VENTILATION EFFICIENCY

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INTRODUCTION

There is a tendency in Scandinavia and many other countries in the world, also USA, to reduce ventilation in order to save energy and lower the cost for HVAC. It looks like there are conflicting interests between hygiene and comfort on the one hand, and energy economy on the other hand. This situation has made us look at ventilation in terms of efficiency. We believe that ventilation can be done more efficient, indicating that decreased ventilation does not necissarily mean decreased air quality. In fact, if one looks at the zone of occupancy, it is possible to increase ventilation air change rate in this zone without increasing the air change rate for the room in question.

The research project reviewed by this paper has the objective of establishing: i. Expressions for ventilation efficiency.

ii. Methods for measuring ventilation efficiency.

iii. Rules for achieving efficient ventilation.

RESULTS

Expressions for Ventilation Efficiency.

The objective of the ventilation precess is to maintain a certain air quality in the zone of occupation, i.e. to control contaminationllevels according to set standards. Heat and cold can in this respect also be treated as contaminations. Contamintaion levels are expressed as concentrations. Concentrations of chemical substances might be ppm or mg/m². Concentrations of heat, etc. are kJ/m^3 (pCpT). Almost all ventilation processes are gouverned by turbulent diffusion of contaminations, i.e. contaminations are spread by the air currents in the room. The turbulent diffusion constants are roughly the same for both heat and respirable airborne chemicals.

The conservation equation for steady state is

$$(C_{E} - C_{S})V = Q$$

(1)

 \hat{Q} = source production rate \hat{V} = ventilation air flow rate C_{E} = steady-state concentration in the exhaust air C_{S} = steady-state concentration in the supply air

The source production rate is the net rate. All local exhaust are subtracted. For heating/cooling Q is the net heating/cooling load handled by the general ventilation system. It is now convenient to separate the zone of occupancy, or the working zone, from the rest of the room. Accordingly equation 1 is transformed introducing C_W , the concentration in the working zone.

$$[(C_{E} - C_{W}) + (C_{W} - C_{S})]y = Q$$
(2)

The part $(C_E - C_W)$ might now be regarded either as an additional local exhaust, or as an additional contamination source, depending on the sign. Required ventilation air flow rate is

$$V = Q / [(C_E - C_W) + (C_W - C_S)]$$
(3)

A convenient referance process is complete mixing. C_E is then equal to C_W . Required ventilation air flow rate for complete mixing is

$$\mathbf{y}_{\rm cm} = \mathbf{Q}/(\mathbf{C}_{\rm W} - \mathbf{C}_{\rm S}) \tag{4}$$

Q, C and C are equal and it is obvious that V /V could be called a steady state ventilation efficiency. The notation ε_{II}^{cm} is used for this efficiency.

$$\varepsilon_{II} = v_{cm} / v = [(C_E - C_W) + (C_W - C_S)] / (C_W - C_S) = 1 + (C_E - C_W) / (C_W - C_S)$$
(5)

It is easy to see that this efficiency (depending on the system lay out) might become greater than 1. A high wall/ceiling supply and exhaust ventilation scheme, run as a warm air heating system, develops thermal stratification. Then $C_{\rm E} > C_{\rm W}$ and $C_{\rm S} > C_{\rm W}$. It is easy to see that $\varepsilon_{\rm II}$ then is less than 1, and accordingly the ventilation system is low-efficient. It is, however, not obvious from the efficiency-expression to conclude the system inefficient to every type of chemical contamination. It depends on the location of the source of contamination and the density of the contamination is similar to the "thermal contamination", efficiency is also equal. It is obvious that steady state efficiency is strongly dependent on the characteristics of the source and the ventilation process, but in general, systems with positive $(C_{\rm E}-C_{\rm W})/(C_{\rm W}-C_{\rm S})$ are favourable and should be looked for.

In real situations transient state is as much important as steady state, and for this reason there is a need for defining a transient ventilation efficiency as well. Our experience from practice is that one often gets thermal stratification which behave very much like a two-box model with complete mixing within each box. A two-box model is for this reason used to arrive at expressions for transient ventilation efficiency [2]. Other have also used this approach (Malmström [1], Sandberg [4]).

