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A NUMERICAL STUDY OF BUOYANCY-DRIVEN FLOWS OF MASS AND ENERGY IN A STAIRWELL.

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SUMMARY

This paper describes a two-dimensional numerical study, by finite-volume method of buoyancy-driven flow in a half-scale model of a stairwell. The stairwell forms a closed system within which the circulation of air is maintained by the supply of heat in the lower floor. The heat loss takes place from the stairwell walls. The mathematical model consists of the governing equations of mass, energy, momentum and those of the $k - \epsilon$ model of turbulence. The predicted flow pattern and the velocity in the stairway are presented and compared with the authors' experimental data.

NOMENCLATURE

A	area (m^2)
a	coefficient of finite-difference equation
C_μ, C_1, C_2, C_3, C_D	constants in the k-ϵ turbulence model
c_p	specific heat ($J\ kg^{-1}\ K^{-1}$)
g	gravitational acceleration ($m\ s^{-2}$)
k	turbulence kinetic energy per unit mass ($N\ m\ kg^{-1}$)
Pe	cell Péclet number
$\dot{q}_w''$	wall heat flux ($W\ m^{-2}$)
S_ϕ	source term for variable ϕ ($S_\phi = b\phi + c$)
T	absolute temperature (K)
T_w	wall temperature (K)
T_P	temperature at node P next to wall (K)
u^+	non-dimensional velocity in wall region ($u^+ = \frac{u}{u_\tau}$)
u_τ	friction velocity ($m\ s^{-1}$)
u, v	mean velocity components in x and y directions, respectively ($m\ s^{-1}$)
y^+	non-dimensional distance from wall ($y^+ = \frac{u_\tau\, y}{\nu}$)
y_P	distance from node P to the adjacent wall (m)

Greek Symbols

β	coefficient of thermal expansion (K^{-1})
Γ_ϕ	diffusion coefficient for variable ϕ ($\Gamma_\phi = \frac{\mu}{\sigma_\phi}$)
ϵ	rate of turbulence energy dissipation per unit mass ($N\ m\ kg^{-1}\ s^{-1}$)
μ	molecular viscosity ($kg\ m^{-1}\ s^{-1}$)

μ_t	turbulent viscosity (kg m^{-1} s^{-1})
μ_{eff}	effective viscosity ($\mu_{eff} = \mu + \mu_t$)
ρ	fluid density (kg m^{-3})
ρ_r	reference density (kg m^{-3})
ν	kinematic viscosity (m^2 s^{-1})
τ_w	wall shear stress (N m^{-2})
ϕ	general dependent variable
$\sigma_T, \sigma_{T,t}$	laminar and turbulent Prandtl number, respectively
σ_ϵ, σ_k	constants of turbulence model

Subscripts

n, e, s, w	Control volume faces
t	turbulent

1. INTRODUCTION

A better understanding of buoyancy-driven flows in stairwells is important in relation to energy saving in buildings, design of air-conditioning systems, architectural design, and fire studies. Until recently this type of flow has received relatively little attention, either experimentally or theoretically.

A number of experimental investigations of stairwell flows have been reported by Feustel et al.,[1] Marshal[2,3,4], Klote and Bodart[5], Zuercher et al.[6], Tamura et al.[7,8], Chu[9] and Maguire[10]. More recently Reynolds[11] and Reynolds et al.[12] developed analytical modelling of the flow processes within stairwells.

In order to improve the understanding of buoyancy-driven flow and the associated energy transfer within the stairwell, the authors have carried out experiments on a half-scale model of a stairwell. These have been reported by Marriott and Reynolds[13], and Zohrabian et al.[14]. Parallel to the experimental investigations, mathematical modelling has also been carried out for prediction of the flow in the stairwell. We have used the k - ϵ model of Harlow and Nakayama[15], as developed by Launder and Spalding[16].

Studies using the k - ϵ model have been applied to a variety of engineering problems similar to that of the stairwell. Neilsen et al.[17] and Alamdari et al.[18] studied buoyancy-affected flows in ventilated rooms. Ideriah[19], Markatos et al.[20] and Fraikin et al.[21] predicted buoyancy-induced flows in cavities. In another study Markatos[22] predicted the air flow and heat transfer in television studios. Markatos et al.[23,24], Cox et al.[25] and Kumar et al.[26] used a Fire Research Station computer program (known as JASMINE) to analyse the smoke movement in enclosures.

