.

2. LIST OF SYMBOLS

h=height of air gap under the floor, m
L=length of building, m
B=width of building, m
K_+=conductance between the ventilated layer and the indoor air, W/m^2 °C
K_-=conductance between the ventilated layer and the ground surface, W/m^2 °C
$\vec{q}_a$= air flow rate, m$_a^3$/m^{2}s (m$_a^3$=m^3 of air)
Q_a= total air flow through a ventilation pipe, m$_a^3$/s
r= radius, m
$\hat{r}$= unit vector pointing in the radial direction
T_a=air temperature in the foundation, °C
T_0=annual average outdoor air temperature, °C
T_+=indoor temperature or temperature above the floor, °C
T_-=ground temperature immediately below the ventilated layer, °C
x=horizontal coordinate, m
y=horizontal coordinate, m
z=complex plane, (x+iy)
Φ=potential function for one or more air point sources in an infinite media, m^2/s
Φ^0=basic potential function from an air point source in an infinite media, m^2/s
Φ_c=complex-valued potential function for one or more air point sources in an infinite media, m^2/s
Φ_c^0=complex-valued basic potential function for an air point source in an infinite media, m^2/s
λ=thermal conductivity of the ground, W/m°C
$\rho_a c_a$=volumetric heat capacity of the air, J/m^3 °C
ϑ_1=Theta function

3. INTRODUCTION

Moisture coming from the ground is a serious problem for some types of Swedish foundations, especially slab on the ground built in the 1970th and at the beginning of the 80's. These houses were often built with the thermal insulation above the concrete slab and without any vapour barrier in the foundation. In another type of slab on the ground the thermal insulation layer is made of a lightweight expanded clay aggregate, placed below the concrete. Unfortunately this layer did not prevent capillary rising water from penetrating into the concrete slab.

If we can prevent the moisture supply from the ground to penetrate the concrete slab or remove the moisture directly above the concrete, and thereby prevent contact between water and organic material in the foundation, the construction will dry out and mould growth and odours will diminish.

One way to remove the moisture supply from the ground is by mechanical ventilation of the foundation, see Figure 1. By such methods the moisture will be transported out from the foundation by the moving air, as long as the air is unsaturated. These methods requires that there is an air-permeable layer below the concrete slab. In a joist floor construction it is often possible, and cheaper, to ventilate above the concrete slab in the floor construction.

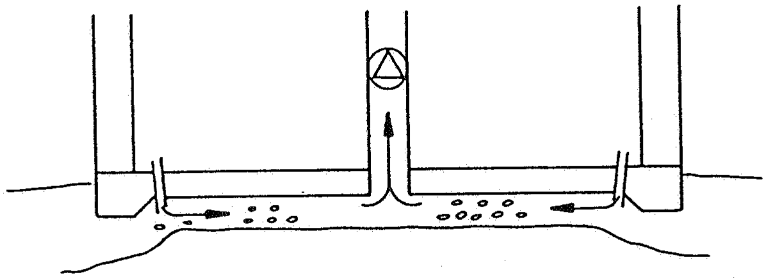


Figure 1: Mechanical ventilation of the foundation

If these methods are going to be successful, certain general conditions must be fulfilled: There has to be a horizontal air gap or an air permeable layer in the construction. The air that is sucked, or pumped into the layer, has to be relatively dry and have a high temperature in order to reduce the risk for condensation. All unventilated connections between the ground and other parts of the building must be as air tight as possible in order to prevent unwanted air leakage.

Because of the condensation risk, it is of great importance to know the temperatures in the ventilated layer, so that condensation can be avoided by using an appropriate air flow intensity.

With the model presented in this paper it is possible to study foundations with two-dimensional air flow coupled to a three-dimensional temperature field below and around the building. The air flow is given by elliptic, complex-valued, double-periodic functions. The three-dimensional temperature field and the energy balance for the air channels are solved by a computer program for time-variable heat conduction based on an explicit finite forward difference method. With this coupled model between air flow and temperature we obtain the temperature distribution below a rectangular building at different depths. The model makes it also possible to investigate how thickness of the thermal insulation, extra insulation outside the outer wall corners, air flow intensity, climate and size of the building influences the temperature distribution in the foundation and in the ground. It is also possible to investigate how positions of the air inlet and air outlet to the vertical rectangular area below a building influence the temperature distribution in the foundation. Figure 2 illustrates two different solutions for a quadratic building. Air inlets, i.e. points where the air are going down into the foundation, are marked $\star$, while air outlets, i.e. points with air leaving from the foundation are marked $\bullet$. In the first case three of the walls are open to air flow. At the middle of the fourth wall, which is closed (air tight), there is an air outlet. In the second case we have a centrally placed air outlet in the building with air inlets near each corner.