The exchange of air between the boxes is determined by the temperature conditions and the location of the air outlets. If contamination is produced in a room with two or more zones of air currents, there is a reason to believe that the concentration will be unequally distributed between the zones. This is due to the unequal supply of fresh air in the zones.

If a room is ventilated by jets, air will be entrained into the jets. After the jets have been dissolved, the air is trasnported back again and into the jets. In this way a circulating zone is established.

Iwo main factors determine whether the whole room is directly influenced by the air current from the jet:

- a) The size of the room.
- b) The temperature of the supply air.

A high wall or ceiling supply and exhaust ventilation scheme as shown in fig. 1, may establish circulation zones as shown in the figure.



Fig. 1. Air currents in a confined room. Supply of warm air. The mixing between the zones is indicated by β.

Supposing the room is filled with a tracer gas. After complete mixing, the decay of the tracer gas is studied.

The differential equations for the decay will be:

$$C_{e}\beta \cdot y - C_{w}\beta y = (1-\kappa) \cdot y \cdot \frac{dC_{w}}{dt} \longrightarrow \frac{dC_{2}}{dt} = a_{21}C_{1}(t) + a_{22}C_{2}(t)$$
(1)

$$C_{w}\beta \cdot V - (1+\beta)C_{e} \cdot V = \kappa \cdot V \frac{dC_{e}}{dt} \rightarrow \frac{dC_{1}}{dt} = a_{11}C_{1}t + a_{12}C_{2}(t)$$
(2)

The differential equations (1) and (2) have a rather simple analytical solution. The general differential equations have a form

$$\frac{dy_{1}}{dt} = a_{11}y_{1} + a_{12}y_{2} \qquad y_{1} = C_{e}(t) \qquad a_{11} = -\frac{1+\beta}{\kappa}n, a_{12} = \frac{\beta}{\kappa}n$$
$$\frac{dy_{2}}{dt} = a_{21}y_{1} + a_{22}y_{2} \qquad y_{2} = C_{w}(t) \qquad a_{21} = \frac{\beta}{1-\kappa}n a_{22} = -\frac{\beta}{1-\kappa}n$$

These equations have a solution if letting

$$y_{1} = k_{1}e^{\lambda t}$$
$$y_{2} = k_{2}e^{\lambda t}$$
$$n = nominel air change rate = \frac{y}{y}$$

The analytical solution gives

$$\lambda = \frac{a_{11} + a_{22} \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11} \cdot a_{22} - a_{12} \cdot a_{22})}}{2}$$

This in turn yields 2 values for each K

$$\frac{K_2}{K_1} = -\frac{a_{11} + a_{21} - \lambda}{a_{12} + a_{22} - \lambda}$$

However, K can be put equal to 1, and a final solution is, if the starting concentrations $C_{e}(t) = C_{w}(t) = C(0)$ for t = 0.

$$C_{e}(t) = C(0) \frac{1 - \frac{2}{k_{2}}}{\frac{1}{k_{2}} - \frac{2}{k_{2}}} e^{\lambda_{1} \cdot t} [1 - \frac{1 - \frac{1}{k_{2}}}{1 - \frac{2}{k_{2}}} e^{(\lambda_{2} - \lambda_{1}) \cdot t}]$$
(3)

$$C_{w}(t) = C(0) \frac{1 - {}^{2}k_{2}}{{}^{1}k_{2} - {}^{2}k_{2}} \cdot {}^{1}k_{2} \cdot e^{\lambda_{1} \cdot t} [1 - \frac{1 - {}^{1}k_{2}}{1 - {}^{2}k_{2}} \cdot \frac{{}^{2}k_{2}}{{}^{1}k_{2}} e^{(\lambda_{2} - \lambda_{1}) \cdot t}]$$
(4)

where ${}^{1}k_{2}$, ${}^{2}k_{2}$ and $\lambda_{\beta}\lambda_{2}$ are functions of κ , n and β . With complete mixing, eq. (3) and eq. (4) simply will reduce to

$$C_{\rm cm}(t) = C(0) e^{-nt}$$
(5)

 $[\lambda_{1}] > [\lambda_{1}]$, indicating that after a certain time, which usual has the order of magnitude close to the nominal time constant for the room, the concentration decay has the same rate for each box. The concentrations however, are different for each box, normally lowest for the "supply box" (depending on the thermal characteristics of the supply air, the nominal "supply box" is not always the real supply box). The concentration difference is strongly dependent on β and so is the λ 's and the K's. In a lin.-log. plot (lin. time, log. conc.) of the concentration decay, λ is the slope of decay curve, indicating how fast contaminations are diluted, fig. 2.



