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**DEVELOPMENT OF AN EFFICIENT CONTROL ALGORITHM
FOR A MULTIZONE CONSTANT CONCENTRATION TRACER GAS
AIR INFILTRATION MEASUREMENT SYSTEM**

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ABSTRACT

A constant concentration tracer gas (CCTG) measuring system needs a control algorithm to calculate, at each sampling time, the required tracer gas injection rate to keep the gas concentration at a target level. A new control algorithm is presented here in full details. Practical considerations concerning modifications to take into account the physical limitations of the CCTG system and the computing of the optimal control parameters are also presented.

1. INTRODUCTION

The constant concentration tracer gas technique (CCTG) is now commonly used for air flow measurement within buildings. This technique requires a control algorithm to keep the gas concentration at a target level by computing the necessary amount of tracer gas to inject. In an inhabited multizone building, the control algorithm has to respond quickly to large variations of interzonal airflows and/or outside air infiltration.

Few authors^{1,2,3} have described in detail the implemented algorithm of their CCTG systems. As far as we know, they all use common control methods such as P, PI or PID.

We present a different control algorithm, used on our CESAR* system^{4,8}. In order to improve this control method, we developed a computer program simulating a room with variable air change rate which takes the real characteristics of the CCTG system into account. This program allowed us to test some modifications of the theoretical control algorithm and to estimate its optimal parameters.

* CESAR : Compact Equipment for Survey of Air Renewal.

2. THEORETICAL APPROACH

2.1 Single zone model

In a first step, we consider a single zone. The equation governing the tracer gas concentration into the zone is :

$$V \frac{dC}{dt} = -FC + S \quad (1)$$

where

- V : effective volume of the zone [m³]
- C : tracer gas concentration [-]
- F : air flow leaving the zone [$\frac{m^3}{s}$]
- S : tracer gas injection into the room [$\frac{m^3}{s}$]

This equation assumes that the outside tracer gas concentration is negligible and that the air has a constant density. A perfect mixing of the tracer gas within the zone is also assumed. Even if these assumptions are not always true in real conditions, this model is sufficiently good for control purposes.

Since the CCTG system analyses the concentrations in the zone by sampling at discrete time, the differential equation (1) should be integrated over one sampling time T_s to give a difference equation :

$$C_{k+1} = a_k C_k + b_k U_k \quad (2)$$

where : C_{k+1}, C_k : tracer gas concentration at time $(k+1) T_s$ and $k T_s$ respectively [-]

$$U_k = \frac{S_k}{V} : \text{injection rate } \left[\frac{1}{s} \right]$$

$$a_k = \exp \left(- \frac{F_k}{V} \cdot T_s \right) \quad [-] \quad (3)$$

$$b_k = V \left(\frac{1 - a_k}{F_k} \right) \text{ if } F_k > 0, \quad b_k = T_s \text{ if } F_k = 0 \quad [s] \quad (4)$$

T_s : sampling time [s]

k : sample number [-]

The air flow F_k and the injection rate S_k are assumed to be constant over the integrating time interval $(k T_s; (k+1) T_s)$.

2.2 Control algorithm

The function of the control algorithm is to compute the necessary injection rate U_k to maintain the concentration at a target level denoted W_k (for complete generality the target level may also change during time : therefore it is also denoted by the sample number k). The injection rate is computed using the expression :

$$U_k = - K_s C_k + K_R X_{Rk} + K_w W_k \quad [s^{-1}] \quad (5)$$

with K_s, K_R, K_w : parameters of the control algorithm $[s^{-1}]$

X_{Rk} : an integrating term defined by the difference equation :

$$X_{Rk+1} = X_{Rk} + W_k - C_k \quad [-] \quad (6)$$

This expression is derived by minimizing the quadratic form :

$$J = \sum_{k=0}^{\infty} [(W_k - C_k) Q_e (W_k - C_k) + X_{Rk} Q_R X_{Rk} + U_k R U_k] \quad (7)$$

where Q_e, Q_R and R are weighting factors (see ⁵ for full details).

Different methods are available for the determination of the parameters K_s, K_R and K_w . We choose a method which consists of imposing the poles of the system⁵. This method is far more practical to use for two reasons :

- It is not necessary to make an arbitrary choice of the weighting factors for the quadratic form
- The computing of K_s , K_R and K_w is easier and doesn't require complex computer code as the other methods.

In fact the two poles of the system depend only on the parameters K_s and K_R . The value of the third parameter K_w doesn't have any influence on the pole location but may be used to compensate the effect of one pole.

The relations between the poles and the parameters are :

First case : Real poles Z_1 and Z_2 (Z_1 , is the compensated pole)

$$K_s = \frac{-Z_1 - Z_2 + a_k + 1}{b_k} \quad \left[\frac{1}{s}\right] \quad (8)$$

$$K_R = \frac{(1 - Z_1)(1 - Z_2)}{b_k} \quad \left[\frac{1}{s}\right] \quad (9)$$

$$K_w = \frac{K_R}{1 - Z_1} \quad \left[\frac{1}{s}\right] \quad (10)$$

Second case : Two complex conjugate poles $Z_{1,2} = \text{Re}(Z) \pm i \text{Im}(Z)$
(the real part of the pole is compensated)

$$K_s = \frac{(-2 \text{Re}(Z) + a_k + 1)}{b_k} \quad \left[\frac{1}{s}\right] \quad (11)$$

$$K_R = \frac{((\text{Re}(Z) - 1)^2 + \text{Im}^2(Z))}{b_k} \quad \left[\frac{1}{s}\right] \quad (12)$$

$$K_w = \frac{K_R}{1 - \text{Re}(Z)} \quad \left[\frac{1}{s}\right] \quad (13)$$

where a_k and b_k are defined by (3) and (4).