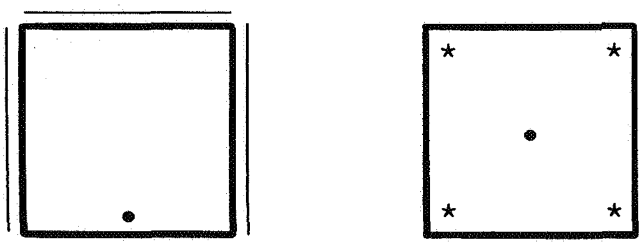


Figure 2: Two different cases for mechanical ventilation of the foundation of a building

These two cases and a third reference case without ventilation of the foundation will be discussed further in Section 6.

Further information about mechanical ventilation of concrete slabs on the ground damaged by moisture, is going to be published in a thesis later this year, by Harderup. General information about repairing methods for concrete slabs on the ground damaged by moisture can be found in Harderup[2,4] and Tobin[6].

4. CALCULATION OF THE AIR FLOW PATTERN

For the considered foundations we will assume that we have an air gap,or a porous layer, of constant height h and constant permeability for flow resistance. The width of the ventilation pipes are assumed to be much smaller than the dimensions of the building. Air inlets to the foundation will be approximated by point sources.

4.1 Radial air flow in an infinite medium

The air flow rate induced by a single air point source in a layer of infinite extension will be studied first. This will give a basic solution that will be used below. Consider an air source located at the origin of coordinates and with the total air flow rate Q_a (m_a^3/s). At the distance r (m) from the center of the air source we get the following air flow rate:

$$\vec{q}_a(r) = \frac{Q_a}{2\pi r}\frac{1}{h}\hat{r} \tag{1}$$

Using the relation

$$\frac{\hat{r}}{r} = \nabla \ln(r), \tag{2}$$

we can write (1) as:

$$\vec{q}_a(r) = \frac{Q_a}{2\pi h}\nabla \ln(r) = \nabla\left\{\frac{Q_a}{2\pi h}\ln(r)\right\} \tag{3}$$

The function in the bracket can be treated as a potential. We introduce the potential of a single air point source in an infinite region:

$$\Phi^0 = \frac{Q_a}{2\pi h}\ln(r) \tag{4}$$

The air flow $\vec{q}_a$ must satisfy the mass balance equation. Using (3) we have:

$$\nabla \cdot \vec{q}_a = 0 \quad \Rightarrow \quad \nabla^2\Phi^0(r) = 0 \qquad r \neq 0 \tag{5}$$

Here $\nabla\cdot$ denotes the divergence operator and ∇^2 denotes the Laplace operator. This Laplace equation for Φ^0 is the same as for steady-state heat transfer problems and electrostatic problems. All results from potential theory are thereby applicable.

4.2 Superpositions of air point sources

For the case with a number of air point sources and sinks located at different places, superposition can be used.

Consider an air point source located at the position (x_n, y_n). The air flow rate at the point (x, y) due to this source becomes:

$$\vec{q}_{a,n}(r) = \nabla\Phi^0(r_n)$$

$$\Phi^0(r_n) = \frac{Q_{a,n}}{2\pi h}\nabla \ln(r_n) \qquad r_n = \sqrt{(x-x_n)^2 + (y-y_n)^2} \tag{6}$$

Here r_n is the distance between the air source and the point (x, y), and $\vec{q}_{a,n}$ is the heat flow rate at the point due to air source number n.

The total air flow rate $\vec{q}_a$ becomes:

$$\vec{q}_a = \sum_{n=1}^{N} \vec{q}_{a,n} = \nabla \left\{ \sum_{n=1}^{N} \Phi^0(r_n) \right\} \tag{7}$$

The foundations we are interested in are of course of finite extension. However, by the use of superposition and the method of images, it is possible to obtain solutions for our cases.