The objective of the present work is to predict the velocity and temperature distributions in the stairwell and to assess the results with the aid of experimental data.

2. THE PHYSICAL MODEL

The half-scale stairwell model geometry is shown in Fig.1. It consisted of a lower and an upper compartment connected by the stairway. The recirculation of air was maintained by continuous supply of heat through a heater positioned in the lower compartment. The area we refer to as the "throat area", shown by the broken line D-D' in Fig. 1, is the area within which most of the measurements were taken. The full details of the experimental rig and the measurement techniques are reported elsewhere[14].

3. THE MATHEMATICAL MODEL

3.1. The governing equations in differential form

In a two-dimensional Cartesian coordinate system, the conservation differential equations for continuity, momentum, energy and those of turbulence energy, k, and its rate of dissipation, ϵ, can be written in the general form

$$\frac{\partial}{\partial x}(\rho u \phi) + \frac{\partial}{\partial y}(\rho v \phi) = \frac{\partial}{\partial x}(\Gamma_\phi \frac{\partial \phi}{\partial x}) + \frac{\partial}{\partial y}(\Gamma_\phi \frac{\partial \phi}{\partial y}) + S_\phi$$

where ϕ, Γ_ϕ and s_ϕ are given in table 1.

The turbulent viscosity, μ_t, in the k-ϵ model is related to k-ϵ (see for example references 15,16,39):

$$\mu_t = C_\mu \, \rho \, \frac{k^2}{\epsilon}$$

Based on the concept of eddy diffusivity, $-\rho\overline{v'T'}$ in the k and ϵ equations can be replaced by

$$-\rho\overline{v'T'} = \Gamma_{T,t} \frac{\partial T}{\partial y}$$

3.2. The Governing Equations in Discretised form and the Solution procedure

The first step in deriving the finite-difference equations is to adopt an appropriate grid system. We have adopted the staggered grid system suggested by Patankar and Spalding[27], as shown in Fig.2. In such a system the scalar variables (p, T, k, ϵ) are stored at the grid nodes, while u and v velocities are stored at the mid-point between the two adjacent nodes.

The governing equations in discretised form are obtained by integration of the differential equations over the corresponding control volumes, with the aid of a discretisation scheme. The schemes chosen for this study are described in section 3.3. The discretised forms of the governing equations of momentum, thermal energy, turbulence energy and energy dissipation rate can be written as:

ϕ - transport (ϕ = T, k, ϵ)

$$(a_p - b)\phi_P = \sum_n a_n \phi_n + c$$

u - momentum

$$(a_P - b)u_P = \sum_n a_n u_n + A_w (p_w - p_P) + c$$

v - momentum

$$(a_P - b)v_P = \sum_n a_n v_n + A_n (p_s - p_P) + c$$

where

$$a_P = \sum_n a_n \quad : \qquad a_n = \rho_n v_n A_n f_n$$

and $\sum_n$ denotes summation over neighbours N,S,E,W.

The symbol f_n represents a weighting factor, which is determined according to the chosen scheme. For example, in the hybrid difference scheme for the north boundary of the cell, it can be written as [28,29,30]:

$$f_n = \begin{cases} \frac{1}{2}(1 + 2\,Pe_n^{-1}) & \text{for} \quad -2 < Pe_n < 2 \\ 1 & \qquad Pe_n \geq 2 \\ 0 & \qquad Pe_n \leq 2 \end{cases}$$

where Pe is the cell Péclet number.