Through a_k and b_k the parameters are functions of the sampling time T_s and of the air change rate $\frac{F_k}{V}$. There is apparently a problem here as the control algorithm requires parameters which depend on the varying air change rates not known before measurement ! In fact the control algorithm needs only an estimation of the air change rate and the parameters are calculated once for the entire measurement procedure. This point and the optimal location of the poles will be discussed further in chapter 3.

At the beginning of the measurement procedure, the integrating term X_{Rk} has to be initialized to a certain value. Under the assumption that the initial concentration C_0 was kept constant in the past, the initial value X_{R0} is defined by :

First case : Real poles Z_1 and Z_2 (Z_1 is the compensated pole)

$$X_{R0} = \frac{1}{1 - Z_2} \cdot C_0 \quad (14)$$

Second case : Two complex conjugate poles (the real part of the poles is compensated)

$$X_{Ro} = \frac{1}{1 - \text{Re}(Z)} \cdot C_o \quad (15)$$

2.3 Advantages of the algorithm

The proposed control algorithm defined by equation (5) looks like a special case of a traditional PI control algorithm. In fact our algorithm present some advantages over the usual PI control method.

First, a faster response to a step change in the target concentration W_k is obtained by the pole compensation (Fig. 1). This property is very useful for measurement procedures using variable target concentrations in multizone buildings.

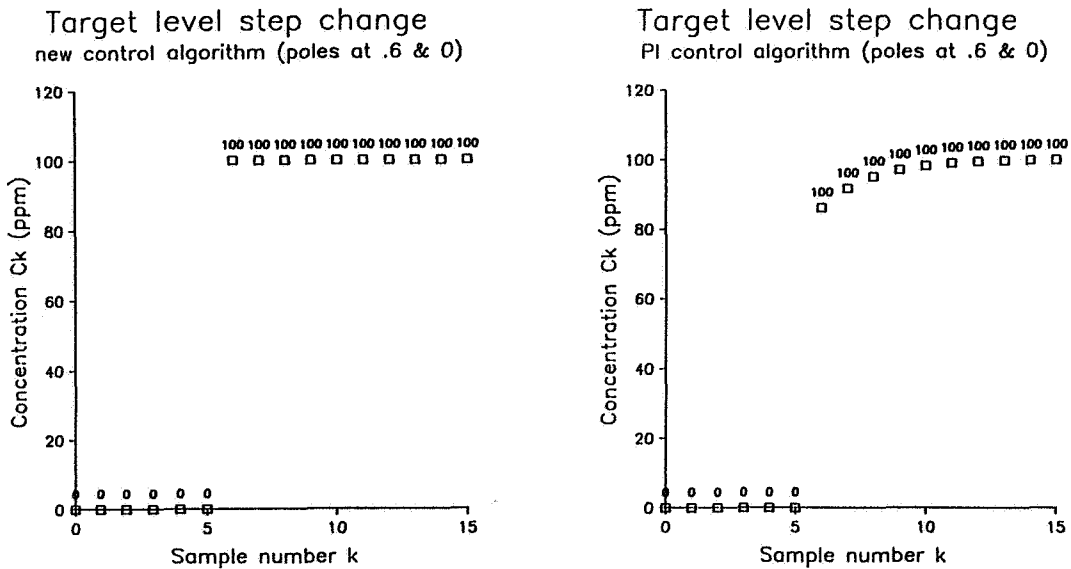


Figure 1 : Response of the control algorithm for a target level step change : the new control algorithm has a faster response than the PI control method. Points are labelled by the actual target level W_k (ppm). (values computed with 1 air change/hour and $T_s = 540$ [s]).

Secondly, the minimization of a quadratic form is well suited to generalize into a multivariable control algorithm. Thus an extension of our algorithm to the case of a multizone measurement system is not difficult. This extension is summarized as follows : equation (1) is rewritten⁶ using matrix notation :

$$V \frac{d}{dt} C = F C + S \quad (16)$$

where	V :	volumes diagonal matrix	$[m^3]$
	C :	concentrations vector	$[-]$
	F :	air flows matrix	$[\frac{m^3}{s}]$
	S :	tracer gas injections vector	$[\frac{m^3}{s}]$

The difference equation (2) becomes :

$$C_{k+1} = A C_k + B U_k \quad (17)$$

where A and B are matrixes derived from the air flows matrix F and the volumes matrix V

$$\text{and } U_k = V^{-1} S_k \quad (18)$$

From equation (17), various methods are available⁵ to compute the parameters matrixes K_s, K_R and K_w necessary for the control algorithm defined (similar to equation (5) by :

$$U_k = - K_s C_k + K_R X_{Rk} + K_w W_k \quad (19)$$

where W_k is the target concentrations vector and X_{Rk} the integrating vector defined by the difference equation :

$$X_{Rk+1} = X_{Rk} + W_k - C_k \quad (20)$$

2.4 Modifications of the algorithm

The tracer gas injection rate the CCTG is able to supply is limited to a certain range : no tracer gas can be removed from the zone and, a maximum injection rate can not be exceeded. Figures 2 and 3 show two cases where these limitations lead to large over- or under-shoots of concentration.

These problems are caused by inappropriate values of the integrating term X_{Rk} . Although the CCTG system cannot supply the necessary injection rate or remove gas to readjust the concentration to target level, the value of X_{Rk} is still incremented as if the system had no limitations. This results in too high or too low values of X_{Rk} , and it takes several steps with over or under concentration to correct the value of this term.