Figure 3 shows the simple case with the superposition of two air point sources of the same air flow rate. Due to the symmetry around the vertical line we obtain a boundary of zero air flow rate.

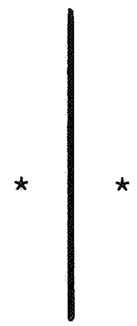


Figure 3: Superposition of two air point sources in order to create a vertical boundary line of zero air flow rate.

To obtain the same result for a rectangular boundary we have to superimpose an infinite number of air point source images of equal air flow rate. Consider the case with a source at the coordinates (x_0, y_0) inside a rectangle with the length L and width B, see Figure 4.

The total potential Φ for the rectangular case with air tight boundaries become:

$$\Phi = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \Phi^0(r_{1mn}) + \Phi^0(r_{2mn}) + \Phi^0(r_{3mn}) + \Phi^0(r_{4mn}) \tag{8}$$

$$r_{1mn} = \sqrt{(x - (x_0 + m \cdot 2L))^2 + (y - (y_0 + n \cdot 2B))^2}$$

$$r_{2mn} = \sqrt{(x - (x_0 + m \cdot 2L))^2 + (y - (2B - y_0 + n \cdot 2B))^2}$$

$$r_{3mn} = \sqrt{(x - (2L - x_0 + m \cdot 2L))^2 + (y - (2B - y_0 + n \cdot 2B))^2}$$

$$r_{4mn} = \sqrt{(x - (2L - x_0 + m \cdot 2L))^2 + (y - (y_0 + n \cdot 2B))^2}$$

For this case we have used sources only. For the cases of physical interest we must have a balanced ventilation, that is the net air flow rate into the foundation must be equal to zero. It should be noted that expression (8) is divergent.

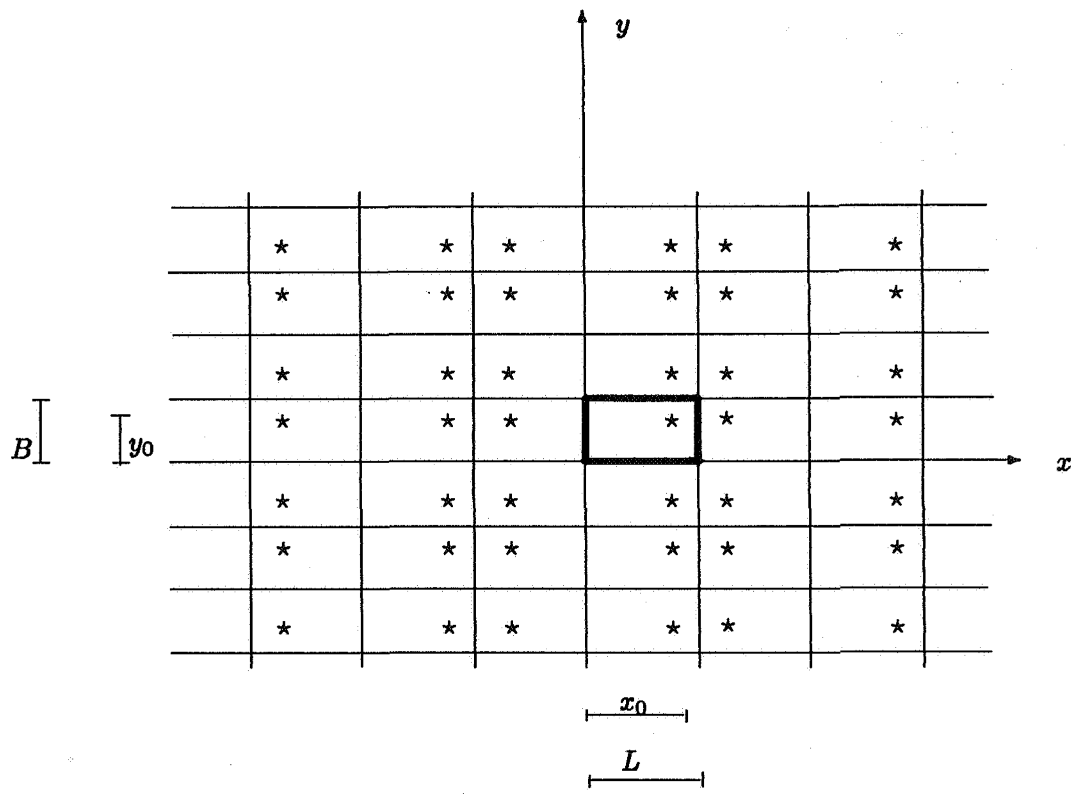


Figure 4: Superposition of an infinite array of point sources in order to obtain zero air flow rate at the rectangular boundary.