The main variables in the above equations are u, v, k, ϵ and T. The remaining unknown variable, i.e., pressure, has no equation of its own. To derive the pressure, a special procedure known as SIMPLE (Semi-Implicit Method for Pressure-linked Equations) was used [27,30]. The procedure is based on an iterative solution of the governing equations, by which the variables, including pressure, are guessed over the entire field of solution and then corrected as the iteration proceeds. In this procedure the continuity equation is used to derive an additional equation known as the pressure-correction equation. The main variable in this equation is the pressure-correction (p'), which when added to the guessed (current) value of the pressure (p^*) results in an improved value of the pressure ($p = p^* + p'$). This equation is written in the same form as the equations for other scalar variables (T,k,ϵ). The six discretised equations are solved simultaneously using the line-by-line method and the Tri-Diagonal Matrix Algorithm, in the following sequence : u, v, p', T, k, ϵ.

3.3. The Discretization Schemes

The detailed description of the different discretisation schemes and their mathematical formulation is described by Patankar[30]. In this study, several schemes have been adopted for comparison. These are central-difference, hybrid and power-law schemes.

In the central-difference scheme a piecewise-linear variation for ϕ is assumed between the grid nodes. According to Patankar[30], the central-difference scheme gives accurate results for $|Pe| < 2$. Outside this limit the scheme is inaccurate[30,31,32].

In the upwind scheme the value of ϕ at an interface of the adjacent nodes is assumed to be equal to the value of ϕ at the grid node on the upwind side.

The hybrid scheme was developed by Spalding[33]. This scheme is a combination of the central-difference and the upwind schemes. The significance of the hybrid scheme is: (i) For $|Pe| < 2$ it is identical with the central-difference, (ii) Outside this range it reduces to the upwind scheme.

The power-law scheme[34] approximates closely the exponential (exact) variation of the property between the two grid nodes. According to Patankar[30], it is premature to ignore the diffusion effects, as soon as the Péclet number exceeds 2, as is the case in upwind scheme. This scheme has the following advantages: (i) it is not expensive to compute, and (ii) at $|Pe| > 10$ the power-law scheme becomes identical with the hybrid scheme.

4. BOUNDARY CONDITIONS

The usual boundary conditions are the non-slip condition for the velocity, and the definition of the wall temperature or the wall heat flux. However, other modifications to the discretised equations are necessary to account for the contribution of the wall to the adjacent cell, for example, in the form of the shear-stress force. Also, the equations for turbulence energy and energy dissipation have to be modified, as the form given in Table 1 is suitable only for high Reynolds number flows. If a wall thermal boundary condition is in the form of a given temperature, then the wall heat flux has to be calculated. Two of the approaches usually adopted for the special treatment near the walls are the low Reynolds number models, as described for example by Jones and Launder[36,37] and the wall-function method[38,16]. The first approach (not adopted in this study) requires a very fine grid within the wall layer, and this makes it unsuitable for complex geometries such as the stairwell. The second approach, that adopted for this study, is based on the assumption that a standard wall layer has formed at the wall. The boundary-layer equations are then used for the calculation of various parameters such as wall shear stress and heat flux. The details of the wall-function method are described in many sources (see for example references 16,38) and are not repeated here. However, the wall treatment for the thermal boundary conditions is of particular importance in this study and is therefore mentioned here.

For the heater the heat flux is known, see Table 2. It was assumed that an equal amount of heat is transferred to the air from each of the two sides, that facing into the room and that facing the wall, see Fig.1. The heat flux was introduced directly in the equations via the source term. For the stairwell walls, the wall temperatures are specified and the heat flux is calculated from the following relations:

If $y^+ \leq 11.63$

$$\dot{q}''_w = \frac{\mu C_p (T_w - T_P)}{\sigma_T y_P}$$

If $y^+ > 11.63$

$$\dot{q}''_w = \frac{\rho C_p C_\mu^{\frac{1}{4}} k^{\frac{1}{2}} (T_w - T_P)}{T^+}$$

where

$$T^+ = \sigma_{T,t} [u^+ + f]$$

and f is a function given by Jayatillaka[35] as :

$$f = 9.24 \left\{ \left(\frac{\sigma_T}{\sigma_{T,t}}\right)^{\frac{3}{4}} - 1 \right\} \left\{ 1 + 0.28 \exp \left(-0.007 \frac{\sigma_T}{\sigma_{T,t}}\right) \right\}$$

5. COMPUTATIONAL DETAILS

A grid of non-uniform intervals was employed, the nature of which can be realised from the vector plot, Fig. 3. The grid size was 56 x 37. The choice of minimum grid size was rather restricted due to the geometry of the flow domain, as at least two grid lines were necessary for each step of the stairs, and a reasonable number for the upper and the lower compartments. The maximum grid size was also limited, if the computing time was to be kept to an acceptable level. The final size of 56 x 37 was chosen after considerable trial.