To avoid these problems, it is necessary to correct the value of X_{Rk} when the system is unable to supply the injection rate the control algorithm has asked for. Among the many possibilities to adjust these values, we present here two procedures we have chosen :

First case : $U_k < 0$

Negative values typically occur after a concentration overshoot due to an abrupt decrease of the air infiltration rate into the zone (Fig. 2). As seen above (equation 5), the injection rate is proportional to $W_k - C_k$ and X_{Rk} . During the whole overshoot while $C_k > W_k$, the integrating term decreases. Then, when the concentration goes below the target level, because of the low value of X_{Rk} , the injection rate remains negative and it takes several steps with $C_k < W_k$ to raise the value of X_{Rk} to its new stationary value allowing positive injection rate to be computed.

To avoid this undershooting, the integrating term could be adjusted just before the decreasing concentration crosses the target level. Thus, our procedure works as follows :

Overshoot (1st case) (target level: 100 ppm)
without modification

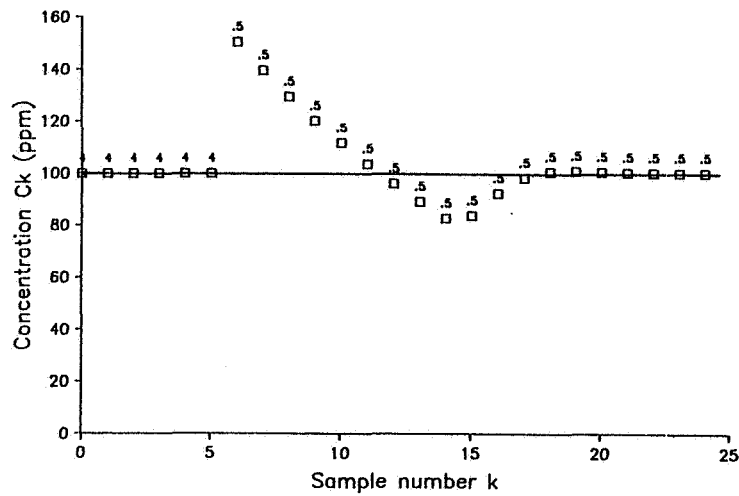


Figure 2 : Concentration overshoot due to an abrupt decrease of the air change rate. Points are labelled by the actual air change rate [h^{-1}] (values computed with $V = 80 \text{ [m}^3\text{]}$ and $T_s = 540 \text{ [s]}$).

Overshoot (2nd case) (target level: 100 ppm)
without modification

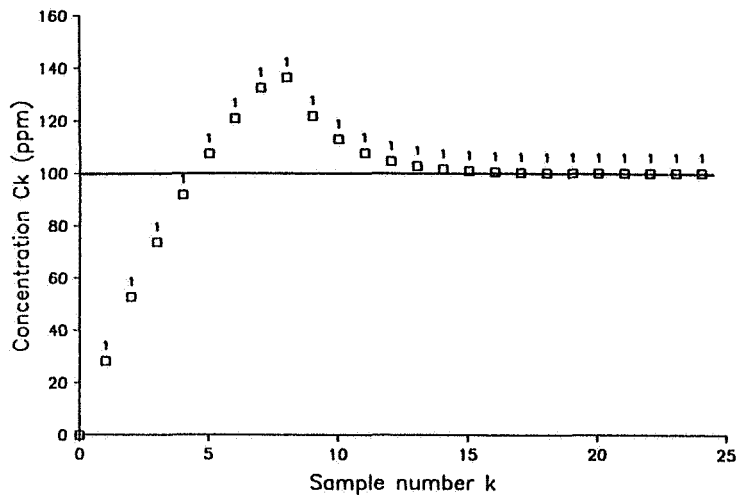


Figure 3 : Concentration overshoot due to the limiting effect of the maximum injection rate U_{max} the CCTG system is able to supply. All points are labelled by the actual air change rate [h^{-1}] (values computed with $V = 200 \text{ [m}^3\text{]}$, $T_s = 540 \text{ [s]}$ and $U_{\text{max}} = 5.67 \cdot 10^{-8} \text{ [s}^{-1}\text{]}$). These values lead to the maximum concentration increase at the first step : $C_1 = b \cdot U_{\text{max}} = 28,4 \text{ (ppm)}$).

Overshoot (1st case) (target level: 100 ppm)
with modification

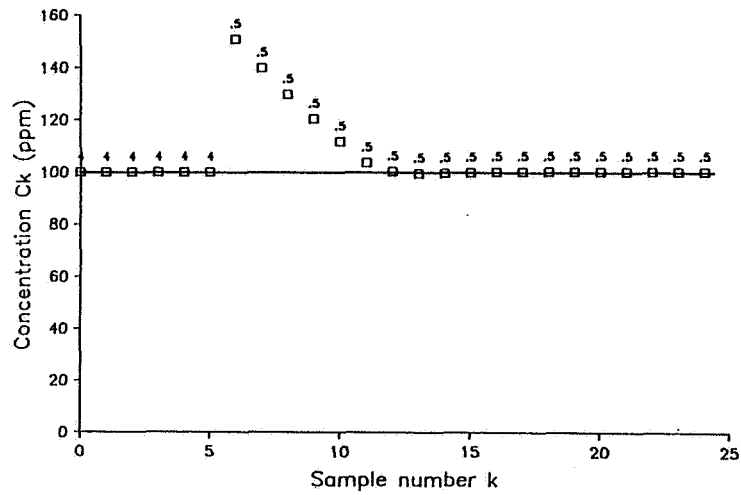


Figure 4 : Effect of the first modification of the control algorithm (compare with fig. 2). All points are labelled by the actual air change rate [h⁻¹]. (values computed with $V = 80 \text{ [m}^3\text{]}$ and $T_s = 540 \text{ [s]}$).