The sum of (8) and a corresponding expression for balancing sinks will together converge properly.

Cases with open boundaries, that is boundaries with a constant potential, may also be treated by superposition technique. For these cases it is not necessary to have balanced ventilation, since the net positive air flow rate will flow out through the boundaries. However, the theory for cases with open boundaries will not be dealt with in this paper.

4.3 Complex-valued formulation of the potential functions. Introduction of the theta function

The basic potential function Φ^0 (4) can be expressed as the real part of a complex-valued analytical function:

$$\Phi_c^0(z) = \frac{Q_a}{2\pi h} \ln(z) \qquad z = x + i \cdot y \tag{9}$$

Here we have used the notation c to mark that the function is complex-valued. Formula (8) then becomes:

$$\Phi_c = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \Phi_c^0(z - z_{1mn}) + \Phi_c^0(z - z_{2mn}) + \Phi_c^0(z - z_{3mn}) + \Phi_c^0(z - z_{4mn})$$

$$\begin{aligned} z_{1mn} &= (x_0 + m \cdot 2L) + i(y_0 + n \cdot 2B) \\ z_{2mn} &= (x_0 + m \cdot 2L) + i(2B - y_0 + n \cdot 2B) \\ z_{3mn} &= (2L - x_0 + m \cdot 2L) + i(2B - y_0 + n \cdot 2B) \\ z_{4mn} &= (2L - x_0 + m \cdot 2L) + i(y_0 + n \cdot 2B) \end{aligned} \tag{10}$$

Here we have used the notation Φ_c for the total complex-valued potential function.

Double-periodic arrays of sinks and sources, and the associated Theta functions are studied in Oberhettinger,Magnus[5] and Whittaker,Watson[7]. These analytical quasi double-periodic functions of z have very interesting properties. The Theta function ϑ_1 is defined by the series:

$$\vartheta_1(z,\tau) = 2\sum_{n=0}^{+\infty}(-1)^n q^{(n+1/2)^2} \sin((2n+1)\pi z) \qquad q = e^{i\pi\tau} \quad \Im(\tau) > 0 \tag{11}$$

As we can see, the convergence of the series are very rapid. The function has zeroes at the points:

$$z_{zero} = n + m \cdot \tau \qquad m, n \text{ integers} \tag{12}$$

A Taylor series of the logarithm of ϑ_1 around any of these points shows that it tends to zero as:

$$\ln(\vartheta_1(z,\tau)) \rightarrow \ln(z - z_{zero}) + \text{constant} \qquad z \rightarrow z_{zero} \tag{13}$$

The real part of the logarithm of the Theta function satisfies the potential equation (5) at all points in the plane, since ϑ_1 is an analytical function of z. At the zeroes of the Theta function, the logarithm $\ln(\vartheta_1)$ has the same behaviour as the basic complex-valued potential Φ_c^0 around $z = 0$. Thus the real part of the logarithmic expression gives air sources at all zeroes of ϑ_1 in the plane. It is therefore possible to give a closed expression for Φ_c by using the Theta function.

Figure 5 shows the case with a source and a sink inside a rectangular boarder. The source is located at $x_0 + iy_0$ and the sink at $x_1 + iy_1$. The total potential function becomes:

$$\Phi_c \cdot \frac{2\pi h}{Q_a} = \ln\left\{\frac{\vartheta_1((z-(x_0+iy_0))/2L)}{\vartheta_1((z-(x_1+iy_1))/2L)}\right\} + \ln\left\{\frac{\vartheta_1((z-(x_0+i(2B-y_0)))/2L)}{\vartheta_1((z-(x_1+i(2B-y_1)))/2L)}\right\} +$$

