PAPER 8

THE APPLICATION OF RECIPROCITY IN TIGHTNESS TESTING

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1. BACKGROUND

The problem of tightness in buildings is currently a very topical and important problem area. When people began to try and improve the energy status of houses they perhaps took too much notice of k values. More recently it has been seen that uncontrolled air-leakage is equally important - sometimes more so. One problem with such air-leakage is the difficulty in measuring quantity. Probably the most accessible method is to base the quantitative determinations of air-leakage on values achieved through tightness testing of the building shell and its sub-areas. In the case of small houses there is a method available today for carrying out such tightness testing on the whole house by using a separate test fan. However, for larger houses there is no method. During the last year, large office blocks have been tested in much the same way as small houses by using the existing fans. This method demands a considerable amount of work and is not always possible to carry out.

Invariably it is not possible to measure the tightness of a section of the facade by subjecting a room, or a limited section of the building, to a positive pressure. The large amount of leakage which thus occurs between dividing walls and joist structures is often too great to allow an acceptable result to be achieved.

The idea of applying the theory of reciprocity was developed in the work concerned with tightness testing of large office blocks, by deciding how the leaks are distributed throughout the sub-sections of the outer shell, irrespective of the size of the building.

The first measurement indicated that it was theoretically possible to use the reciprocity theorem in conjunction with refined tightness testing. The thesis was published in a paper "Draughts or ventilation" in Byggmästaren 1977 no. 7-8. To quote an example, the method means that each room is pressurised in turn and the sum of the leakages is then measured thus indicating the true air leakage through the outer shell of the building.

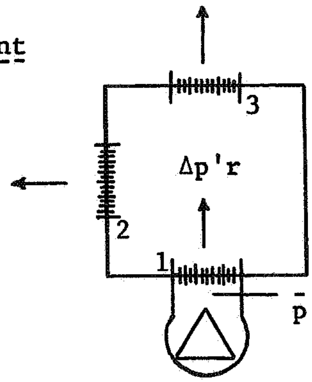
After carrying out theoretical studies, an experimental study was carried out in a laboratory. The experimental work was carried out by Sven Strand and published in a dissertation "Application of a reciprocity theorem in the tightness testing of buildings". Sven Strand Division of Building Technology, Royal Institute of Technology, Stockholm 1979. This report discusses the basic theory and an example of its application.

2. THEORETICAL derivation

The perviousness of a building's outer shell can be expressed in the form of a relationship between the flow through the outer shell and the pressure difference across the shell. In the case of a house, this relationship has the basic form of a function, $q = f(\Delta p)$, i.e. the air flow is a function of the pressure difference and this function can be written as $q = k(\Delta p)^{\beta}$, where k is a constant, Δp is the pressure difference and β is an exponential constant. For pure laminar air flow, $\beta = 1.0$ and in the case of pure turbulent flow, $\beta = 0.5$. The theoretical basis for the method is given below.

<u>Laminar air flow, $\beta = 1.0$</u>

<u>First measurement</u>



Assume that the room in the figure is completely tight apart from three (1, 2 and 3) leakages. Leakage 1 is subjected pressure $\bar{p}$. The pressure across 1 is then $\bar{p} - \Delta p'_r$, where $\Delta p'_r$ is the pressure which occurs in the room. $\Delta p'_r$ is also the pressure drop across leakages 2 and 3. The flows through the respective leakages can be written as follows:

$$q_1' = k_1(\bar{p} - \Delta p'_r)$$

$$q_2' = k_2 \Delta p'_r$$

$$q_3' = k_3 \Delta p'_r$$

The flow in through 1 is equal to the total flow out through 2 and 3.

$$q_1' = q_2' + q_3' \Leftrightarrow k_1(\bar{p} - \Delta p_r') = k_2 \Delta p_r' + k_3 \Delta p_r'$$

The equilibrium pressure in the room is then

$$\Delta p_r' = \frac{k_1 \bar{p}}{k_1 + k_2 + k_3}$$

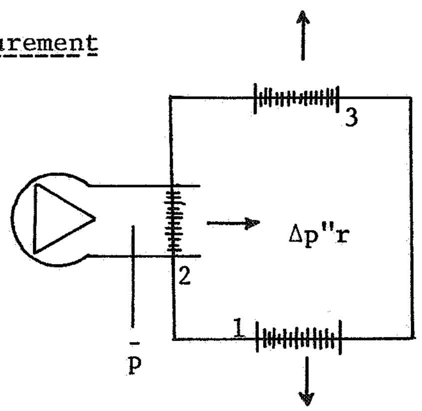
and the flows

$$q_1' = k_1(\bar{p} - \frac{k_1 \bar{p}}{k_1+k_2+k_3}) = k_1 \bar{p}(1 - \frac{k_1}{k_1+k_2+k_3})$$

$$q_2' = \frac{k_2 k_1 \bar{p}}{k_1 + k_2 + k_3}$$

$$q_3' = \frac{k_3 k_1 \bar{p}}{k_1 + k_2 + k_3}$$

Second measurement



Leakage 2 is now subjected to $\bar{p}$ and using the analogy of the above:

$$k_2(\bar{p} - \Delta p_r'') =$$

$$= k_1 \Delta p_r'' + k_3 \Lambda p_r''$$

The equilibrium pressure in the room is then

$$\Delta p_r'' = \frac{k_2 \bar{p}}{k_1 + k_2 + k_3}$$

and the flows

$$q_1'' = \frac{k_1\, k_2\, \bar{p}}{k_1 + k_2 + k_3}$$

$$q_2'' = k_2\, \bar{p}(1 - \frac{k_2}{k_1 + k_2 + k_3}$$

$$q_3'' = \frac{k_3\, k_2\, \bar{p}}{k_1 + k_2 + k_3}$$

It can thus be seen that $\underline{q_2' = q_1'' = \dfrac{k_1\, k_2\, \bar{p}}{k_1 + k_2 + k_3}}$

The flow out through leakage 2 in the first test, as a result of the external load pressure $\bar{p}$, is therefore equal to the flow out through the leakage 1 in the second test for the same $\bar{p}$. This means that the flow through 3 in the first test is the difference between the measured flows through 1 in the first test and second test.

$$\therefore\ \underline{q_3'} = q_1' - q_2' = \underline{q_1' - q_1''}$$

<u>Turbulent air flow, $\beta = 0.5$</u>

In the same way as before we get the following:

<u>First measurement</u>

$$q_1' = k_1(\bar{p} - \Delta p_r')^{0.5}$$

$$q_2' = k_2(\Delta p_r')^{0.5}$$

$$q_3' = k_3(\Delta p_r')^{0.5}$$

$$q_1' = q_2' + q_3' \Leftrightarrow k_1(\bar{p} - \Delta p_r')^{0.5} = (k_2 + k_3)\ (\Delta p_r')^{0.5}$$

$$\Delta p_r' = \frac{k_1^2\ \bar{p}}{k_1^2 + k_2^2 + k_3^2 + 2k_2k_3}$$

$$q_1' = k_1 \sqrt{\bar{p} - \frac{k_1^2\ \bar{p}}{k_1^2 + k_2^2 + K_3^2 + 2k_2k_3}}$$

$$q_2' = k_2k_1 \sqrt{\frac{\bar{p}}{k_3^2 + k_2^2 + k_3^2 + 2k_2k_3}}$$

$$q_3' = k_3k_1 \sqrt{\frac{\bar{p}}{k_1^2 + k_2^2 + k_3^2 + 2k_2k_3}}$$

<u>Second measurement</u>

$$k_2(\bar{p} - \Delta p_r')^{0.5} = (k_1 + k_3)\ (\Delta p_r'')^{0.5}$$

$$\Delta p_r'' = \frac{k_2^2\ \bar{p}}{k_1^2 + k_2^2 + k_3^2 + 2k_1k^3}$$

$$q_1'' = k_1k_2 \sqrt{\frac{\bar{p}}{k_1^2 + k_2^2 + k_3^2 + 2k_1k_3}}$$

$$q_2'' = k_2 \sqrt{\bar{p} - \frac{k_2^2\ \bar{p}}{k_1^2 + k_2^2 + k_3^2 + 2k_1k_3}}$$

$$q_3'' = k_3k_2 \sqrt{\frac{\bar{p}}{k_1^2 + k_2^2 + k_3^2 + 2k_1k_3}}$$

For comparison's sake we can give the expressions for q_2' and q_1'' i.e.

$$q_2' = k_2 k_1 \sqrt{\frac{\bar{p}}{k_1^2 + k_2^2 + k_3^2 + \underline{2k_2k_3}}}$$

$$q_1'' = k_1 k_2 \sqrt{\frac{\bar{p}}{k_1^2 + k_2^2 + k_3^2 + \underline{2k_1k_3}}}$$