Overshoot (2nd case) (target level: 100 ppm)
with modification

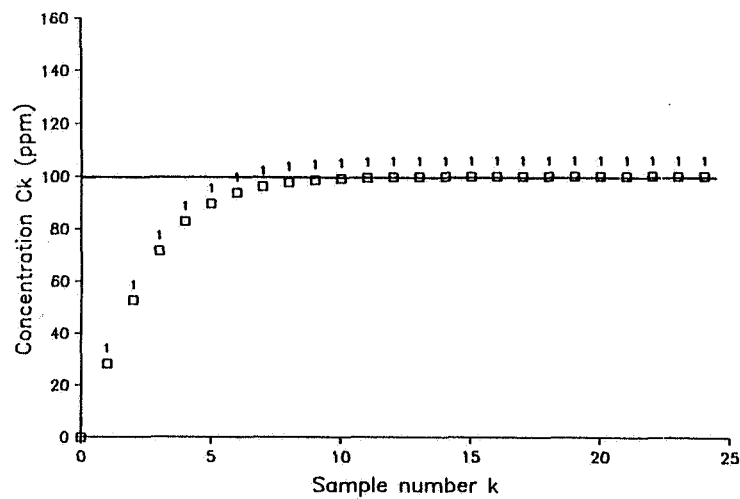


Figure 5 : Effect of the second modification of the control algorithm (compare with fig. 3). All points are labelled by the actual air change rate [h⁻¹]. (values computed with $V = 200 \text{ [m}^3\text{]}$, $T_s = 540 \text{ [s]}$ and $U_{\max} = 5.67 \cdot 10^{-8} \text{ [s}^{-1}\text{]}$).

Whenever the control algorithm computes a negative injection rate, the actual infiltration rate is estimated and the concentration for the next step can be extrapolated. If, without any gas injection, the extrapolated concentration is still above target level, nothing is done. But in the other cases, the integrating term is forced to a value leading, through the use of control equation (5), to the necessary injection rate to reach the target level. The following relations are used for this procedure :

(assuming $U_k < 0$)

estimated actual air change rate :

$$n_{est} = \frac{1}{T_s} \cdot \text{Ln} \left(\frac{C_{k-1} + T_s \cdot \max(0; U_{k-1})}{C_k} \right) \quad \left[\frac{1}{s} \right] \quad (21)$$

(if $U_{k-1} > 0$ n_{est} underestimates the true air change rate)

predicted concentration :

$$C_{p\ k+1} = C_k \exp(-n_{est} \cdot T_s) \quad [-] \quad (22)$$

necessary injection rate (if $C_{p\ k+1} < W_{k+1}$) :

$$U_k^* = \frac{1}{T_s} C_k \left(\frac{W_{k+1}}{C_{p\ k+1}} - 1 \right) \quad \left[\frac{1}{s} \right] \quad (23)$$

new value for the integrating term :

$$X_{Rk}^* = \frac{U_k^* + K_s C_k - K_w W_k}{K_R} \quad [-] \quad (24)$$

Since the actual air change rate is computed over one sampling time only, its value is sensitive to any measurement error. Therefore equation (23) gives in fact an underestimate of the necessary injection rate to avoid large effects from small measurement errors (it is always easier to add some gas at the next step than to remove some !).

A comparison between fig. 2 and fig. 4 shows the effect of this procedure.

Second case $U_k > U_{max}$

This case may occur at the beginning of a measurement on a large zone or when the air infiltration rate is very high. For extreme cases, the target concentration will never be reached.

Figure 3 illustrates the problem : the increase of the concentration is limited by U_{max} while the integrating term increases. When the concentration reaches the target level, the high integrating term value forces the control algorithm to inject too much gas. This leads to a concentration overshoot until the integrating term finds its stationary value again.

To avoid this phenomenon, each time the control algorithm asks for an injection rate higher than U_{\max} , the integrating term is forced to a proper value. This value is calculated so that, when used in the control equation (5), the maximum possible injection rate U_{\max} results.

The new value of the integrating term is then :

(assuming $U_k > U_{\max}$)

$$X_{Rk}^* = \frac{U_{\max} + K_s C_k - K_w W_k}{K_R} \quad [-] \quad (25)$$

As shown in figure 5, this procedure gives good results.

3. ALGORITHM TESTING

3.1 Simulation program

In order to test the algorithm, a simulation program was developed⁷. The program simulates a zone with variable air change rate and takes into account the real characteristics of the CCTG system (e.g. time delays due to the pipes, gaz analyzer response time, random errors on the measured concentration, and maximum injection rate U_{\max}).

The tracer gas mixing process within the zone is also simulated using a simple model : The tracer gas is injected in a small fictious volume which is connected to the zone by a constant air flow rate. The small fictious volume and the zone volume are both assumed perfectly mixed. A good mixing into the zone is achieved when the concentration within the fictious volume is close enough to the concentration within the zone. The mixing time constant is defined by :

$$\tau_{mx} = \frac{v}{f} \quad [s] \quad (26)$$

where

v :	fictious volume	[m ³]
f :	constant air flow between the fictious volume and the zone	[$\frac{m^3}{s}$]

According to Sandberg and Blomqvist³, this time constant typically lies between 30 [s] and 180 [s]. It can be adjusted by giving proper values to v and f. By assuming f equal to the flow rate produced by the small mixing fan used in the zone ($f \cong 0,015 \frac{m^3}{s}$ for the fans we use) the fictious volume lies in the interval between 0,45 and 2,7 [m³].

3.2 Results

Chapter 2 describes the control algorithm but gives no information on the location of the poles and the estimated air change rate. They are both required to compute the three parameters K_S , K_R and K_w . To investigate these topics, we used our program to systematically try all the possible poles locations for different estimated air change rates. All simulations were performed under the operating conditions listed in table 1.