$$\ln\left\{\frac{\vartheta_1((z-(2L-x_0+i(2B-y_0)))/2L)}{\vartheta_1((z-(2L-x_1+i(2B-y_1)))/2L)}\right\} + \ln\left\{\frac{\vartheta_1((z-(2L-x_0+iy_0))/2L)}{\vartheta_1((z-(2L-x_1+iy_1))/2L)}\right\}$$

$$\tau = iB/L \tag{14}$$

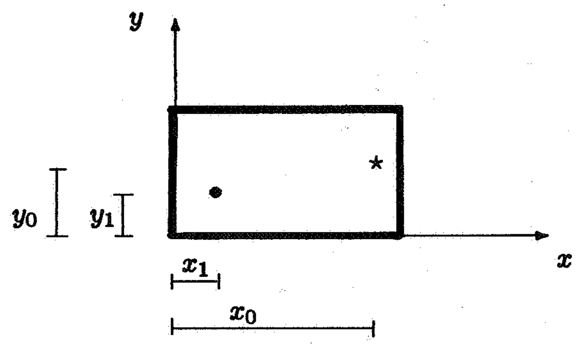


Figure 5: One air point source and one air point sink inside a rectangular boundary of zero air flow rate.

5. COUPLED AIR FLOW AND HEAT TRANSFER FROM THE GROUND

5.1 Balance equation for the air along a stream line

The air flow pattern from the air sources to the sinks is given in Section 4. The air flows in a number of well-defined stream tubes with height h and delimited in the horizontal plane by two stream lines.

Figure 6 shows an air stream tube. The length coordinate along one stream line is denoted by s (m). The width of the stream tube at s is denoted by $b(s)$ (m). The product $q_a \cdot b$ is constant along a stream tube. It gives the air flow rate m_a^3/s in the stream tube. The air temperature is denoted by $T_a(s,t)$. The convective-diffusive heat balance for the air between s and $s + ds$ is:

$$b \cdot ds \cdot K_+(T_+ - T_a) + b \cdot ds \cdot K_-(T_- - T_a) - \rho_a c_a q_a \cdot hb \cdot \frac{\partial T_a}{\partial s} \cdot ds = 0 \quad (15)$$

Here K_+ (W/m^2 °C) is the conductance per unit area, between the indoor temperature T_+ and the air, and K_- the conductance to the center of the first cell in the ground with the temperature T_-. These conductance may be variable along the stream tube. We neglect horizontal heat conduction in the air and the capacity term $(\rho_a c_a \cdot hb \cdot ds \cdot \partial T_a/\partial t)$.

The temperature field in the ground is calculated for time-step after time-step. At each step, equation (15) is solved analytically for every stream tube in the following way. We introduce the average temperature T_m and the length l for the considered section of the stream tube:

$$T_m = \frac{K_+T_+ + K_-T_-}{K_+ + K_-} \quad (16)$$

$$l = \frac{\rho_a c_a q_a hb}{b(K_+ + K_-)} \quad (17)$$

Equation (15) becomes:

$$\frac{\partial T_a}{\partial s} = -\frac{1}{l}(T_a - T_m) \tag{18}$$

For each stream tube we get the same equation as (4) in Harderup,Claesson,-Hagentoft[3]. The stream tube is divided into a number of cells. Cell number i is defined by the area in the stream tube between $s = s_i$ and $s = s_{i+1}$. The quantities l and T_m are piece-wise constant for each cell, $s_i \leq s < s_{i+1}$. The temperature along the stream tube cell becomes:

$$T_a(s,t) = T_{m,i} + (T_a(s_i,t) - T_{m,i})e^{-(s-s_i)/l_i} \tag{19}$$

The air temperature at the air source is given. The outlet temperature becomes the inlet temperature to the next cell, and so on.

When the air temperatures in all air stream tubes are calculated, these will become boundary temperatures for the ground temperature at the next time-step.

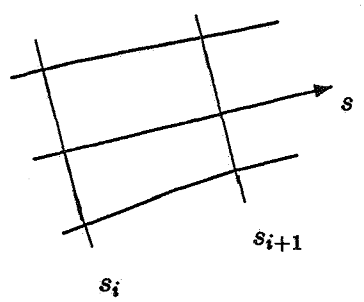


Figure 6: Air stream tube between two stream lines.