Fig. 2. Decay of tracer gas in a room with two zones. Single logarithmic diagram.

The dilution rate is the same for the whole room except for a constant ratio between the concentration in each box. It is obvious that the box with the lowest concentration in total has the best ventilation. Nevertheless, λ_1 is a measure for the overall performance of the ventilation system and should be compared with n, the nominal air exchange rate for the room. The ratio $-\lambda_1/n$ is adopted as a transient ventilation efficiency, ε_1 .

$$\varepsilon_{T} = -\lambda_{T}/n \tag{8}$$

Malmström [1] has shown that ε_1 is equal to the ratio between the concentration in the exhaust air, C, and the mean concentration for the room in question, C, during transient conditions, which is the same as the definition of ventilation efficiency used by Rydberg (1947), [3].

The efficiency could also be defined as the ratio between the concentration in the exhaust air and the working zone, similar to steady state.

$$\varepsilon_{IV} = \frac{C_{e}(t)}{C_{w}(t)} = 1/{}^{1}k_{2} \left(\frac{1 - \frac{1 - k_{2}}{1 - 2k_{2}} e^{(\lambda_{2} - \lambda_{1}) \cdot t}}{1 - \frac{1 - k_{2}}{1 - 2k_{2}} \cdot \frac{2k_{2}}{k_{2}} \cdot e^{(\lambda_{2} - \lambda_{1}) \cdot t}} \right)$$
(9)

After the period of stabilization, eq. (9) can be written as

$$\varepsilon_{IV} = 1/k_2$$
 (10)

Since this ratio does not relate to any reference condition, only is simply an expression for the concentration distribution in the room, it is not a useful tool to describe ventilation efficiency. ε_{IV} and ε_{I} are, however, closely connected.

Now, if the exhaust opening is moved to box 2-low, fig. 3, a,"diagonal" ventilation scheme is produced.



Fig. 3. Flow patterns for a two-box model, supply and exhaust in different boxes.

The same scheme is produced also if supply opening is moved to box 2-low istead of the exhaust. This process is governed by the differential equations (1) and (2).

Fig. 4 shows the calculated efficiency ε_{1} and ε_{1} as a function of β for different ventilation and contamination source schemes. Note that the diagonal ventilation scheme, supply and exhaust in different boxes, is the most efficient. And furthermore: Complete mixing is the worst situation for the diagonal scheme, while best for the "conventional scheme". This indicates that complete mixing is not what one always should aim at, and that it should



Fig. 4. Different efficiencies as a function of β. "Diagonal scheme" in the upper part, "short circuiting scheme" in the lower part.

be possible to improve ventilation efficiency by simply changing to diagonal solutions. Best results are achieved if one supports the natural tendency for thermal stratification and use diagonal bottom fed ventilation schemes. This is what we successfully have done in industrial plants in Norway the last few years. It is of course important here to avoid supplying air with higher temperature than the air in the working zone. Other conclusions from fig. 4 is firstly that $\varepsilon_{\rm T}$ does not depend on the source, contrary to $\varepsilon_{\rm TT}$, and secondly: It is no prerequisit to do tests with uniform initial concentrations in the room.

Laboratory Tests

Experimental technique

Tests have been carried out in a room as shown in Fig. 5.

Four different locations of the air exhaust were used. The air supply was a slot with a width of 20 mm mounted just beneath the ceiling. To simulate a contaminated convection current, the tracer gas was supplied near a heat source of 60 W as shown in the figure.



Fig. 5. The model which was used for the laboratory investigations.

A number of probes for tracer gas measurements were placed on a column (rig) which could be placed at different positions in the room. The probes were placed in 5 different levels, 0.12, 0.57, 1.07, 1.60 and 2.0 m above the floor. One probe was placed in the exhaust duct.