Sampling time	T_s	= 540	[s]	
Volume of the zone	V	= 80	[m ³]	
Initial concentration	C_o	= 0	(ppm)	
Target concentration	W	= 100	(ppm)	
Fictitious volume	v	= 0.9	[m ³]	} $\tau_{mx} = 60$ [s]
Air flow induced by the fan	f	= 0.015	[$\frac{m^3}{s}$]	
Measurement random error	gaussian noise	$\mu = 0$	(ppm)	
		$\sigma = 1$	(ppm)	
Time delays due to the pipes		5.2	[s]	
Analyzer response time		8	[s]	
Maximum injection rate	U_{max}	= $1.42 \cdot 10^{-7}$	[$\frac{1}{s}$]	

Table 1 : Operating conditions used for our simulations.

To remain close to the conditions observed in inhabited buildings, the simulations were performed over 320 samples ($320 \cdot 540$ [s] = 48 hours) with variable air change rates as shown in fig. 6.

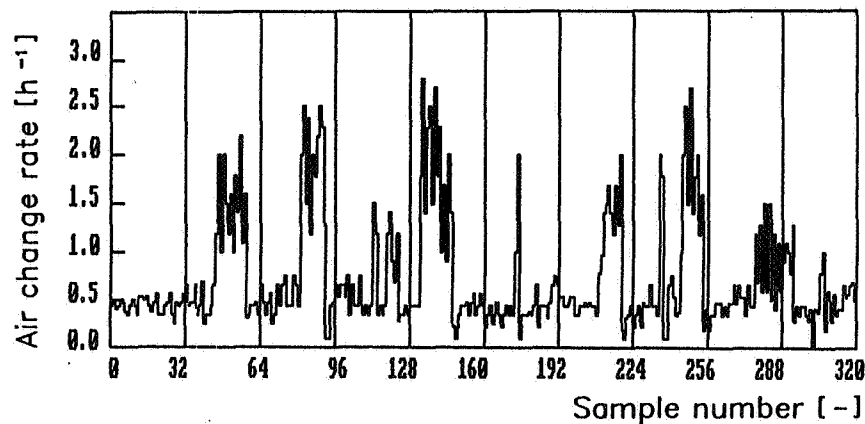


Figure 6 : Variable air change rate used for our simulations.

The simulation results were characterized by three numbers :

- a) the difference between the mean measured concentration and the target level :

$$d = \langle C \rangle - W \quad [-] \quad (27)$$

- b) the concentration standard deviation from the target level :

$$s = \sqrt{\langle (C - W)^2 \rangle} \quad [-] \quad (28)$$

- c) the total negative tracer gas volume the control algorithm wanted to remove from the zone :

$$v_{neg} = V \cdot T_s \cdot \sum_{k=1}^{320} \min(0; U_k) \quad [m^3] \quad (29)$$

($\langle \rangle$ denotes the "mean" operator = $\frac{1}{N} \sum_{k=1}^N$ with N the number of samples)

It is clear that the optimal control algorithm would lead to ($d = 0, S = 0, v_{neg} = 0$) ! The set of simulations can then be investigated to find out the poles location leading to the smallest vector (d, s, v_{neg}). Figures 7 to 12 present different maps showing the values d, s and v_{neg} for all locations of the poles in real and complex planes.

By studying the maps, in the complex plane (fig. 7 to 9) no complete overlapping appears for the best regions for d, s and v_{neg} . Therefore it is not advisable to locate the poles somewhere in the complex plane.

In the real plane (fig. 10 to 12), the three best regions lie closer to each other. We choose the poles location $Z_1 = 0,6$ (compensated pole) and $Z_2 = 0$ as the "best" values to compute the parameters of our algorithm.

It also appeared that the estimated air change rate required to compute the parameters has only a small influence on the simulation results. It is possible to understand this by looking at figure 13 which shows the dependance of the parameters from the estimated air change rate.

Parameter K_w is the most sensitive parameter but it does not have a strong influence because the dynamic properties of the control algorithm are essentially determined by K_s and K_R which are much less sensitive to the estimated air change rate. It is anyhow important to keep in mind that K_w has a strong influence when a target level step change occurs (e.g. at the beginning of the measurement procedure). It is therefore advisable to choose the minimum expected air change rate for the considered zone.

These conclusions still apply in case of bad mixing ($\tau_{mx} = 180$ [s]) and for other sets of air change rates if their values lie between 0 and 4 [h^{-1}]. For larger variations of air change rates, a recalculation of the control parameters should sometimes be undertaken during the measurement. This procedure and the criteria to apply it, will be investigated during the continuation of this work.

Figure 14 shows the concentrations during 12 hours of a real CCTG measurement using our algorithm with poles located at $Z_1 = 0,6$ $Z_2 = 0$, with an estimated air change rate of 0.1 [h^{-1}]. A fast response to changes in air flow rates can be observed.

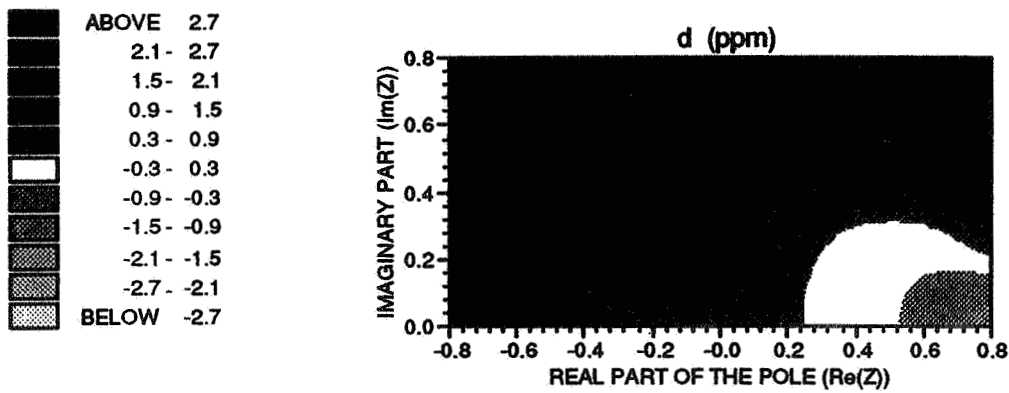


Figure 7 : Difference between the mean measured concentration and the target level in (ppm) for poles located in the complex plane. (Estimated air change rate used for the calculation of the control parameters : 0.5 [h⁻¹]).