In the general case, the stream tube cell is coupled with more than one computational cell in the ground with its rectangular mesh. For this case K_- is the mean conductance between the air and the cells below, and T_- is a weighted ground temperature.

A more detailed treatment of general convective heat flow problems is given in Claesson,Bennet[1].

6. EXAMPLES

The three-dimensional computer program handles time-dependent boundary temperatures and air flows. However, for simplicity we will only show some results from steady-state calculations. All calculations have been performed on an IBM PS/2 386/25 with *87 math co-processer. For the ground we have used a rectangular mesh with 9240 cells. The minimum cell is a cube with the side length 0.2 m. The quadratic building in Figure 2 is used in the calculations.

We have the following data:

$T_+ = 20\ °C$	L=10.4 m	$K_+ = 0.6\ W/m^2 °C$
$T_a(0,t) = 20\ °C$	B=10.4 m	$K_- = 4.6\ W/m^2 °C$
$T_0 = 6.7\ °C$	Q_a=100.0 m_a^3/h	$K_w = 0.5\ W/m^2 °C$
$\lambda = 1.5\ W/m°C$		

During the calculations we have constant indoor temperature T_+. The temperature of the air entering into the foundation, $T_a(0,t)$, has the same value as the indoor air. Outside the building the temperature is T_0, which is the annual average outdoor temperature in Stockholm. The dimensions of the building is $L \times B$, and Q_a is the total air flow sucked out from the foundation. Below the outer walls there are thermally insulated foundation walls, with a depth of 0.6 m. The corresponding conductance is denoted K_w. The thermal conductivity for the soil is denoted by λ.

In the example to the right in Figure 7, (A), three of the outer walls are open to air flow from the inside of the building. At the middle of the fourth wall, which is airtight, the air outlet is an exhaust air fan connected to the floor.

In the example to the left, (B), we have a centrally placed exhaust air fan combined with air supply devices (air inlets) at the outer corners. The inlet to the exhaust air fan tube is placed in the ventilated layer below or above the concrete slab. The air supply devices are in direct contact with the indoor air. All connections between the walls and the floor is assumed to be airtight.

A third case, (C), has also been calculated, as a reference. In this case there is no mechanical ventilation of the foundation.

In Figure 7, the streamlines to the exhaust air fan are shown. In the first case, with three open boundaries, the ventilation intensity is poor at the two corners opposite to the airtight wall. For the second case, with air supply devices only at the outer corners, the mid regions near the outer walls are poorly ventilated.

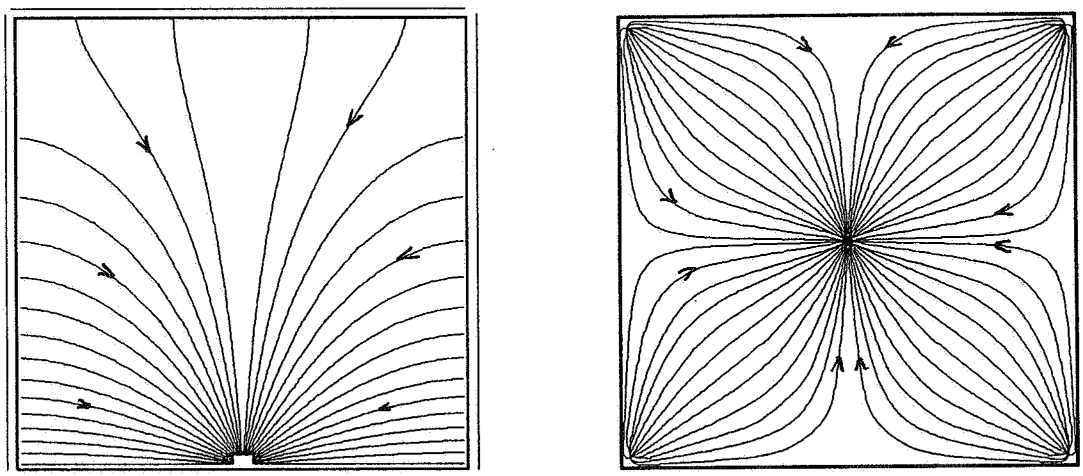


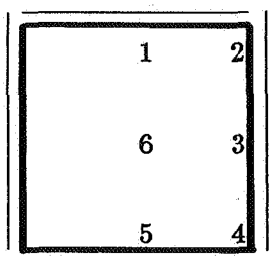
Figure 7: Air stream lines for two types of ventilated foundations. The first building (A), left, has open boundaries at three sides and an exhaust air fan in the middle of the fourth side. The second one (B), right, has air supply devices at the corners and an exhaust air fan in the middle.