If $k_1 = k_2$, we get $q_2' = q_1''$

If $k_1 \neq k_2$ and if the difference is not too great we get

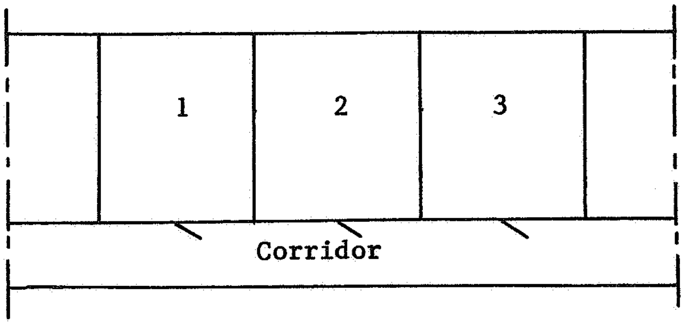
$$q_2' \approx q_1''$$

and therefore

$$\underline{q_3' \approx q_1' - q_1''}$$

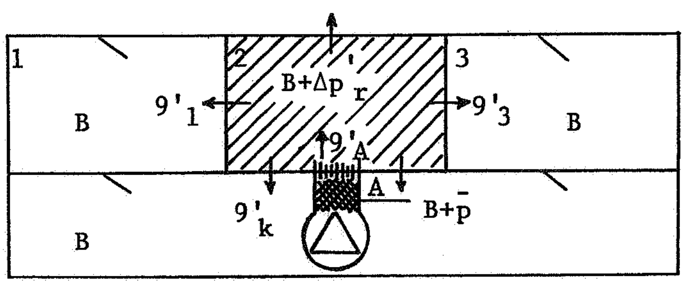
3. AN APPLICATION EXAMPLE

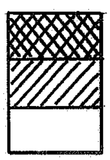
On the basis of the previous derivations, the reciprocity theorem's application in practice can be illustrated in the following manner.



In this situation we assume a row of offices with offices 1, 2, 3, etc. and a corridor. For simplicity's sake we shall limit the example to one floor. The tightness of the external walls is to be calculated, i.e. the relationship between flow - pressure difference. The following procedure describes the method

1





primary pressure load, $B + \bar{p}$

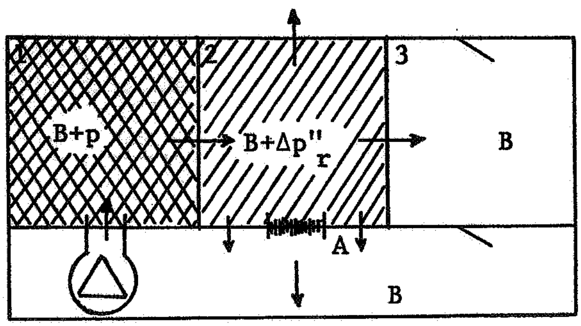
secondary pressure increase, $B + \Delta p_r$

barometric pressure, B

A is a calibrated flowmeter. This is subjected to pressure $\bar{p}$, wherein the pressure in room 2 assumes the value of $\Delta p'_r$. Flow q'_A through meter A is registered. There are three leakages, one to room 1, one to room 3 and one to the corridor. The flow through the shell to be calculated is

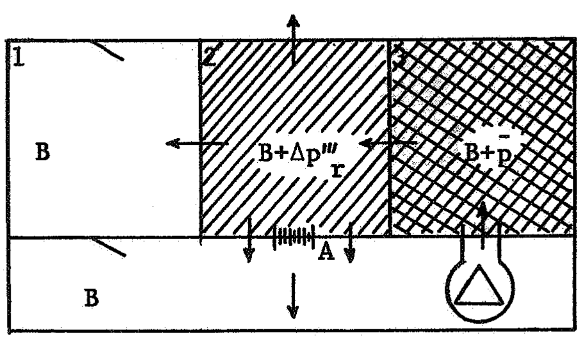
$$q' = q_A' - q_1' - q_3' - q_k' \tag{1}$$

2.



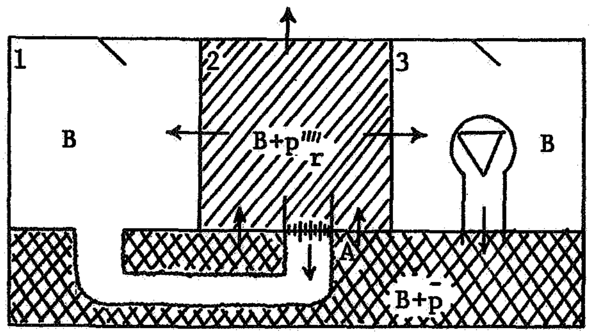
If we subject room 1 to $\bar{p}$ and measure p_r'' we get from the calibration curve for the flowmeter A a value of q_A'' which, according to the derivations, is approximately q_1'.

3.



In the same way as above, room 3 is subjected to positive pressure $\bar{p}$, $\Delta p_r'''$ is measured and we get $q_A''' \approx q_3'$

4.



In order to determine the final leakage, the corridor is subjected to $\bar{p}$, $\Delta p_r''''$ is measured and we get $q_A'''' \approx q_k'$. In this case A has been connected to a tube which, for the purposes of the test, has a negligible flow resistance, and which is connected to room 1. From this we get the pressure drop across A as $\Delta p_r''''$

5. Equation (1) now gives the load to be calculated

$$q' = q_a' - q_A'' - q_A''' - q_A''''$$

When tightness testing large houses it is best to use the existing ventilation fans and subject the whole building to positive or negative pressure. The pressure difference across the external walls and the flow in the main ducts is registered.

When it is not possible to carry out such measurement - the ventilation system may be a self-exhausting system for example - it is possible to use the method described and work through room by room in order to get a measurement of the building's total tightness. Of course, the method is equally applicable for several leakages, i.e. on several floors. The method above is applied in order to correct for these leakages. It is best to regard the method as a means of carrying out random tightness checks of localised external wall sections, preferably in combination with a total test using the "existing fan system".