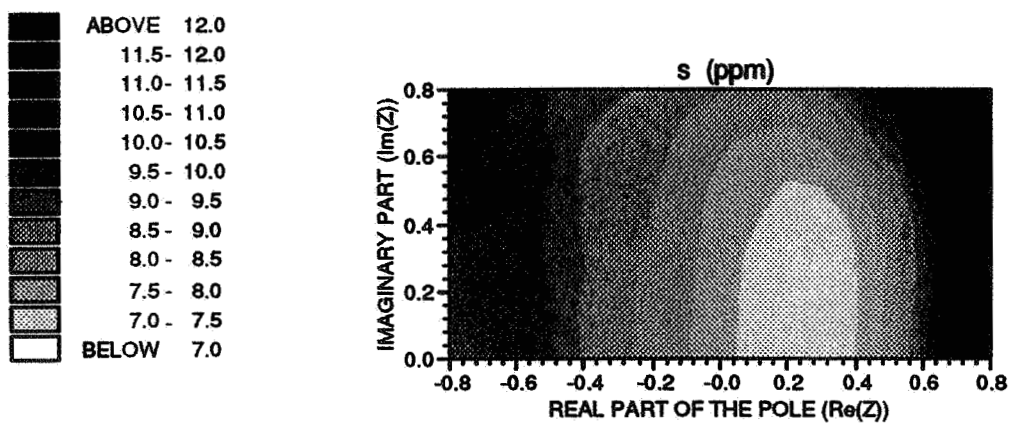


Figure 8 : Concentration standard deviation from the target level in (ppm) for poles located in the complex plane. (Estimated air change rate used for the calculation of the control parameters : 0.5 [h⁻¹]).

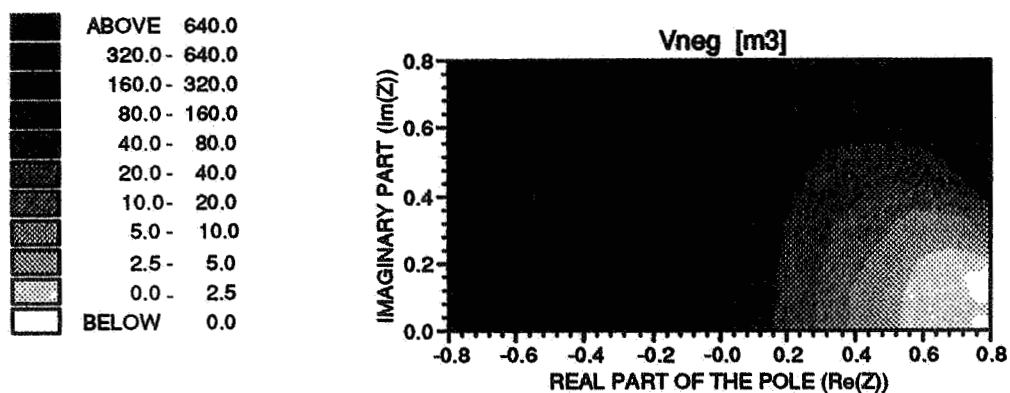


Figure 9 : Total negative tracer gas volume the control algorithm wanted to remove from the zone for poles located in the complex plane. (Estimated air change rate used for the calculation of the control parameters : 0.5 [h⁻¹]).

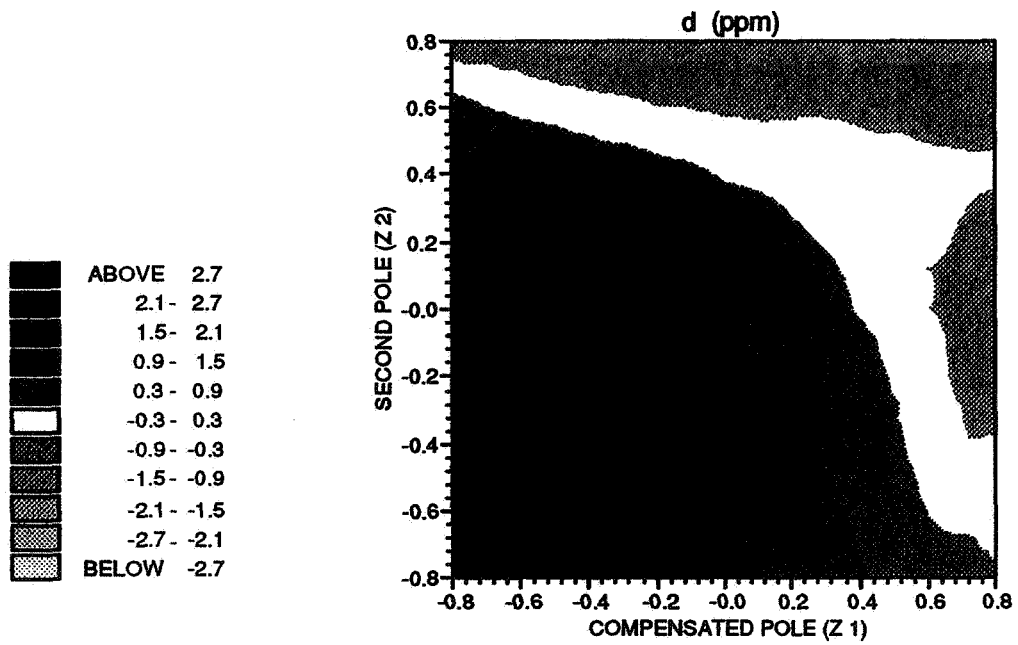


Figure 10 : Difference between the mean measured concentration and the target level in (ppm) for poles located in the real plane. (Estimated air change rate used for the calculation of the control parameters : $0.5 \text{ [h}^{-1}\text{]}$).