The computing time per iteration was 4 to 5 seconds on a Pyramid 9820 computer. This merely gives a general idea of the computing time involved because the computing time, in general, depends on many factors. The type of computer and grid size are obviously important. But it also depends on how efficiently the computer program is written and how the initial conditions are defined. Using information from previous runs for the initial conditions reduces the computing time considerably.

The criterion for convergence in studies of this type is to check the gradual reduction of the sum of the residual sources of all the cells (an exact solution would give zero residuals). Also, the computed values of each variable should reach steady values. The sum of the residual sources is normally compared with a suitable reference value. For example, for an open system the sum of mass residuals is compared with the inflow of mass into the flow domain. In this study, in the absence of such an obvious reference value, the variation of the sum of the residual sources was plotted against the number of iterations, and examined. The results showed considerable fluctuations at first, followed by a gradual stabilisation and decrease. The computation stopped when the sum of residual sources for each equation reduced to a relatively small value.

The initial conditions for the variables were as follows :
u was set to 0.1 m/s at the extreme end of the lower compartment (AC in Fig. 1). The u velocities in other locations were computed from the continuity equation. Temperature was set to 20^{o}C. v and p were set to zero. The turbulence energy and energy dissipation rate were obtained from

$$k = 0.03\, u^2$$

$$\epsilon = \frac{k^{3/2}}{0.005\ell}$$

where ℓ is the height of the stairwell.

6. RESULTS AND DISCUSSION

The stairwell model we have examined here is a simple configuration compared with the various designs used in buildings. Our experimental work on a half-scale stairwell model[14], showed that the flow was three-dimensional and unsteady. There were a number of separation and recirculation zones. What is more, the relative importance of viscosity varies dramatically through the field, being very large in corners and near the steps, and relatively small in the body of the flow. An additional difficulty in modelling of the flow is the great variety of heat transfer processes which must be described in adequately realistic fashion. There is heat transfer from every surface and the nature of the flow changes profoundly around the boundaries of the field.

The two-dimensional approach we have adopted here can, therefore, serve to give only a general picture of the actual flow. A three-dimensional approach should provide a better solution. However, any model is based on a set of assumptions which may be violated in one way or another in these wide-ranging conditions. The flow pattern predicted is shown in Fig.3. This shows close agreement with the pattern established by the authors' experimental work[14], as shown in Fig.4. The main features of the flow are the rising column of warm air along the heated walls of the heater, followed by a nearly parallel flow along the ceiling of the lower compartment. As the air flows into the upper compartment, part of it forms a recirculation zone and the other part moves towards the ceiling and, after a circulation in the upper compartment, flows down along the steps to the lower part of the heater.

The predicted velocity profiles at the throat area, using two discretisation schemes, are shown in Fig.5. The experimental values are also included. This figure shows that the velocity values at the throat area are underpredicted in the upper region of the upflow, and in the downflow. However, in comparing the two, one should bear in mind that the experimental results are obtained in three-dimensional flow. The difference may also be attributed, apart from the inadequency of the mathematical model, to the experimental error inherent in the data. Also it should be noted that the measurements of the wall temperatures showed that they varied along each wall, while average values were adopted in this study. This approach was chosen because a single average value is probably what is available to a designer.

The same argument is valid when comparing the temperatures. The range of temperatures in the throat area obtained by computation was 62 to 76^{o}C. This was high compared with the measured values, which were in the range of 34 to 48^{o}C. In the experimental rig heat losses took place from the sides of the stairwell. This was obviously absent in the two-dimensional model. The over-prediction of the temperature may also be related to the turbulence model, as similar overprediction has been reported by Alamdari et al[18]. They suggested that a factor might be introduced into the dissipation term of the turbulence energy equation to reduce the rate of dissipation of turbulence energy near the walls.