The column was placed at six different positions, of which it is only referred to three in this paper. At each positions a number of samplings were taken at the different levels.

The sampling, analyzing of measurements and moving of the rig were done automatic, based on a microprocessor.

The studies started with a "clean" room. The tracer gas supply and the sampling were started at the same time. The gas concentration increased until the steady-state was obtained. In the steady-state situation the column was moved to the five other positions. Then it returned to the starting position, and the tracer gas supply was turned off before starting the transient measurements.

N₂O was used as a tracer gas. The measurements were done with an infrared gas analyzer type URAS 1. Multipoint measurements with one analyzer was conducted by means of an auxilliary pumping system as shown in fig. 6. Test air for testing was continously sucked through the 6 tubes from the measuring points. The solenoid valves were located close to the analyzer and the pump and the analyzer sucked the gas through each tubes in sequence. The valve switching was controlled by the microprocessor. After each switching there was a certain time log due to a short distance between the solenoid manifold and the analyzer.



To analyser

Fig. 6. Tubing and value arrangements for auxiliary suction from measuring points.

Test results

Fig. 6 shows a typical test record. The ventilation scheme for this record is high wall supply and exhaust ("short circuiting scheme"). The concentrations are increasing gradually from zero up to steady state level. The measured concentrations are fluctuating very much, except for the exhaust air measurements. This is due to the source characteristics which together with the ventilations system creates much turbulence.



Fig. 7. Tracer gas test record. "Short-circuiting scheme". $\Delta T = +9^{\circ}C$, n = 6 airchanges per hour.

The fluctuations continue for the same reason through the steady state period. In the decay period (tracer gas supply is shut off), the concentrations decrease gradually to zero. During this period the measurements are not fluctuating. The reason for this is that the concentrations are distributed more even when the source is shut off. The curve fitting regression coeffisients for all measurements are satisfactory in spite of the fluctuations. It should be mentioned that the heat source is not switched off when the tracer gas supply is cut.

In fig. 8 a lin.-log plot of the concentrations is made for the decay period.



Six main features in these measurements should be pointed on:

1. Concentration levels are different for different levels in the room.

2. The log.-lin.plot of the decay curves in concentration - time diagram tends to become parallell straight lines.

3. The difference in concentration between various locations for steady state and transient state are not equal.

4. There are great fluctuations in concentration levels when the concentration source is present.

Fig. 8. Log.-lin. plot of decay period in fig. 7.

5. Transient state decay does not depend on the source, steady state does.

6. Efficiences are lower than unity.

These findings correspond reasonably well with the two-box model.

Measurements of Malmström [5] show that, during the building up transient period the slope of the lin.-log curves, taking the difference between steady state concentration levels and the measured concentrations for each point, are equal to the slope during the decay period. This means that $\varepsilon_{\rm I}$ might be calculated from the first period. We have not done that because of the great fluctuations. Our calculations of $\varepsilon_{\rm r}$ are based on the decay period.

A still more pronounced difference between transient and steady state is shown in fig. 9.



Fig. 9. Tracer gas test record. "Diagonal scheme". $\Delta T = +9^{\circ}C$, n = 6 airchanges per hour.

This is a test record from a test with a "diagonal scheme" as shown in the figure. During transient decay period there is a very little difference in concentration levels between the zone of occupation and the exhaust air. The slope of the decay curve is much steeper than for the test shown in fig. 7. The nominal air change rate is the same for those two tests. This corresponds again very well with the theoretical two-box model predictions.

Some important tests are tabulated in table 1. Tests shown are carried out with supply air temperature above exhaust air temperature. However, conclusions might be drawn from these tests with respect to what will happen when supply air temperature is lower than the room temperature.

"Diagonal scheme"

Calculation of $\boldsymbol{\varepsilon}_{T}$ and $\boldsymbol{\varepsilon}_{TT}$ are shown in fig. 10.

 $\varepsilon_{\rm TI}$ is the arithmetic mean value from measurements in position 1, 2 and 3. This is done because of the large local variations caused by the source. We believe that the mean values give a reasonable representation of the overall conditions. The local variations can be read from table 1.