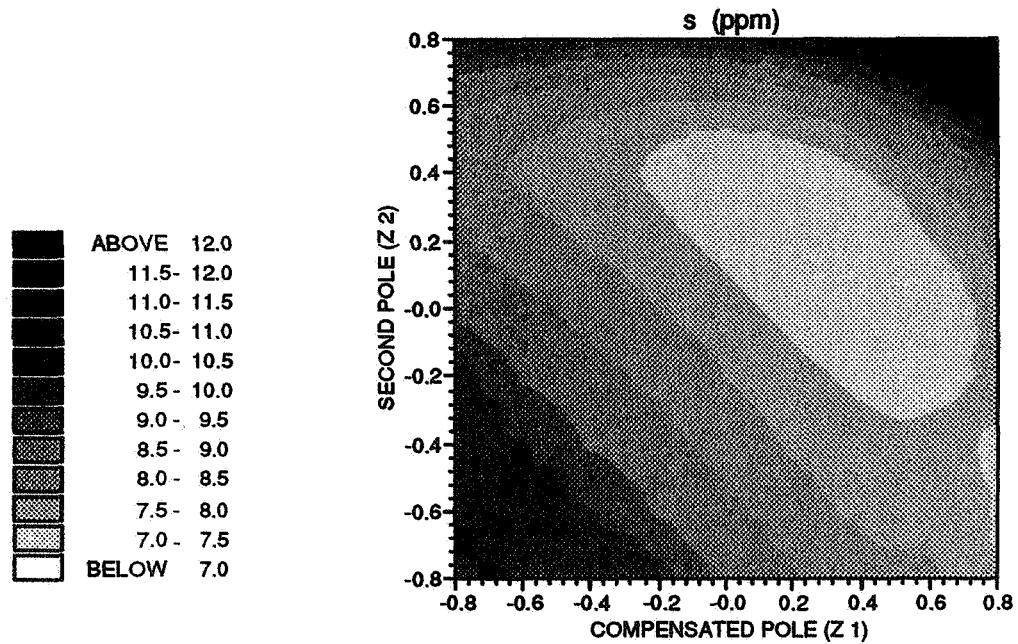


Figure 11 : Concentration standard deviation from the target level in (ppm) for poles located in the real plane. (Estimated air change rate used for the calculation of the control parameters : $0.5 \text{ [h}^{-1}\text{]}$).

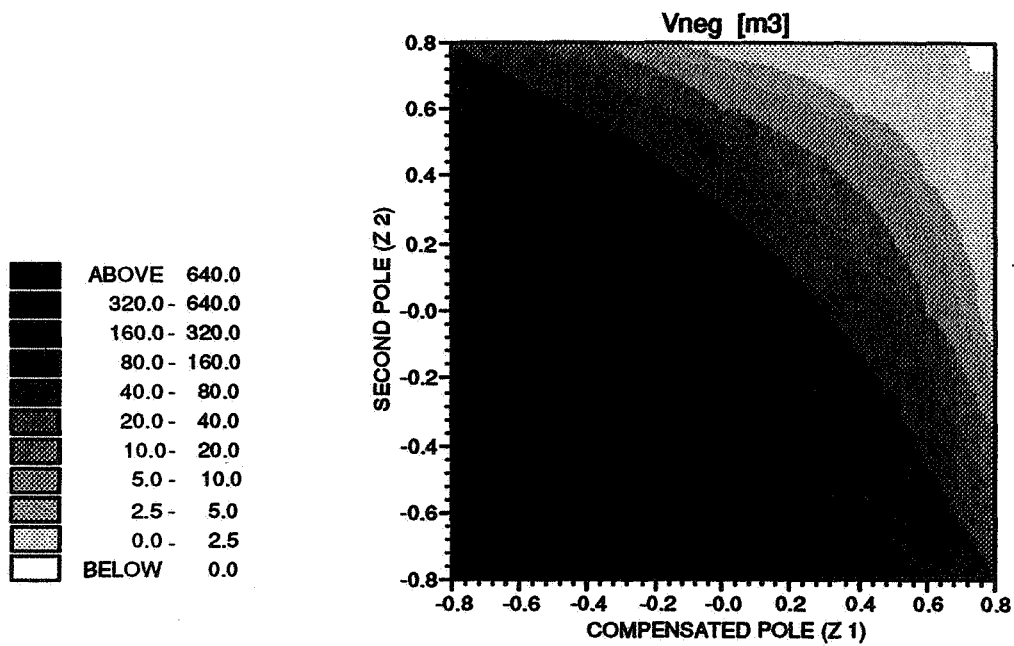


Figure 12 : Total negative tracer gas volume the control algorithm wanted to remove from the zone for poles located in the real plane. (Estimated air change rate used for the calculation of the control parameters : $0.5 \text{ [h}^{-1}\text{]}$).