An examination of the experimental results for heat losses from the walls showed that about two-thirds of the heat loss took place through the upper compartment. This is in close agreement with our prediction.

We have presented the results shown in Fig. 5 using two different discretization schemes, namely, the power law and the hybrid schemes. The former should lead to more accurate results in the expense of more computer time. The results shown in Fig.5 indicate that the difference between the two predicted results is not appreciable.

We have also used the central-difference scheme. However this did not lead to a converged solution. The choice of discretisation scheme is closely related to the range of cell Péclet numbers. They were in the range of -20 to 20 in the lower compartment, and -10 to 10 in the upper compartment. Higher values were obtained near the heater and close to the walls, and much smaller values in the central regions of the recirculation zones. The range of Péclet numbers indicates the reason for failure of the central-difference scheme.

6.1. The Case of an "Open Stairwell"

Work is underway on a so-called "open stairwell". This situation may arise when air enters the stairwell, for example, through cracks. To simulate this case experimentally, two openings (10mm wide) were introduced, one in the lower and one in the upper compartment as shown in Fig. 6. The experiments conducted so far indicate that air is pulled through the opening A, due to the temperature difference between inside and outside of the stairwell, flows a short distance along the floor and then rises along the heater walls. The same amount of air obviously leaves the stairwell from the opening B. The general flow pattern in the stairwell was similar to that shown in Fig.4. Prediction of the flow is also progressing. A uniform velocity of 0.9 m/s was introduced at the opening A. The results indicate that the jet penetrates farther along the floor than is observed experimentally, apparently due to slow mixing of the jet. This behaviour needs further investigation and the results will be reported elsewhere.

7. CONCLUSIONS

The two-dimensional model, based on the k-ϵ model of turbulence, appears to be adequate in obtaining some basic information on the flow characteristics in the stairwell. The predicted flow pattern was in good agreement with the pattern established by experiment on a half-scale stairwell model rig. The predicted velocities were reasonable. The proportion of the heat loss from the upper compartment was also in good agreement with the experiment.

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Equation	ϕ	Γ_ϕ	S_ϕ
Continuity	1	0	-
u-momentum	u	μ_{eff}	$-\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial v}{\partial x}\right)$
v-momentum	v	μ_{eff}	$-\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu_{eff}\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial v}{\partial y}\right) + g(\rho_r - \rho)$
Turbulent Energy	k	$\frac{\mu_{eff}}{\sigma_k}$	$G_k - C_D \rho \epsilon + \rho g \beta \overline{v'T'}$
Energy Dissipation	ϵ	$\frac{\mu_{eff}}{\sigma_\epsilon}$	$C_1 \frac{\epsilon}{k} G_k - C_2 \rho \frac{\epsilon^2}{k} + C_3 \rho \frac{\epsilon}{k} g \beta \overline{v'T'}$
Energy equation	T	Γ_{eff}	S_T

$$\Gamma_{eff} = \frac{\mu}{\sigma_T} + \frac{\mu_t}{\sigma_{T,t}}$$

$$G_k = \mu_t \left\{ 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right\}$$

The empirical constants have been adopted from Launder and Spalding[16] and take the following values :

$C_\mu = 0.09$, $C_D = 1.0$, $C_1 = 1.44$, $C_2 = 1.92$, $C_3 = 1.0$, $\sigma_k = 1.0$, $\sigma_\epsilon = 1.3$

TABLE 1. The differential equations of the mathematical model .

WALL	AB	BC	CD	DE	EF	FG	GH	HI	IJ	JA
Temperature °C	60	40	40	28	30	27	27	27	30	40
Heat Flux from the two walls of the heater = 1600 W/m^2										