TEST	TES	ST CONDITIONS	AIRI	NLET/O	UTLET 1	POSITIONS	S EFFICIENCY STEADY STATE					EFFICIENCY		
	AIR EXCH. RATE	SUPPLY TEMP./TEMP. ABOVE EXHAUST TEMP.	b	a	C C	a	ε _{IIwI}	€ _{IIwII}	ε _{IIwIII}	£1121	ε ^{II5II}	ε _{II2III}	MEAN VALUE E _{II}	$\frac{\varepsilon_1 = -\frac{\lambda_1}{n}}{\varepsilon_1}$
1	6	42/10	x				0,38	0,33	0,32	0,42	0,39	0,37	0,37	0,33
2	6,81	40,6/9		x			0,68	1,03	3,20	0,88	0,71	1,54	1,34	1,66
3	6	29,5/ 3	x				0,91	1,06	1,16	1,07	1,26	1,36	1,14	0,53
4	6,44	·23,9/ 3		x			0,86	1,11	1,08	0,90	1,06	1,12	1,02	1,16
5	3,96	29,4/9		x			0,29	0,63	1,66	0,51	0,64	1,22	0,83	1,29
6	4,01	36,0/9	x				0,26	0,28	0,30	0,25	0,28	0,30	0,29	0,37
7	4,02	23,9/ 3		x			1,00	1,09	1,15	1,02	1,02	1,02	1,05	1,31
8	4,44	24,4/3	x	•			0,83	0,95	0,97	0,81	0,94	0,95	0,91	0,47
9	4,08	38,4/15		x			0,54	1,32	2,76	0,71	0,69	1,20	1,20	1,38
10	3,3	23,2/3			x		0,99	1,03	0,99	0,98	1,04	1,03	1,01	1,27
11	3,0	25,9/6			x		0,98	1,26	1,18	1,02	1,13	1,11	1,11	1,40
13	3,18	38,7/15		-	x		0,80	0,95	2,38	0,44	0,83	0,86	1,03	1,28
14	3,36	44,3/20			x		0,74	0,93	1,21	0,55	0,77	0,71	0,82	1,20
15	2,94	47,3/15				x	0,31	0,29	0,29	0,33	0,33	0,31	0,31	0,45
16	2,94	- / 9				x	0,40	0,49	0,70	0,65	0,75	0,69	0,61	0,51
17	3,48	23,3/ 3				x	1,12	1,27	1,29	1,23	1,24	1,28	1,24	1,16 *
17A	3,12					x		-	-		-	-	-	0,58
12	3,12	30,6/ 9			x		0,63	1,12	1,10	0,83	0,98	1,18	0,93	1,59 *
12A		·			x									1,56
18	3,18	21 /-6				x	0,98	0,91	1,01	0,97	0,94	1,00	0,97	1,04

Table 1.

* Under influence of convection from bulb.

No. 17A: Same cond. as No. 17, but bulb switched off.

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air inlet temp. above room temp(°C) Fig. 10. Calculation of $\varepsilon_{\rm I}$ and $\varepsilon_{\rm II}$ from measurements as a function of ΔT . Diagonal scheme with two different locations of exhaust opening.

It seems that the transient ventilation efficiency is rather independent on whether the exhaust is located on the same or on the opposite side compared to the supply side. The efficiencies are greater than or equal to one, which corresponds with the predictions. For the lower air change rates the steadystate efficiencies decreases with increasing temp. difference. This is a little difficult to explain. One explanation could be that the W-levels for the lower temperature differences are partly located in the "high box", a situation which gradually changes toward a more complete "low box" level for the higher temperature differences. For the higher air change rates, the Wlevels are every one located in the "high box". The change in transient efficiency can be explained by a change in the relative size. of the "box" wolumes. In general the flow conditions are very complicated, where convection currents on the walls play an important but still unquantitied role.

It should be mentioned that all tests are run with a slot width that produces a momentum flux which generates appropriate air velocities for comfort reasons.

Fig. 11 and 12 show photos from smoke tests supplying smoke at the heat and tracer gas source. The photos are taken some time after start of supply, showing the shape of the circulation boxes. Note the shift in the ability of the source to penetrate into the high box when the temperature difference increases.



Fig. 11. "Diagonal scheme". $\Delta T = 3^{\circ}C$, n = 3. Plume penetrates into the "high box".