Point	Surface temperatures °C		
	Case		
	A	B	C
1	11.9	11.7	11.5
2	10.5	18.1	10.1
3	16.0	11.7	11.5
4	16.9	18.1	10.1
5	13.0	11.7	11.5
6	14.4	14.1	14.2

Table 1: Surface temperatures in six discrete points from steady-state calculations

The influence of the air flow, on the steady state temperature at some discrete points (1-6) is shown in Table 1. These points are marked in Figure 8. The presented values are the temperatures at the surface of a layer immediately below the ventilated horizontal layer.

Low ventilation intensity, near the outer walls, results in small temperature differences between the unventilated case and a ventilated case. From Table 1 it can be seen that the surface temperatures near the outer walls are strongly dependent on how the system is designed. In the undisturbed case (C) without ventilation, the temperature is lowest near the corners. With a ventilation system of type (B), the annual average temperature increased by 8 °C in the corners, points 2 and 4. From the studied cases it can also be seen that the temperature increase, at the middle of the outer walls, is greatest for case (A). It should be noted that temperatures below the center of the building is very stable, see point 6.



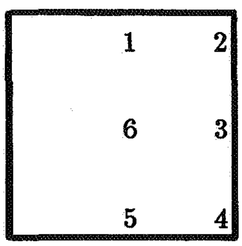


Figure 8: Locations of calculated temperatures given in Table 1.

For a certain building with a certain air flow pattern the surface temperatures can be raised by higher air flow intensity or by thermal insulation in the ground, near the outer walls outside the building.

Before an installation of such a mechanical ventilation system is made, the program can be used to investigate how different locations of the supply air devices and exhaust air devices will influence the temperature at a certain point in the foundation, or immediately outside the building.

7. ACKNOWLEDGEMENTS

This theoretical work has been initiated by Dr. Johan Claesson at the Department of Building Technology in Lund. The computer programming has been carried out by Johan Bennet at the Department of Mathematical Physics in Lund. Their support has been of great value.

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Discussion

Paper 11

Frank D. Heidt (University of Siegen, FRG)

a) Did you consider the effect of evaporating water on air and surface temperatures?

b) There should be an influence due to latent heat transfer.

Carl-Eric Hagentoft (Building Technology, Lund, Sweden)

In the presented model with a two-dimensional air-flow coupled to a three-dimensional temperature field we do not take the effects of evaporating water and latent heat into account. In a similar model with one-dimensional air-flow coupled to a two-dimensional temperature field the effect of evaporating water and latent heat are taken into account. In this model the vapour concentration in the outdoor air, the moisture supply from the ground and the moisture supply to the inside air are accounted for. With this model we can calculate the relative humidity and the temperature along the ventilated layer. All facilities in the model with one-dimensional air flow are going to be incorporated in the other model too.

Mike Holmes (Ove Arup, London, UK)

Temperature predictions require knowledge of heat transfer coefficient. Could you say what values were used for the surface convection coefficient, and if radiant exchange within the cavity was considered to be important?

Carl-Eric Hagentoft (Building Technology, Lund, Sweden)

We assume a constant temperature within the stream tubes and thereby neglect the radiation. The surface resistances are usually small compared with the overall resistance between the air in the stream tube and the ground/indoor temperature. So we have just used standard values if I remember it right.

$1/\alpha$ in the tube is around $0.1\ m^2U/m$ for the examples I have shown.

Alfred Moser (ETH, Switzerland)

For potential flow in a cavity potential lines are not lines of constant pressure in general. (They are in special cases such as radial flow from a single source in infinite space).

Carl-Eric Hagentoft (Building Technology, Lund, Sweden)

In forced convection we neglect the temperature influence on the air flow pattern. We have the following assumption:

$q_a \sim \nabla p$

q_a = *air flow rate*

p = *pressure*

This is the same definition as for our potential discussed in the paper. This means that the pressure will become proportional to the potential function: $p \sim \phi$.