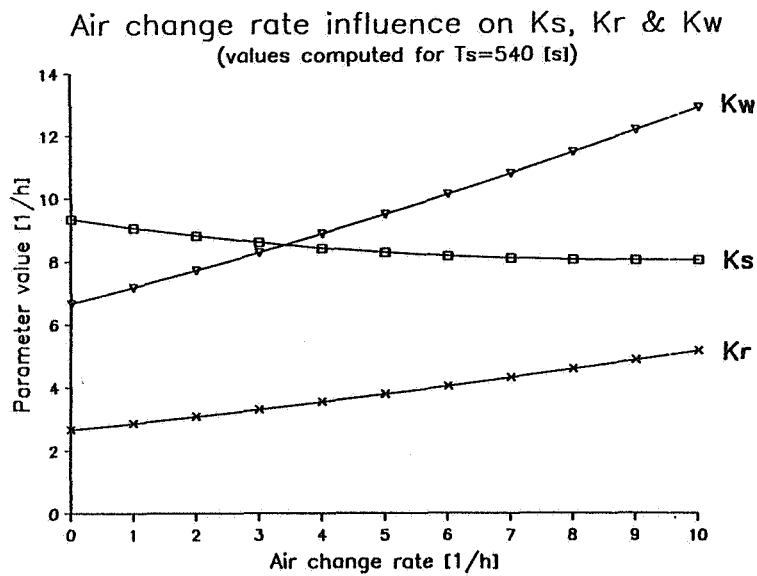


Figure 13 : Values (expressed in $[\text{h}^{-1}]$) of the control parameters for various estimated air change rates (poles located at $Z_1 = 0.6$ and $Z_2 = 0$).

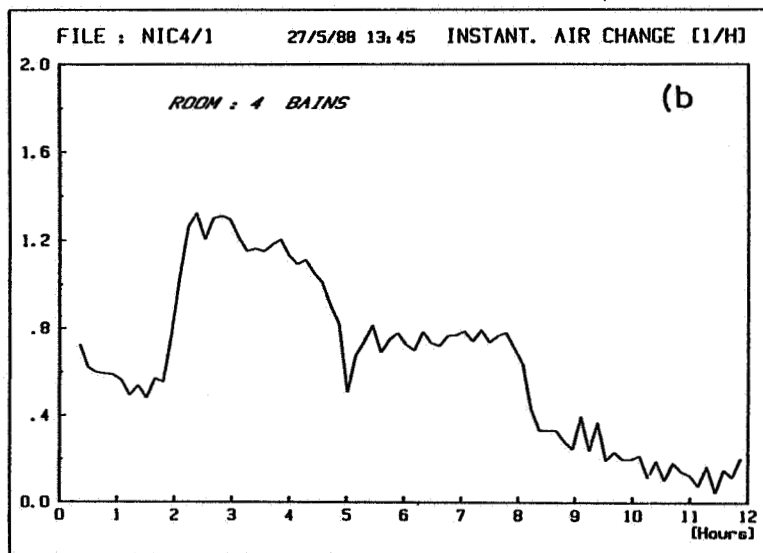
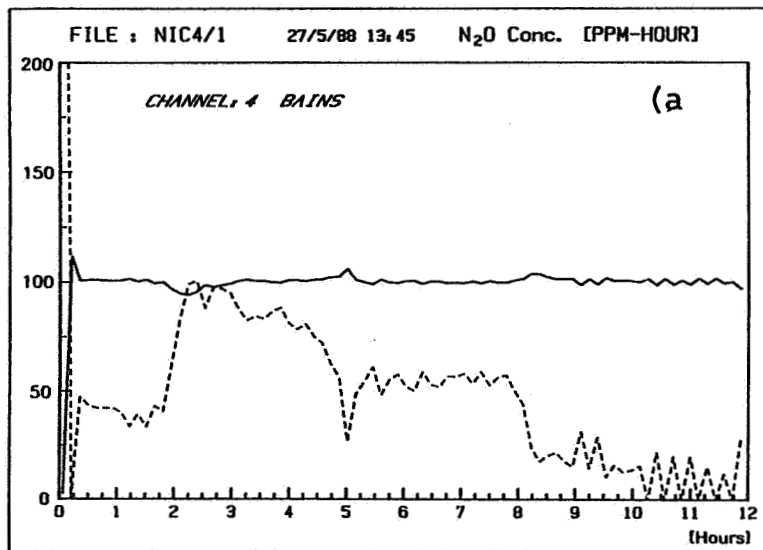


Figure 14 :

- a) Concentrations measured during 12 hours within a bathroom ($V = 14.1 \text{ [m}^3\text{]}$, target level = 100 ppm, $T_s = 525 \text{ [s]}$). Dotted line shows the tracer injection rate (arbitrary units).
- b) Air change rates deduced from the CCTG measurement. Large air change rate variations due to the mechanical ventilation system can be observed.

We also tried to use the multizone control algorithm briefly presented in chapter 2.3. The first trial did not improve significantly the control ability. There are two reasons for this result :

- the control matrixes K_S K_R and K_w are diagonally dominant : the off-diagonal elements lie one order of magnitude under the diagonal elements which are close to the values of the single zone algorithm.
- The CCTG system is only able to analyze and to dose with tracer gas one zone at a time, which implies that the multizone control algorithm is not completely used. A CCTG system able to analyze and dose many zones simultaneously would take advantage of this multizone algorithm.

Although further work should be useful to refine these findings, it appears that the single zone control algorithm can be used for multizone CCTG system : all zones are simply independently controlled.

4. CONCLUSIONS

A complete control algorithm has been developed and the values required to compute its parameters have been systematically investigated. Some improvements of the theoretical algorithm were developed to avoid inappropriate control behaviour due to the physical limitations of the CCTG system.

The presented algorithm is now implemented on our multizone CCTG system CESAR^{4,8}. The control algorithm considers each zone independently.

Further work is necessary to develop an automatic parameters recalculation procedure when large variations in air change rate are encountered. The possibility for the algorithm to be extended to a multivariable control algorithm also needs further investigations. It is probable that this algorithm would be useful for multizone CCTG system able to analyze and dose many zones simultaneously.

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