Fig. 12. "Diagonal scheme". $\Delta T = 15^{\circ}C$, n = 3. Plume does not penetrate into the "high box".

"Shortcircuiting scheme"

 ε_{I} and ε_{II} are shown in fig. 13. There is a marked drop in efficiencies compared to the "diagonal" scheme. When exhaust and supply is located on the same side, the drop does not occur until the temperature difference exceeds 5°C. This is because the contamination plume in this case, due to the direction of the recirculating currents enters more or less concentrated and stratified into the exhaust opening. This effect disappears when stratification increases, because the temperature of the plume is to low to allow the plume to enter the "high" box. During the decay period this, because the heat source is not switched off, in fact works as if exhaust take place from the low box, which in turn produces transient efficiency larger than 1.



Fig. 13. Calculation of ε_{I} and ε_{I} from measurements as a function of ΔT . "Shortcircuiting scheme" with two different locations of exhaust opening.

Fig. 14 and 15 show photos from smoke tests, which visualize the shape of the circulation boxes. In addition to confirming the applicability of the two box model, the tests using a shortcircuiting scheme demonstrate how sensible the behaviour of a ventilation system can be to the mutual lay-out of the location of supply and exhaust openings, and the location and strength of the heat sources. Note here too the same shift in flow patterns with increasing temperature difference as for fig. 11 and 12.



Fig. 14. "Shortcircuiting scheme". $\Delta T = 3^{\circ}C$, n = 3. Plume enters exhaust opening and accordingly the "high box".

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Fig. 15. "Shortcircuiting scheme". $\Delta T = 15^{\circ}C$, n = 3. Plume does not penetrate into the "high box".

If now, both heat and tracer gas supply are shut off, transient efficiency should drop. Fig. 16 shows a plot where this is done. As we can see the predicted drop actually take place. General comments



Tests with supply air temperature lower than the room air temperature (cooling) are not shown here. A few tests are run of which one is tabulated on table 1, test No. 18. The results are indicating that a shortcircuiting scheme tends to behave like a diagonal scheme and vice verse with high wall/ceiling air supply. Moving air supply to floor level, it is obvious that supplying air with a temperature lower than the room air temperature produces similar results as with high supply of heated air. However, we expect that efficiences for a diagonal scheme will be higher for all contaminations having a density lower than the room air. All contaminations having "normal" concentrations, will become lighter than the room air when heated only some tenth of l^OC.

Fig. 16. Results with the shortcircuiting scheme when the heat source is switched off.

CONCLUSIONS

Expressions for ventilation efficiency are derived using a two-box theoretic model. The definition of ventilation efficiency arrived at in this manner can be used for practical measurements, and seems to be valid also for multi-box schemes.

Different methods of measuring and defining ventilation efficiency give essential differences in the results. The ratio between λ_{1} and n, ε_{I} , gives a measure of the average speed at which a ventilating system delutes contaminations braught into a room compared to complete mixing delution rate.

The ratio between the concentration in the exhaust air and the working zone at steady-state, ε_{II} , gives a measure of the ability of the ventilating system to remove contaminations from the working zone. The latter efficiency is the most interesting one at normal conditions. ε_{TV} , the ratio between the concentra-

tion in the exhaust air and the working zone at transient conditions does not tell much about how well a ventilating system removes contaminations from the working zone. The transient methods do not consider the fact that the contaminations are usually supplied from single sources.

The tests show that the efficiencies varies considerably between different ventilating systems. One conclusion so far states that with the air supply located just beneath the ceiling and the air exhaust near the floor, we will obtain the best system for warm air ventilation (air heating). Then it should be obvious that a system with air supply near the floor and exhaust beneath the ceiling should be the best system for cool air ventilation (cooling). That is, "diagonal schemes" seems to be the most efficient, and more efficient than having complete mixing.

It should be mentioned that diagonal schemes with diffuse air supply (not applied for heating) in the working zone have successfully been used in industrial plants in Scandinavia the last few years.

In future research, we are going to explore more thoroughly "diagonal schemes" with low level air supply, both for small and large rooms. We will also carry out field measurements to make out how ventilation efficiencies are in existing buildings, and of course explore the usefulness of the described methods of measurement.

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