TABLE 2. Thermal Boundary Conditions

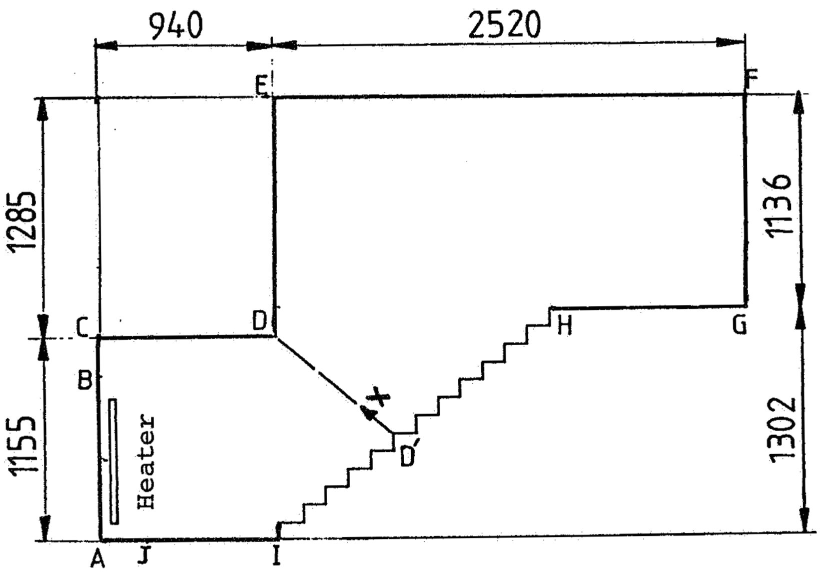

Figure 1. Stairwell model geometry. DD' indicates the throat area. (Dimensions are in mm).

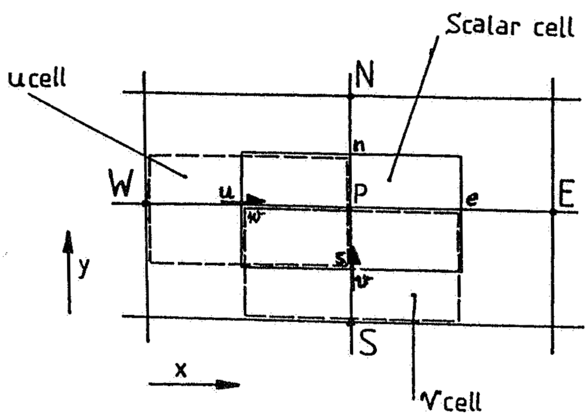

Figure 2. Control volumes of staggered-grid system.

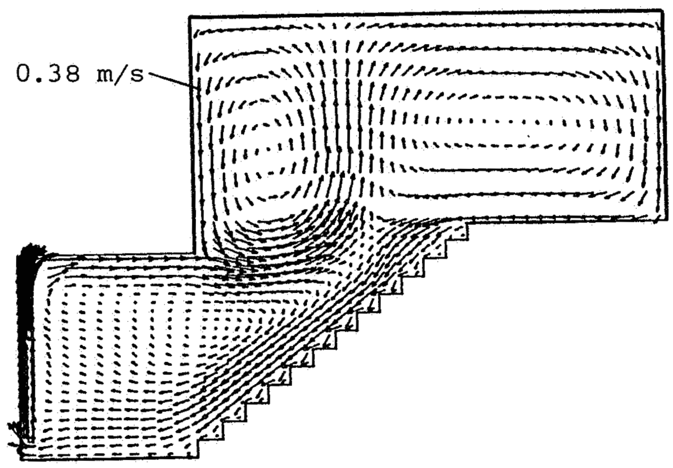

Figure 3. Predicted vector plot of the flow field.

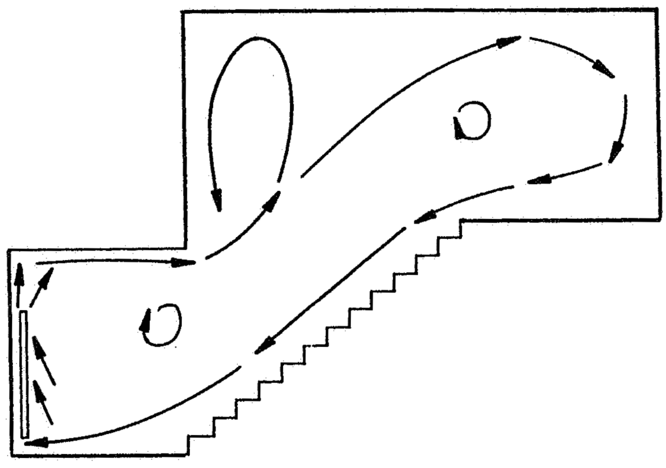

Figure 4. A two-dimensional view of the flow pattern in the stairwell.

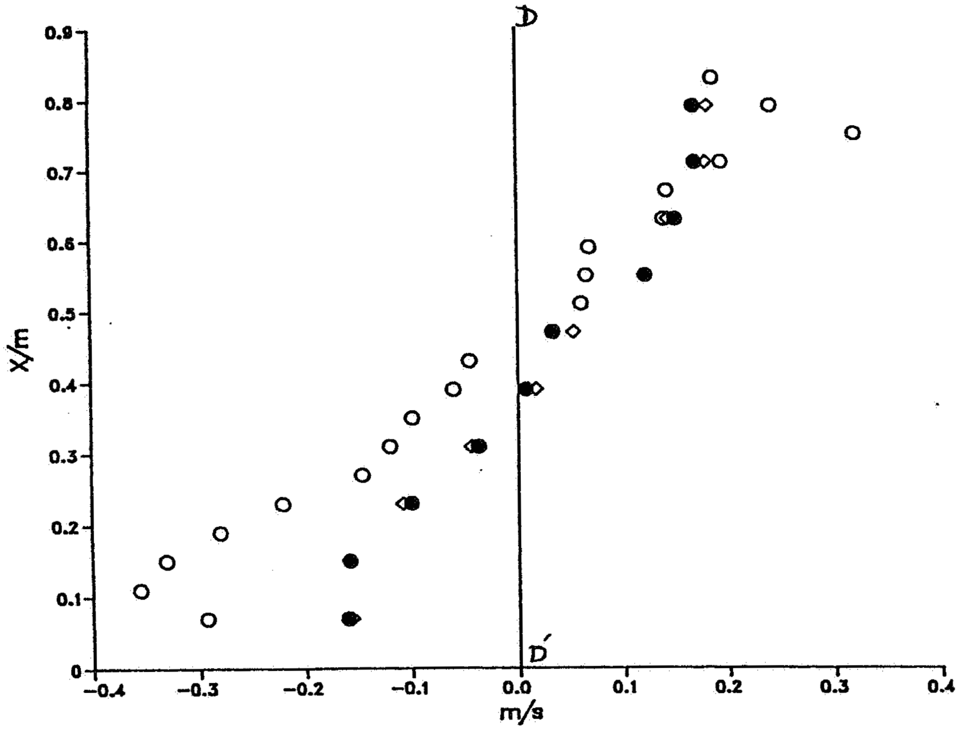

Figure 5. Predicted (2D) and measured (3D) profiles of the component of velocity (Perpendicular to DD') at the throat area.

O - experiment ● - prediction (hybrid scheme)

◊ - prediction (power-law scheme).

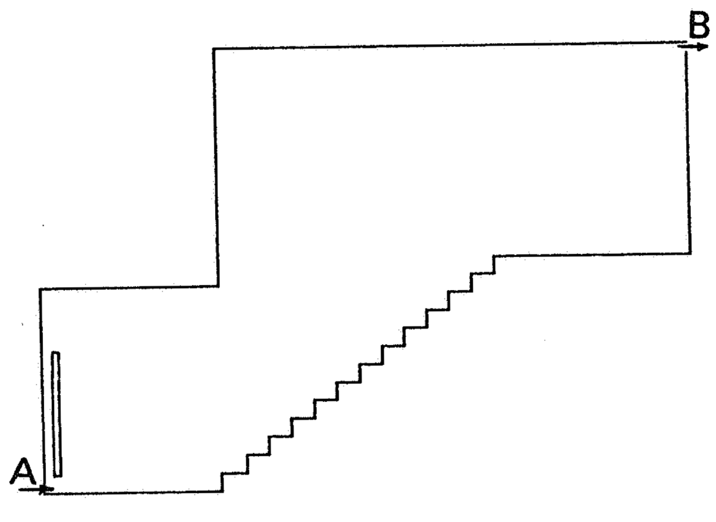

Figure 6. Schematic diagram of the open stairwell.